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Adaptive Fuzzy Sliding Mode Control of Underwater Robotic Vehicles

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Abstract: Sliding mode control is a very attractive control scheme because of its robustness against modelling imperfections and external disturbances. It has been successfully employed to the dynamic positioning of remotely operated underwater vehicles. In such situations, the discontinuities in the control law must be smoothed out to avoid the undesirable chattering effects. The adoption of properly designed boundary layers have proven effective in completely eliminating chattering, however, leading to an inferior tracking performance. This work describes the development of a dynamic positioning system for underwater robotic vehicles. The adopted approach is based on the sliding mode control strategy and enhanced by an adaptive fuzzy algorithm for uncertainty/disturbance compensation. Numerical results are presented in order to demonstrate the control system's performance.

Keywords: Adaptive Algorithms, Fuzzy Logic, Nonlinear Control, Sliding Modes, Underwater Robotic Vehicles.

INTRODUCTION

The control system is one of the most important elements of an underwater robotic vehicle, and its characteristics (advantages and disadvantages) play an essential role when one has to choose a vehicle for a specific mission. Unfortunately, the problem of designing accurate positioning systems for underwater robotic vehicles still challenges many engineers and researchers interested in this particular branch of engineering science. A growing number of papers dedicated to the position and orientation control of such vehicles confirms the necessity of the development of a controller, that could deal with the inherent nonlinear system dynamics, imprecise hydrodynamic coefficients, and external disturbances.

It has already been shown (Yuh, 1994; Goheen and Jefferys, 1990) that, in the case of underwater vehicles, the traditional control methodologies are not the most suitable choice and cannot guarantee the required tracking performance. On the other hand, sliding mode control, due to its robustness against parameter uncertainty and external disturbance, has proven to be a very attractive approach to cope with this problems (Bessa et al., 2006; Pisano and Usai, 2004; Guo et al., 2003; Kiriazov et al., 1997; Christi et al., 1990; Healey and Lienard, 1985; Yoerger and Slotine, 1985). But a well known drawback of conventional sliding mode controllers is the chattering effect. To overcome the undesired effects of the control chattering, Slotine (1984) proposed the adoption of a thin boundary layer neighboring the switching surface, by replacing the sign function by a saturation function. This substitution can minimize or, when desired, even completely eliminate chattering, but turns *perfect tracking* into a *tracking with guaranteed precision* problem, which in fact means that a steady-state error will always remain. In order to enhance the tracking performance inside the boundary layer, some adaptive strategy should be used for uncertainty/disturbance compensation.

Due to the possibility to express human experience in an algorithmic manner, fuzzy logic has been largely employed in the last decades to both control and identification of dynamical systems. In spite of the simplicity of this heuristic approach, in some situations a more rigorous mathematical treatment of the problem is required. Recently, much effort (Bessa, 2005; Liang and Su, 2003; Wong et al., 2001; Ha et al., 2001; Yu et al., 1998) has been made to combine fuzzy logic with sliding mode methodology.

An appealing option is to embed an adaptive fuzzy inference system inside the boundary layer of a sliding mode controller, to cope with the uncertainties and disturbances that can arise. This control strategy has already been successfully applied to the depth regulation of remotely operated underwater vehicles (Bessa et al., 2006). In this article, a rigorous demonstration of the stability and convergence properties of the closed-loop systems by means of Lyapunov stability theory and Barbalat's lemma was also presented.

In this work, the aforementioned control scheme is employed for full positioning system, *i.e.*, in the regulation of all controllable degrees of freedom. The adoption of a reduced order mathematical model for the underwater vehicle and the development of control system in a decentralized fashion, neglecting cross-coupling terms, is discussed. Numerical results are presented in order to demonstrate the control system performance.

VEHICLE DYNAMICS MODEL

A reasonable model to describe the underwater vehicle's dynamical behavior must include the rigid-body dynamics of the vehicle's body and a representation of the surrounding fluid dynamics. Such a model must be composed of a system of ordinary differential equations, to represent rigid-body dynamics, and partial differential equations to represent both tether and fluid dynamics.

In order to overcome the computational problem of solving a system with this degree of complexity, in the majority of publications (Bessa et al., 2006; Smallwood and Whitcomb, 2004; Hsu et al., 2000b; Kreuzer and Pinto, 1996; Yoerger and Slotine, 1985) a lumped-parameters approach is employed to approximate vehicle's dynamical behavior. In this way, the equations of motion for underwater vehicles are commonly presented, with respect to the body-fixed reference frame, in the following vectorial form:

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{k}(\boldsymbol{\nu}) + \mathbf{h}(\boldsymbol{\nu}) + \mathbf{g}(\mathbf{x}) + \mathbf{p} = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{\nu} = [v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]^{\mathrm{T}}$ is the vector of linear and angular velocities in the body-fixed reference frame, $\mathbf{x} = [x,\ y,\ z, \alpha, \beta, \gamma]^{\mathrm{T}}$ represents the position and orientation with respect to the inertial reference frame, $\mathbf{M}$ is the 6×6 inertia matrix, which accounts not only for the rigid-body inertia but also for the so-called hydrodynamic added inertia, $\mathbf{k}(\boldsymbol{\nu})$ is the vector of generalized Coriolis and centrifugal forces, $\mathbf{h}(\boldsymbol{\nu})$ represents the hydrodynamic quadratic damping, $\mathbf{g}(\mathbf{x})$ is the vector of generalized restoring forces (gravity and buoyancy), $\mathbf{p}$ stands for occasional current disturbances, and $\boldsymbol{\tau}$ is the vector of control forces and moments.

It must be noted that, in the particular case of remotely operated underwater vehicles (ROVs), the disturbances caused by the umbilical or tether cable should also be taken into account. The umbilical can be treated as a continuum, discretized with the finite element method or modeled as multibody system (Bevilacqua et al., 1991; Pinto, 1996). However, the adoption of any of these approaches requires a computational effort that would be prohibitive for on-line estimation of the control action. A common way to surmount this limitation is to consider the forces and moments exerted by the tether as random, and incorporate them into the vector $\mathbf{p}$.

The most relevant forces and moments acting on underwater vehicles are discussed in the following subsections.

Restoring forces

The forces and moments due to gravity ($\mathcal{M}g$) and buoyancy ($\rho g \nabla$), where $\mathcal{M}$ is the vehicle's mass, g is the acceleration of gravity, ρ is the water's density and ∇ is the displaced volume of water, can be expressed by the vector $\mathbf{g}(\mathbf{x}) \in \mathbb{R}^6$:

$$\mathbf{g}(\mathbf{x}) = \left[\begin{array}{c} \mathbf{R}^{\mathrm{T}}\ [0\ \ 0\ \ (\rho g \nabla - \mathcal{M}g)]^{\mathrm{T}} \\ \mathbf{r_f} \times \mathbf{R}^{\mathrm{T}}\ [0\ \ 0\ \ \rho g \nabla]^{\mathrm{T}} - \ \mathbf{r_g} \times \mathbf{R}^{\mathrm{T}}\ [0\ \ 0\ \ \mathcal{M}g]^{\mathrm{T}} \end{array} \right] \tag{2}$$

where $\mathbf{R} \in \mathbb{R}^{3\times 3}$ stands for the rotation matrix.

The vectors $\mathbf{r_g}$ and $\mathbf{r_f}$ represent the gravity and buoyancy center with respect to the body-fixed reference frame.

Hydrodynamic forces

Remotely operated underwater vehicles typically operate with velocities never exceeding 2 m/s. Consequently, the hydrodynamic forces (F_h) can be approximated using the *Morison equation* (Newman, 1986):

$$F_h = C_D \frac{1}{2} \rho A v |v| + C_M \rho \nabla \dot{v} + \rho \nabla \dot{v}_w \tag{3}$$

where v and $\dot{v}$ are, respectively, the relative velocity and the relative acceleration between rigid-body and fluid, $\dot{v}_w$ is the acceleration of underwater currents, A is a reference area, C_D and C_M are coefficients that must be experimentally obtained.

The last term of Eq. (3) is the so-called *Froude-Kryloff force* and will not be considered in this work due the fact, that at normal working depths, the acceleration of the underwater currents is negligible. In this way, the coefficient $C_M \rho \nabla$ of the second term will be called hydrodynamic added mass. The first term represents the nonlinear hydrodynamic quadratic damping. Experimental tests (Kleczka et al., 1992) show that Morison equation describes with sufficient accuracy the hydrodynamic effects due to the relative motion between rigid-bodies and water.

Quadratic Damping

The effects of the hydrodynamic damping $\mathbf{h}(\boldsymbol{\nu})$ over the vehicle, due not only to the translational but also to rotational motions, can be described in the body-fixed reference frame by (Pinto, 1996; Hsu et al., 2000a):

$$\mathbf{h}(\boldsymbol{\nu}) = \begin{bmatrix} \frac{1}{2}C_{D_x}(\boldsymbol{\upsilon})\rho\nabla^{\frac{2}{3}}|\boldsymbol{\upsilon}|^2 \\ \frac{1}{2}C_{D_y}(\boldsymbol{\upsilon})\rho\nabla^{\frac{2}{3}}|\boldsymbol{\upsilon}|^2 \\ \frac{1}{2}C_{D_z}(\boldsymbol{\upsilon})\rho\nabla^{\frac{2}{3}}|\boldsymbol{\upsilon}|^2 \\ \frac{1}{2}C_{D_{\alpha t}}(\boldsymbol{\upsilon})\rho\nabla|\boldsymbol{\upsilon}|^2 + \frac{1}{2}C_{D_{\alpha r}}\rho\nabla^{\frac{5}{3}}\omega_x|\omega_x| \\ \frac{1}{2}C_{D_{\beta t}}(\boldsymbol{\upsilon})\rho\nabla|\boldsymbol{\upsilon}|^2 + \frac{1}{2}C_{D_{\beta r}}\rho\nabla^{\frac{5}{3}}\omega_y|\omega_y| \\ \frac{1}{2}C_{D_{\gamma t}}(\boldsymbol{\upsilon})\rho\nabla|\boldsymbol{\upsilon}|^2 + \frac{1}{2}C_{D_{\gamma r}}\rho\nabla^{\frac{5}{3}}\omega_z|\omega_z| \end{bmatrix} \tag{4}$$

where $\boldsymbol{\upsilon} = [\upsilon_x, \upsilon_y, \upsilon_z]^{\mathrm{T}}$ is a vector containing only the first three components of $\boldsymbol{\nu}$, *i.e.*, the translational velocities.

The parameters C_{D_x}, C_{D_y}, C_{D_z}, $C_{D_{\alpha t}}$, $C_{D_{\beta t}}$, $C_{D_{\gamma t}}$, $C_{D_{\alpha r}}$, $C_{D_{\beta r}}$ and $C_{D_{\gamma r}}$ depend on the geometry of the vehicle and should be experimentally obtained in a wind tunnel (Pinto, 1996), or on-line estimated with adaptive algorithms in a water tank (Smallwood and Whitcomb, 2003).

Added inertia

Considering that an underwater vehicle typically operates at low speeds, the added inertia matrix, $\mathbf{M}_A \in \mathbb{R}^{6\times 6}$, could be assumed as diagonally dominant and described as follows:

$$\mathbf{M}_A = \mathrm{diag}\left\{C_{M_x}\rho\nabla\ ,\ C_{M_y}\rho\nabla\ ,\ C_{M_z}\rho\nabla\ ,\ C_{M_\alpha}\rho\nabla\ ,\ C_{M_\beta}\rho\nabla\ ,\ C_{M_\gamma}\rho\nabla\right\} \tag{5}$$

As with the computation of the hydrodynamic damping, the coefficients C_{M_x}, C_{M_y}, C_{M_z}, C_{M_α}, C_{M_β} and C_{M_γ} should be experimentally determined. The matrix $\mathbf{M}_A$ must be combined with the rigid-body inertia matrix in order to obtain the matrix $\mathbf{M}$ of Eq. (1).

Thruster forces

Underwater vehicles are commonly equipped with electrically actuated bladed thrusters. As previously addressed in many works (Bessa et al., 2005; Whitcomb and Yoerger, 1999; Healey et al., 1995; Yoerger et al., 1990), at low speeds, the dynamic behavior of the vehicle can be greatly influenced by the nonlinear dynamics of the thruster system.

Regarding the dynamic behavior of the thruster, the following first order nonlinear model, with propeller's angular velocity (Ω) as state variable, can be adopted (Bessa et al., 2005; Yoerger et al., 1990):

$$J_{msp}\dot{\Omega} + k_v\Omega|\Omega| = \frac{k_t}{R_m}V_m \tag{6}$$

$$F_p = C_T\,\Omega|\Omega| \tag{7}$$

where J_{msp} is the motor-shaft-propeller inertia, V_m is the motor input voltage, F_p is the resulting thruster force and C_T is a function of the advance ratio. The constants k_t and R_m, which represents the motor torque constant and winding resistance, respectively, can be obtained from motor's data-sheet. The values of k_v and C_T depends on constructive characteristics of each thruster and must be experimentally determined.

Furthermore, the effect on the vehicle of the force produced by everyone of the N thrusters can be described in body-fixed reference frame by

$$\boldsymbol{\tau} = \mathbf{B}\mathbf{F_p} \tag{8}$$

where $\mathbf{F_p} \in \mathbb{R}^N$ is a vector containing the force produced by each thruster and $\mathbf{B} \in \mathbb{R}^{6\times N}$ is a matrix which represents the distribution of the thrust forces on the vehicle.

DYNAMIC POSITIONING SYSTEM

The dynamic positioning of underwater robotic vehicles is essentially a multivariable control problem. Nevertheless, as demonstrated by Slotine (1983), the variable structure control methodology allows different controllers to be separately designed for each degree of freedom (DOF). Over the past decades, decentralized control strategies have been successfully applied to the dynamic positioning of underwater vehicles (Smallwood and Whitcomb, 2004; Da Cunha et al., 1995; Kiriazov et al., 1997; Yoerger and Slotine, 1985).

Considering that the control law for each degree of freedom can be easily designed with respect to the inertial reference frame, Eq. (1) should be rewritten in this coordinate system.

Remembering that

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{x})\boldsymbol{\nu} \tag{9}$$

where $\mathbf{J}(\mathbf{x})$ is the Jacobian transformation matrix, it can be directly implied that

$$\boldsymbol{\nu} = \mathbf{J}^{-1}(\mathbf{x})\dot{\mathbf{x}} \tag{10}$$

and

$$\dot{\boldsymbol{\nu}} = \dot{\mathbf{J}}^{-1}\dot{\mathbf{x}} + \mathbf{J}^{-1}\ddot{\mathbf{x}} \tag{11}$$

Therefore, the equations of motion of an underwater vehicle, with respect to the inertial reference frame, becomes

$$\bar{\mathbf{M}}\ddot{\mathbf{x}} + \bar{\mathbf{k}} + \bar{\mathbf{h}} + \bar{\mathbf{g}} + \bar{\mathbf{p}} = \bar{\boldsymbol{\tau}} \tag{12}$$

where $\bar{\mathbf{M}} = \mathbf{J}^{-\mathrm{T}}\mathbf{M}\,\mathbf{J}^{-1}$, $\bar{\mathbf{k}} = \mathbf{J}^{-\mathrm{T}}\mathbf{k} + \mathbf{J}^{-\mathrm{T}}\mathbf{M}\,\dot{\mathbf{J}}^{-1}\dot{\mathbf{x}}$, $\bar{\mathbf{h}} = \mathbf{J}^{-\mathrm{T}}\mathbf{h}$, $\bar{\mathbf{g}} = \mathbf{J}^{-\mathrm{T}}\mathbf{g}$, $\bar{\mathbf{p}} = \mathbf{J}^{-\mathrm{T}}\mathbf{p}$ and $\bar{\boldsymbol{\tau}} = \mathbf{J}^{-\mathrm{T}}\boldsymbol{\tau}$.

It should be noted that in the case of remotely operated underwater vehicles (ROVs), the metacentric height is sufficiently large to provide the self-stabilization of roll (α) and pitch (β) angles. This particular constructive aspect also allows the order of the dynamic model to be reduced to four degrees of freedom, $\mathbf{x} = [x,\ y,\ z, \gamma]^{\mathrm{T}}$, and the vertical motion (heave) to be decoupled from the motion in the horizontal plane. This simplification can be found in the majority of works presented in the specialized literature (Zanoli and Conte, 2003; Guo et al., 2003; Hsu et al., 2000b; Kiriazov et al., 1997; Pinto, 1996; Da Cunha et al., 1995; Yoerger and Slotine, 1985).

Thus, the positioning system of a ROV can be divided in two different parts: Depth control (concerning variable z), and control in the horizontal plane (variables x, y and γ).

Considering the aforementioned assumptions and using the Euler angles parameterization, the Jacobian transformation matrix associated to the horizontal plane becomes

$$\mathbf{J}(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

where $\mathbf{J}(\gamma)$ is orthogonal, and consequently $\mathbf{J}^{-1}(\gamma) = \mathbf{J}^{\mathrm{T}}(\gamma)$.

Adaptive fuzzy sliding mode controller

Let $S_i(t)$ be a sliding surface defined in the state space of each degree of freedom (x_i) by the equation $s_i(t, x_i, \dot{x}_i) = 0$, with $s_i : \mathbb{R}^2 \to \mathbb{R}$ satisfying

$$s_i(t, x_i, \dot{x}_i) = \dot{\tilde{x}}_i + \lambda_i \tilde{x}_i \tag{14}$$

where $\tilde{x}_i = x_i - x_{di}$ is the tracking error associated to each DOF, $\dot{\tilde{x}}_i$ is the time derivative of $\tilde{x}_i$, x_{di} is the correspondent desired trajectory and λ_i are strictly positive constants.

So, given the main characteristics of the system to be controlled and assuming that the restoring forces can be previously compensated (Kiriazov et al., 1997), adaptive fuzzy sliding mode controllers (AFSMC) are proposed to regulate each degree of freedom of the underwater robotic vehicle:

$$\bar{\tau}_i = \hat{\bar{k}}_i + \hat{\bar{h}}_i + \hat{\bar{m}}_i\left(\ddot{x}_{di} - \lambda_i\dot{\tilde{x}}_i\right) + \hat{d}_i(s_i) - K_i \mathrm{sat}\left(\frac{s_i}{\phi_i}\right)\,, \qquad i = 1, 2, 3, 4 \tag{15}$$

where $\mathrm{sat}(\cdot)$ is the saturation function, ϕ_i are strictly positive constants, $\bar{\tau}_i$ represents the components of vector $\bar{\boldsymbol{\tau}}$, $\hat{\bar{k}}_i$ and $\hat{\bar{h}}_i$ are, respectively, the components of vectors $\hat{\bar{\mathbf{k}}}$ and $\hat{\bar{\mathbf{h}}}$, which stands for estimates of vectors $\bar{\mathbf{k}}$ and $\bar{\mathbf{h}}$.

Concerning $\hat{\bar{m}}_i$, it represents in the depth controller the mass of the vehicle plus the respective added mass. In the horizontal plane, estimates of the main diagonal terms of $\mathbf{J}^{-\mathrm{T}}\mathbf{M}\,\mathbf{J}^{-1}$, with $\mathbf{J}$ defined according to Eq. (13), may be attributed to the correspondent $\hat{\bar{m}}_i$. To ensure the stability of the closed-loop system, estimates of the off-diagonal terms of $\mathbf{J}^{-\mathrm{T}}\mathbf{M}\,\mathbf{J}^{-1}$ should be incorporated in the vector $\bar{\mathbf{p}}$, as will be discussed further in the paper.

It should be emphasized that the lumped parameters approach, adopted to describe the hydrodynamic effects (quadratic damping and added inertia), represents a simplification, and hence only estimates of the actual phenomena are available. Due to the presence of the term $\mathbf{J}^{-\mathrm{T}}\mathbf{M}\,\dot{\mathbf{J}}^{-1}\dot{\mathbf{x}}$, the vector $\bar{\mathbf{k}}$ cannot be exactly known.

The gain K_i of each controller should be carefully determined in order to ensure the global stability of the closed-loop system, and robustness with respect to disturbances and uncertainties. According to the sliding mode methodology, K_i must be defined as follows (Bessa et al., 2006; Bessa, 2005):

$$K_i \geq \mathcal{P}_i + \hat{\bar{m}}_i \mathcal{G}_i \eta_i + |\hat{d}_i(s_i)| + \hat{\bar{m}}_i(\mathcal{G}_i - 1)|\ddot{x}_{di} - \lambda_i \dot{\tilde{x}}_i| \tag{16}$$

where η_i are strictly positive constants related to the reaching time of each controller.

Defining $\hat{\bar{m}}_i = \sqrt{\bar{m}_{\max}\bar{m}_{\min}}$ and $\mathcal{G}_i = \sqrt{\bar{m}_{\max}/\bar{m}_{\min}}$ automatically implies that

$$\mathcal{G}_i^{-1} \leq \frac{\hat{\bar{m}}_i}{\bar{m}_{\ i}} \leq \mathcal{G}_i \tag{17}$$

Regarding $\mathcal{P}_i$, this term should be defined for each controller in order to compensate the uncertainties of the respective components of vectors $\bar{\mathbf{k}}$ and $\bar{\mathbf{h}}$, and perturbations provided by $\bar{\mathbf{p}}$, *i.e.*,

$$\left|\Delta \bar{k}_i + \Delta \bar{h}_i + \bar{p}_i\right| \leq \mathcal{P}_i \tag{18}$$

Returning to the control law, Eq. (15), the adoption of a saturation function, sat$(\cdot)$, instead of the well-known sign function, sgn$(\cdot)$, leads to the formation of a thin boundary layer neighboring each switching surface $S_i(t)$. The incorporation of this boundary layer can minimize or, when desired, even completely eliminate chattering, but turns *perfect tracking* into a *tracking with guaranteed precision* problem, leading to an inferior tracking performance.

In order to enhance the tracking performance, in this work, an adaptive fuzzy inference system is embedded inside the boundary layer, to cope with the uncertainties and disturbances that can arise.

The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang), whose rules can be stated in a linguistic manner as follows:

If s *is* S_r *then* $\hat{d}_r = \hat{D}_r$; $r = 1, 2, \cdots, R$

where S_r are fuzzy sets, whose membership functions could be properly chosen, and $\hat{D}_r$ is the output value of each one of the R fuzzy rules.

Considering that each rule defines a numerical value as output $\hat{D}_r$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{d}(s) = \frac{\sum_{r=1}^{R} w_r \cdot \hat{d}_r}{\sum_{r=1}^{R} w_r} \tag{19}$$

or, similarly, but now for every degree of freedom,

$$\hat{d}_i(s) = \hat{\mathbf{D}}_i^{\mathrm{T}} \mathbf{\Psi}_i(s_i) \tag{20}$$

where, $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, \ldots, \hat{D}_N]^{\mathrm{T}}$ is the vector containing the attributed values $\hat{D}_r$ to each rule r, $\mathbf{\Psi}(s) = [\psi_1(s), \psi_2(s), \ldots, \psi_N(s)]^{\mathrm{T}}$ is a vector with components $\psi_r(s) = w_r / \sum_{r=1}^{N} w_r$ and w_r is the firing strength of each rule.

In order to obtain the most suitable values for $\hat{d}_i(s)$, the vectors of adjustable parameters will be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{D}}}_i = -\varphi_i\, s_i\, \mathbf{\Psi}_i(s_i) \tag{21}$$

where φ_i are strictly positive constants related to the adaptation rate.

For a more detailed discussion about the stability and convergence properties of the proposed control law, the reader is referred to Bessa (2005) and Bessa et al. (2006).

Now, given the required control force $\bar{\boldsymbol{\tau}}$ from Eq. (15) and the thruster's arrangement on the vehicle, the force that should be produced by every thruster can be determined.

Nowadays, most ROVs have the standard thruster configuration in the horizontal plane, with four thrusters in a square arrangement and angled at 45º, which provides a better actuation over x, y and γ. So, from Eq. (8), it can be easily verified that

$$\mathbf{F}_{p_{xy}} = \mathbf{B}^{\mathrm{T}}(\mathbf{B}\mathbf{B}^{\mathrm{T}})^{-1}\mathbf{J}(\gamma)^{\mathrm{T}}\bar{\boldsymbol{\tau}} \tag{22}$$

where $\bar{\boldsymbol{\tau}} = [\bar{\tau}_x, \bar{\tau}_y, \bar{\tau}_\gamma]^{\mathrm{T}}$ and $\mathbf{B}^{\mathrm{T}}(\mathbf{B}\mathbf{B}^{\mathrm{T}})^{-1} \in \mathbb{R}^{4\times 3}$ is the pseudo-inverse of matrix $\mathbf{B} \in \mathbb{R}^{3\times 4}$.

Regarding the vertical motion, the required thrust force can be directly obtained by $F_{p_z} = \bar{\tau}_z / N_z$, where N_z is the available number of thrusters in the vertical direction.

SIMULATION RESULTS

The numerical simulations were performed with an implementation in C, with sampling rates of 500 Hz for control system and 1 kHz for dynamic model. The differential equations of the dynamic model were numerically solved with the fourth order Runge-Kutta method.

In order to simplify the design process, some parameters of the controller were chosen identical for all degrees of freedom, $\lambda_i = 0.6$, $\phi_i = 0.05$ and $\varphi_i = 1 \times 10^3$. Concerning the fuzzy system, the same triangular and trapezoidal membership functions, with the central values defined as $C_i = \{-3\ ;\ -1\ ;\ -0{,}5\ ;\ 0\ ;\ 0{,}5\ ;\ 1\ ;\ 3\}$, were adopted for each DOF. The vectors of adjustable parameters were initialized to zero, $\hat{\mathbf{D}}_i = \mathbf{0}$, and automatically updated according to Eq. (21). For the dynamic model, the following values were adopted: $\mathbf{M} = \mathrm{diag}\,\{80\ \mathrm{kg},\ 80\ \mathrm{kg},\ 100\ \mathrm{kg},\ 8\ \mathrm{kg\,m}^2\}$ and $\mathbf{h} = [125\,\upsilon_x|\upsilon_x|,\ 175\,\upsilon_y|\upsilon_y|,\ 250\,\upsilon_z|\upsilon_z|,\ 12{,}5\,\omega_z|\omega_z|]^{\mathrm{T}}$. The disturbance force was chosen to vary randomly in the range of ± 3 N. The random nature of the disturbance was simulated using the functions `rand()` and `srand()` of the C Standard Library. For controller design, the vehicle's parameters were chosen based on the assumption that exact values are not known, but with a maximal uncertainty of $\pm 25\%$.

To evaluate the control system performance, two different numerical simulations were performed. In the first case, the underwater robotic vehicle was intended to move only in the XY plane, from his initial position/orientation at rest, $\mathbf{x}_0 = [0, 0, 0, 0]^{\mathrm{T}}$, to the desired final position/orientation $\mathbf{x_d} = [2.5\ ,\ 2\ ,\ 0\ ,\ \pi/2]^{\mathrm{T}}$. Once this final position/orientation is reached, it should stay there indefinitely, besides the disturbance forces. The obtained results were presented in Fig. 1 and Fig. 2.

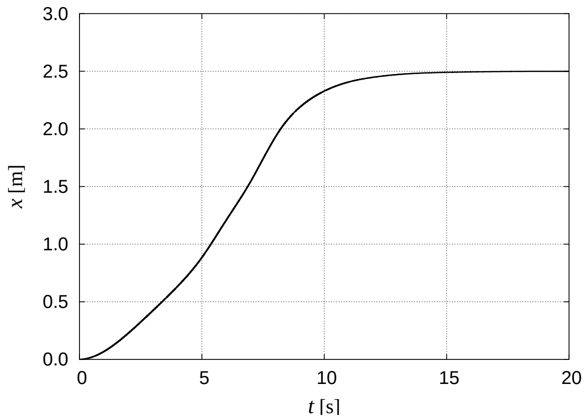
Figure 1 shows the obtained response in the time domain. These results confirm that the proposed control strategy was able to regulate and stabilize the dynamical behavior of the underwater vehicle in the horizontal plane. As observed in Fig. 1(b), Fig. 1(d) and Fig. 1(f), the adaptive fuzzy sliding mode controller was also efficient in minimizing the undesirable chattering effect.

The propeller's angular velocity, associated to the control problem in the horizontal plane, are presented in Fig. 2. It should be noted that the maximal angular velocity, ≈ 420 rad/s, is related to the maximal voltage admitted by the thruster's DC motors.

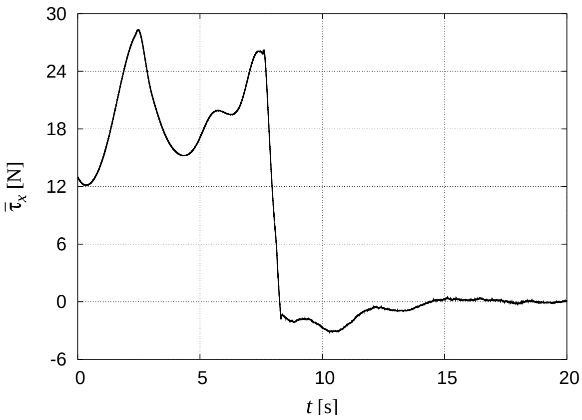
Finally, the second case was a trajectory tracking in $\mathbb{R}^3$. Here, from the initial position $\mathbf{x}_0 = [0, 0, 0, 0]^{\mathrm{T}}$ at rest, the vehicle was forced to move to the following desired positions: $\mathbf{x}_1 = [0, 3, 3, 0]^{\mathrm{T}}$, $\mathbf{x}_2 = [3, 3, 3, 0]^{\mathrm{T}}$, $\mathbf{x}_3 = [3, 3, 0, 0]^{\mathrm{T}}$, $\mathbf{x}_4 = [1, 3, 0, 0]^{\mathrm{T}}$ and $\mathbf{x}_5 = [1, 1, 0, 0]^{\mathrm{T}}$, where $t_0 = 0$ s, $t_1 = 30$ s, $t_2 = 60$ s, $t_3 = 90$ s, $t_4 = 120$ s, $t_5 = 150$ s. During the entire path, the yaw angle should be kept constant, $\gamma = 0$. The obtained results were presented in Fig. 3 and Fig. 4. By observing both figures, it can be verified that, with the proposed control system, the vehicle could follow the desired trajectory, in spite of the disturbance forces. It can be also observed, Fig. (4(d)), that the yaw angle (γ) was held within the acceptable bounds, defined by the chosen width of the boundary layer, $\phi_\gamma = 0.05$.

CONCLUDING REMARKS

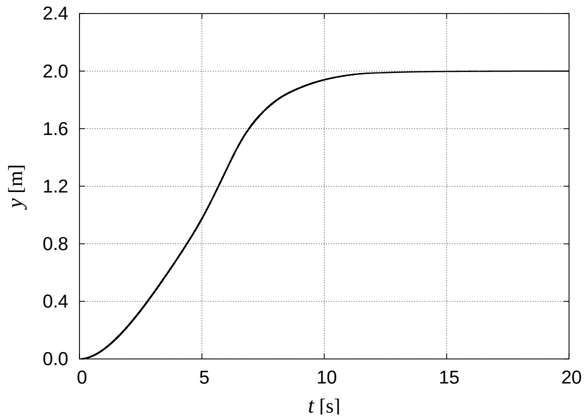
In this paper, the problem of compensating uncertainty/disturbance in the dynamic positioning system of underwater robotic vehicles was considered. An adaptive fuzzy sliding mode controller was implemented to deal with the stabilization and trajectory tracking problems. The adoption of a reduced order mathematical model for the underwater vehicle and the development of a control system in a decentralized fashion, neglecting cross-coupling terms, was discussed. By means of numerical simulations, it could be verified that the proposed strategy was able to cope with both the disturbances, that can typically arise in the subaquatic environment, and uncertainties in hydrodynamics coefficients. As observed, the incorporation of an adaptive fuzzy algorithm within the boundary layer made the better trade-off between tracking performance and chattering possible.



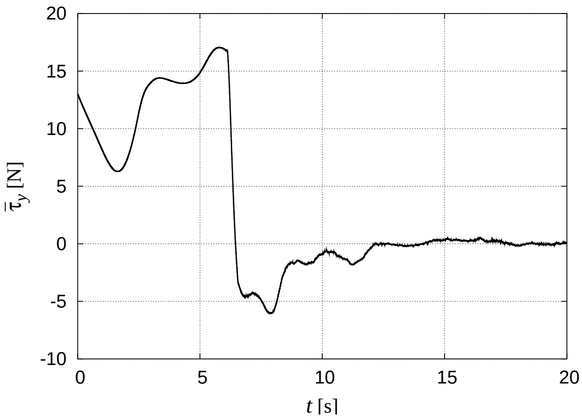
(a) State variable x.



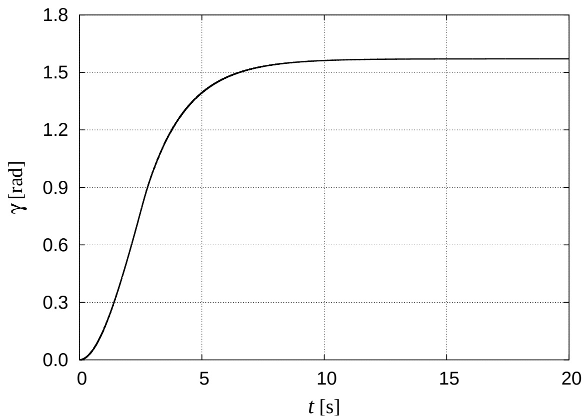
(b) Thruster force $\bar{\tau}_x$.



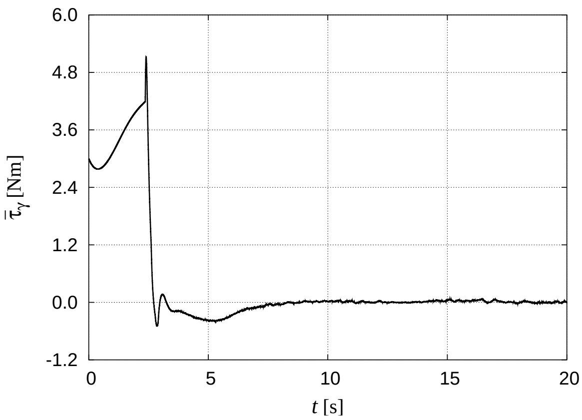
(c) State variable y.



(d) Thruster force $\bar{\tau}_y$.

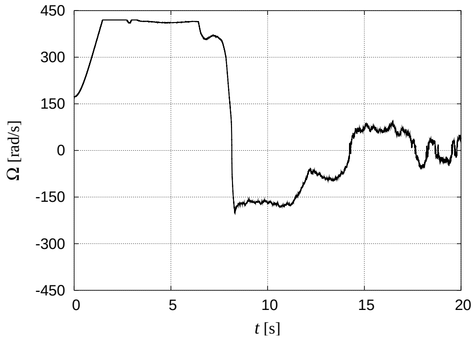


(e) State variable γ.

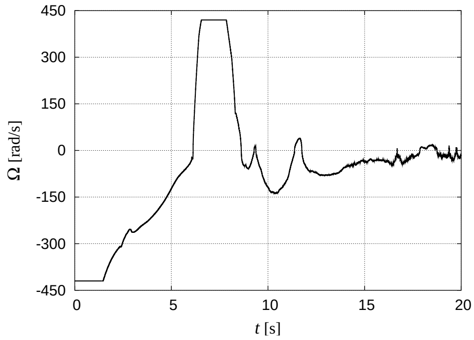


(f) Thruster force $\bar{\tau}_\gamma$.

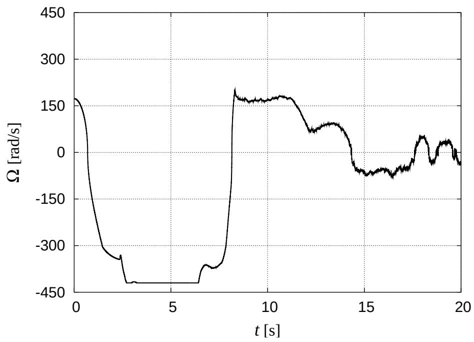
Figure 1 – Dynamic positioning of the vehicle in the horizontal plane.



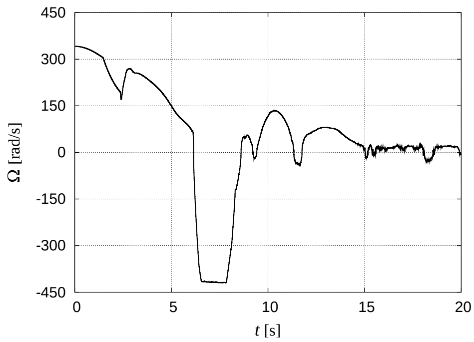
(a) Thruster 1.



(b) Thruster 2.



(c) Thruster 3.



(d) Thruster 4.

Figure 2 – Propeller's angular velocity, related to the dynamic positioning in the horizontal plane.

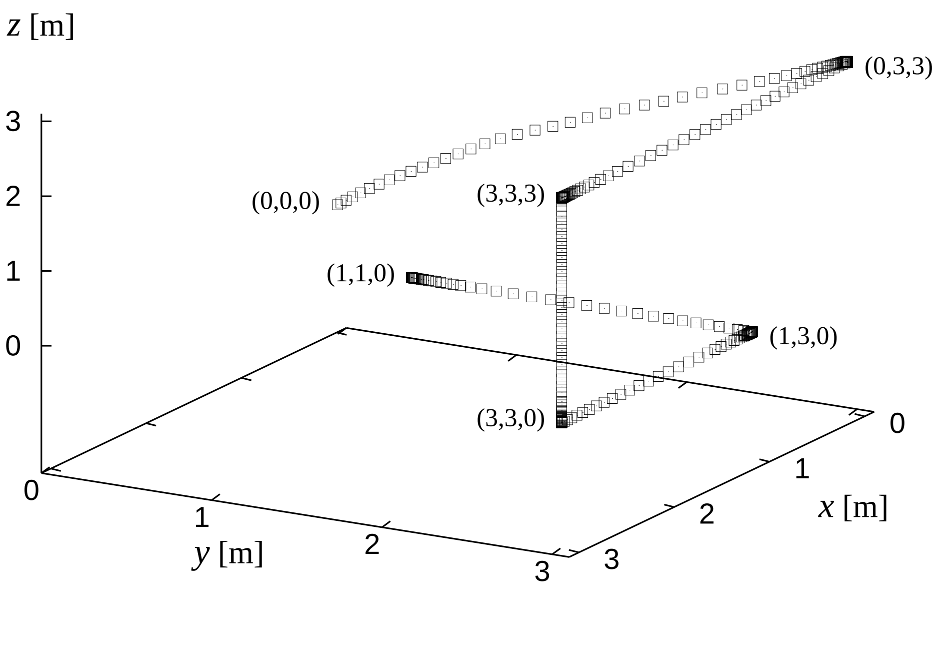
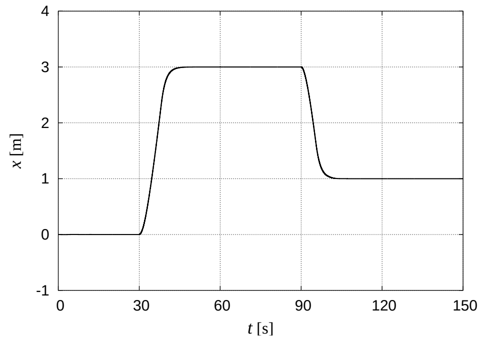
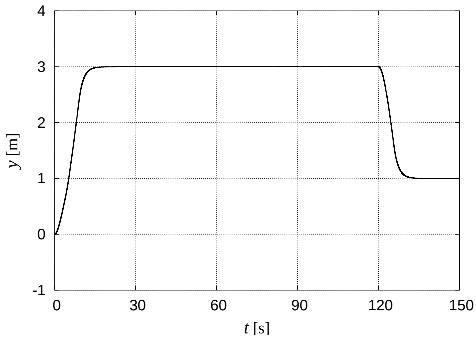


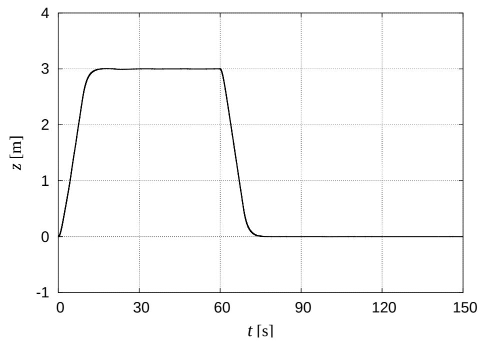
Figure 3 – Dynamic positioning of the vehicle in $\mathbb{R}^3$



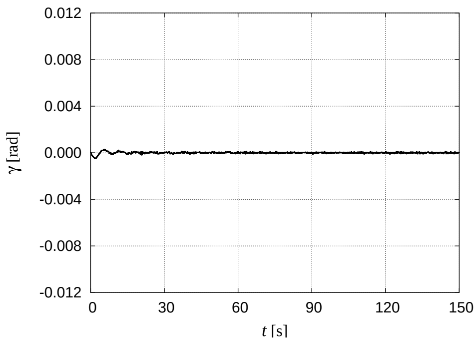
(a) State variable x.



(b) State variable y.



(c) State variable z.



(d) State variable γ.

Figure 4 – State variables in the time domain, associated to the dynamic positioning in $\mathbb{R}^3$

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