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On Non-Ideal and Nonlinear Vibrations of a Shaft Carrying a Disk through Main Resonance

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Abstract: The transverse Non-ideal and nonlinear vibrations of a shaft carryinga an unbalanced disk are analyzed. We suppose that only one natural frequency is related to the two coordinates of the vibrating system (due to the symmetry of the shaft section), so that, with the equation governing the interaction with the energy source(non-ideal), there will be three differential equations of motion for the considered dynamical system. Good agreements between both numerical and analytical solutions were observed.

Keywords: non-ideal motors, non-linear dynamics, Sommerfeld effect

NOMENCLATURE

x = displacement of shaft, m
y = displacement of shaft, m
m = mass of disc, kg
I = moment of inertia, kgm^2

φ = rotational angle, rad.
ρ = eccentricity, m
ω = natural frequency, rad/s

Greek Symbols

INTRODUCTION

We may summarize the research on nonstationary rotor vibration problems into three groups: the first one studies nonstationary vibrations of linear models of the rotor passing through resonance with constant acceleration; the second one studies nonstationary vibrations of nonlinear models and the third one studies phenomena in systems which have mutual interaction between the driving source and the rotor motion (Non-Ideal Systems).

Here, we deal with a Non-ideal system kind of vibration problem.

We remark that when a dynamic system is driven by a power source such as a motor of limited power (Non-ideal), we may have interaction between the motor output and the system response. This interaction manifests itself as a modification of the motor frequency or regime of operation near the resonance and changes in the stable-unstable portions of the dynamical system response. Since most of real motors are of limited power (non-ideal), the results here obtained render descriptions which are closer to the real situations encountered in practice and it has been considered a major challenge in theoretical and practical engineering research. Near resonance, an increase in power will usually be accompanied by an increase in oscillations amplitude without significant increase in frequency. Only after the maximum amplitude of oscillations has been reached will there be a significant alteration in the frequency.

Sommerfeld observed such relationships between the alteration of frequency, amplitude of oscillations and motive power. Usually, this dynamic process is called Sommerfeld effect. He suggested that the structural non-ideal response or non-ideal vibrations provide an “energy sink”. Thus, we spend energy to vibrate our structure rather than operate the machinery. One of the problems often faced by designers is how to drive a system through resonance and avoid the “energy sink” described by Sommerfeld (1904). This kind of problem was described in the classical book of (Kononenko, 1969). Recently, complete reviews of this kind of vibrations can be found in (Balthazar et al., 2003, 2004). Here, we will clearly illustrate this kind of Non-Ideal problem taking into account the nonlinear vibrations of non-ideal shaft carrying an unbalanced disc, described before by (Kononenko, 1969), presenting some new results.

MATHEMATICAL MODEL

We will clearly illustrate this kind of Non-Ideal problem considering a mathematical model of a non-ideal shaft carrying an unbalanced disc, shown by Fig. 1 (Kononenko, 1969). This model can be applied to real turbine rotors.

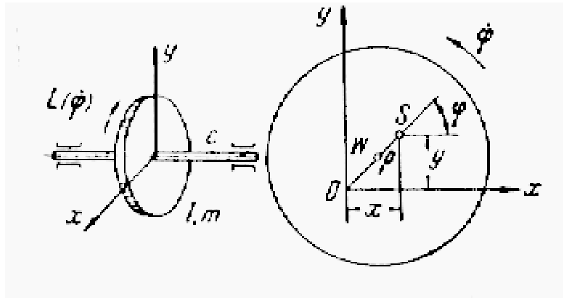

Figure 1 - A Non-Ideal Shaft Carrying a Disc (Kononenko, 1969)

Here, the mathematical model consists of a weightless elastic beam resting on two supports and having a disc of mass m and moment of inertia I rigidly fixed to its mid-point. The center of mass m of the disk S is displaced by the eccentricity ρ relative to the point w where the disc is fixed to the shaft. The shaft has a bending stiffness c, which is the same for all radial directions. We consider that the system studied here is excited by a DC motor of limited power supply. The torque is applied directly to the rotor. Torsion vibrations are neglected. The coordinates of the system are the displacements x and y of the center of mass of the disc relative to a fixed system with origin at point O coinciding with the axis of the undeformed shaft, and the angle φ turned through by the disc. We also consider: W fixing point, S mass centre of deformed shaft; O mass centre of undeformed shaft and s the distance from O to S.

Next, we will discuss the derivation of the governing equations of motion. We will neglect the gravitational force because it does not give an important contribution on the whirling of the rotor in the region of main critical speed.

Equations of Motion

The kinetic energy T and the potential energy are

$$T = \frac{1}{2}(I + m\rho^2)\dot{\varphi}^2 + \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

$$V = \frac{1}{2}cr^2 + mgs\sin\varphi + \frac{1}{2}c[(x-\rho\cos\varphi)^2 + (y-\rho\sin\varphi)^2] + mgs\sin\varphi$$

There are two kind of damping forces: *External,* proportional to the velocity of the centre of the disc S $\chi\dot{x}$ and $\chi\dot{y}$; *Internal*: assumed to be proportional to the velocity of the bending deformation of the shaft $k(\dot{x}+\dot{\varphi}y)$ and $k(\dot{y}+\dot{\varphi}x)$. We also consider the torque of the motor $L(\dot{\varphi})$; the torque of the force resisting the rotational motion $q\dot{\varphi}$ and the torque of the elastic force and the force of friction about the point S.

The problem is to study the motion of non-linear system and in particular the interaction of the coordinates x e y with φ. Then the Lagrange's equations of motion are obtained as (Kononenko**,** 1969)

$$\begin{aligned} m\ddot{x} + cx &= c\rho^*\cos\varphi - (k^* + \chi^*)\dot{x} - k^*\dot{\varphi}y \\ m\ddot{y} + cy &= c\rho^*\sin\varphi - (k^* + \chi^*)\dot{y} - k^*\dot{\varphi}x \end{aligned} \qquad (1)$$

$$I\ddot{\varphi} = L^*(\dot{\varphi}) - c\rho^*(x\sin\varphi - y\cos\varphi) - k^*(\dot{x}y + \dot{y}x) - k^*\dot{\varphi}(x^2 + y^2)$$

The forces caused by unbalance of the rotor as well as damping forces are small. The angular velocity φ does not vary very much. Hence all terms on the right-hand side of equation of motion are small, that is,

$$\rho^* = \varepsilon\rho\,,\ k^* = \varepsilon k\,,\ \chi^* = \varepsilon\chi\,,\ L^*(\dot{\varphi}) = \varepsilon(L(\dot{\varphi}) - q\dot{\varphi})$$

where ε is a small parameter that is a measure of the amplitude of vibration. It is used as a bookkeeping device and set equal to unity if the amplitudes are taken to be small (Nayfeh, 1981). Hence all terms on the right-hand side of equation of motion are small, and for the $\varepsilon = 0$ the system decomposes into three independent equations with natural frequency $\omega = \sqrt{c/m}$:

$$\dot{\varphi} = \Omega,\ \ddot{x} + \omega^2 x = 0\,,\ \ddot{y} + \omega^2 y = 0$$

Note that this above expression can be used as the basic single frequency motion. Then, we may write it in the compact form: $z = C \cos\omega t + D \sin\omega t$, using the classical substitution $z = x + iy$. Thus, taking the expressions $z = A\cos\varphi + B\sin\varphi$ and $\dot{z} = -\omega A\sin\varphi + B\omega\cos\varphi$ as the form for the solution of the Eq. (1) for the x and

y coordinates with the complex quantities A, B are determined as unknown functions of time with a given degree of accuracy.

According to (Kononenko, 1969) we express the variables $\{x, \dot{x}, y, \dot{y}\}$ in terms of A e B and their complex conjugates $\bar{A}$ and $\bar{B}$. Moreover, we introduce the substitution $\frac{d\varphi}{dt} = \Theta$ and confine ourselves to the *region of the fundamental resonance*, assuming that $\omega - \Theta = \varepsilon\alpha_0$. After transformation by these substitutions, Eq. (1) become

$$\frac{dA}{dt} = f_1, \quad \frac{dB}{dt} = f_2, \quad \frac{d\Theta}{dt} = f_3 \tag{2}$$

where

$$f_1 = \varepsilon \alpha_0 B - \frac{\varepsilon}{m\omega}[(k+\chi)(A \omega \sin\varphi - B \omega \cos\varphi) + i k \Theta (A \cos\varphi + B \sin\varphi) + c \rho e^{i\varphi}]\sin\varphi,$$

$$f_2 = -\varepsilon \alpha_0 A + \frac{\varepsilon}{m \omega}[(k+\chi)(A \omega \sin\varphi - B \omega \cos\varphi) + i k \Theta(A \cos\varphi + B \sin\varphi) + c \rho e^{i\varphi}]\cos\varphi ,$$

$$f_3 = \frac{\varepsilon}{I}\{M(\Theta) - \frac{i}{2} k \omega(\bar{A} B - A \bar{B}) - k \Theta[A \bar{A} \cos^2\varphi + B \bar{B} \sin^2\varphi + (B \bar{A} + A\bar{B}) + -\frac{c \rho}{4}[i(A-\bar{A}) + (B+\bar{B}) + $$
$$+ i(A + i B) e^{-2i\varphi} - i(\bar{A} - i \bar{B}) e^{2i\varphi}]\}$$

$\varepsilon\alpha = \Delta \quad \Delta = \omega - \Omega$.

Next we will obtain an analytical solution to Eq. (2).

APPROXIMATE ANALYTICAL SOLUTION NEAR THE MAIN RESONANCE REGION, BY USING A BOGOLIUBOV AVERAGING METHOD

We seek an approximate solution of Eq. (2), in the following form:

$$A = A_1 + \varepsilon U_1(t, A_1, B_1, \Omega); \; B = B_1 + \varepsilon U_2(t, A_1, B_1, \Omega); \; \Theta = B \Omega + \varepsilon U_3(t, A_1, B_1, \Omega) \tag{3}$$

where the quantities A_1, B_1, Ω are determined from the first approximation equations, obtained by averaging the right- hand sides of Eq. (2) for φ. this is equivalent to averaging for t, since for this step $\varphi = \Omega t$ where Ω is assumed to be constant over the period of vibration (Nayfeh, 1981). As result of the averaging process (Kononenko, 1969) we get:

$$\frac{dA_1}{dt} = -\varepsilon \alpha B_1 - \frac{\varepsilon}{2 m \omega}[(k+\chi) \omega A_1 + i k \Omega B_1 + i c \rho],$$

$$\frac{dB_1}{dt} = -\varepsilon \alpha A_1 - \frac{\varepsilon}{2 m \omega}[(k+\chi) B_1 - i k \Omega A_1 - c \rho], \tag{4}$$

$$\frac{d\Omega}{dt} = \frac{\varepsilon}{I}\{M(\Omega) + \frac{i}{2} k \omega (A_1 \bar{B}_1 - \bar{A}_1 B_1) - \frac{k \Omega}{2}(A_1 \bar{A}_1 + B_1 \bar{B}_1)$$
$$- \frac{c \rho}{4}[i (A_1 - \bar{A}_1) + (B_1 + \bar{B}_1)]\}$$

Next we will analyze the steady state solutions of Eq. (4).

Steady state solutions

The stationary motions take place under the conditions $\frac{dA_1}{dt} = 0$, $\frac{dB_1}{dt} = 0$, $\frac{d\Omega}{dt} = 0$. Note that the quantities A_1, B_1, Ω in the stationary conditions of the vibratory motion are determined from the equations:

$$2\ \omega\ \alpha\ B_1 - [(k+\chi)\ \omega\ A_1 + i\ k\ \Omega\ B_1 + i\ c\ \rho] = 0$$

$$2\ \varepsilon\ \alpha\ A_1 + [(k+\chi)\ B_1 - i\ k\ \Omega\ A_1 - c\ \rho] = 0$$

$$\{M(\Omega) + \frac{i}{2}\ k\ \omega\ (A_1\ \bar{B}_1 - \bar{A}_1\ B_1) - \frac{k\ \Omega}{2}(A_1\ \bar{A}_1\ + B_1\ \bar{B}_1) - \frac{c\ \rho}{4}[i\ (A_1\ - \bar{A}_1\ ) + (B_1\ + \bar{B}_1)]\} = 0$$

where ω is in the resonance region, that is, we can assume that (kononenko, 1969) $2\ \varepsilon\ \alpha\ \omega = \omega^2 - \Omega^2$, approximately. Using Eq. (4) and taking $A_1 = A_{11} + i\ A_{12}$ and $B_1 = B_{11} + i\ B_{12}$ (*noting that* $B_1 = i\ A_1$)

$$B_1 = \frac{c\rho\{(k+\chi)\omega - k\Omega\_im(\omega^2-\Omega^2)\}}{m^2(\omega^2-\Omega^2)^2 + [(k+\chi)\omega - k\Omega]^2} \quad A_1 = \frac{c\rho\{m(\omega^2-\Omega^2) - i[(k+\chi)\omega - k\Omega]\}}{m^2(\omega^2-\Omega^2)^2 + [(k+\chi)\omega - k\Omega]^2}$$

and reverting from the above variables to the initial variables x and y, we will obtain:

$$x = a\ \cos(\varphi\ + \xi),\ y = a\ \operatorname{sen}(\varphi\ + \xi) \tag{5a}$$

$$a = \omega^2\ \rho\ \frac{1}{\sqrt{h}}\ ,\ h = (\omega^2 - \Omega^2)'^2 + [(k+\chi)\frac{\omega}{m} - k\frac{\Omega}{m}]^2 \tag{5b}$$

and ξ is defined as a function of the constants of the problem.

We also note from Eq. (5b) that

$$L(\Omega) - S(\Omega) = 0\ ,\ \ S(\Omega) = q\ \ \Omega + \chi\ \ \omega\ \ a^2 = 0 \tag{6}$$

We remark that the third term in Eq. (6) is twice greater than the one described in (Kononenko, 1969) for one degree of freedom system, and that this term shows the interaction between the amplitude of motion and the characteristic curves of the dc motor. Here we take the characteristic curves of the dc motor (energy source) as straight lines, defined by $L = \eta_1\ - \eta_2\ \ \Omega$ where Ω is obtained from Eq. (6). The parameter η_1 is related to the tension in the motor armature and η_2 to the properties of the dc motor taken into account. L represents the driving torque of the rotor. Eq. (5b) represents the amplitude of the non-ideal vibrations. q is the resisting torque. Note also that the quantity $\chi\omega a^2$, that is, the moment of the force resisting the vibrating motion, *depends only on the external friction* χ, the natural frequency and amplitude of vibration of the rotor. Note that stationary non-ideal vibrations of the considered rotor are affected by the eccentricity ρ.

Stability of motion analysis

We assume that the stability conditions for free vibrations of the rotating shaft system (fig 1) are completely satisfied i.e. $(k^* + \chi^*)\omega - k^*\Omega > 0$ (Dimemtberg, 1961). Instead of Eq. (4) we consider the dynamical system in real coordinates

$$\begin{aligned}
\frac{dA_{11}}{dt} &= -\varepsilon a A_{12} + \frac{\varepsilon}{2m\omega}[-(k+\chi)\omega + k\Omega]A_{11} \\
\frac{dA_{12}}{dt} &= \varepsilon a A_{11} + \frac{\varepsilon}{2m\omega}[-(k+\chi)\omega + k\Omega] - \frac{c\rho}{2m\omega} \\
\frac{d\Omega}{dt} &= \frac{\varepsilon}{I}\{M(\Omega) + k(\omega - \Omega)(A_{12}{}^2 + A_{11}{}^2) + c\rho A_{12}\}
\end{aligned} \tag{7}$$

or

$$\frac{d\Omega}{dt} = \varepsilon\Phi_1(\Omega, A_{11}, A_{12})\ ,\ \frac{dA_{11}}{dt} = \varepsilon\Phi_2(\Omega, A_{11}, A_{12})\ ,\ \frac{dA_{12}}{dt} = \varepsilon\Phi_3(\Omega, A_{11}, A_{12})\ ,$$

by taking (Ω_c, a_c, ξ_c) the stationary points and considering a small perturbation of them $(\Omega = \Omega_c + \Omega_1\ ; a = a_c + a_1; \xi = \xi_0 + \xi_1)$ and expanding Eq. (7) in a Taylor series and considering the linear part (the Jacobian matrix) we will obtain that

$$(\Omega = \Omega_c + \Omega_1\ \ ; a = a_c + a_1; \xi = \xi_0 + \xi_1)\ \ \frac{dA_{11}}{dt}\ = b_{21}\Omega + b_{22}A_{11} + b_{23}A_{12}\ \ \frac{dA_{12}}{dt}\ = b_{31}\Omega + b_{32}A_{11} + b_{33}A_{12} \tag{8}$$

and the eigenvalues are solutions of the third order polynomial (characteristic) equation $\lambda^3 + B_1\lambda^2 + B_2\lambda + B_3 = 0$ with $B_i = B_i(b_{i,j})$. According to the classical Routh-Hurwitz stability criterium (RH), the necessary and sufficient conditions to stability are that the coefficients of the characteristic equation obey the following three conditions: $B_1 > 0;\ \ B_1B_2 - B_3 > 0;\ \ B_3 > 0$. We remark that the coefficients $(b's\ ;B's)$ are the same ones obtained by (Kononenko, 1969) page 205. A clear qualitative explanation for the stability of the motion will be summarized next. The conditions of stability (first condition $B_1 > 0$) is always satisfied for an energy source characteristic, since the gradient is always negative and (third conditions and dominant condition $B_3 > 0$) is written as the derivative of the driving torque and the resisting torque with respect to Ω , that is $\frac{d}{d\Omega}[L(\Omega) - S(\ \Omega)] < 0$. This is similar to the stability conditions to be satisfied by the driving torque for stable steady operation of any rotating machine (Tondl, 1965).

We remark that on the resonance curve (torque vs. rotational frequency, that is $S(\Omega)$ vs. Ω), constructed according to the stability conditions ($B_1 > 0;\ \ B_1B_2 - B_3 > 0;\ \ B_3 > 0$), we obtain jumps (saddle-node bifurcations), that is, the points on the rising branch ($\Omega < \omega$) lying to the left of resonance peak $(\Omega = \omega)$ correspond to stable stationary states of vibrations. On the falling part lying to the right side of the resonance peak we have points that correspond to unstable stationary points of motion. We remark that any conclusion based on stability criterion (RH) depends on the slope of the characteristic of the energy sources. These jumps are the reason why in many practical cases the realization of parts of resonance curve is not possible. The boundary points T and R are found from $\frac{d}{d\Omega}[L(\Omega) - S(\ \Omega)] = 0$ which are the points of contact with the graph $S(\Omega)$. The positions of the boundary points T and R are determined by these two points of contact. Note that the resonance curve depends not only on the type of motor and parameters of the system but also on the method of control of the motor power. If we use a motor of greater power it can be reduced or completely removed the instability of the stationary conditions (ideal case). In this case the slopes of characteristic curves of the motor in form of straight lines are nearly perpendicular to the axis Ω .

NUMERICAL SIMULATION RESULTS

In this section we present some numerical simulations in order to analyze some phenomena such as Passage through Resonance, Influence of the Torque, Regular and Irregular motions by using suitable values of the structural and DC motor parameters. Here we take the same numerical values that were used before by (Kononenko, 1969). They are given by

$$m = 0.5Kg;\ \ c = 28.4KN;\ \ q = 7.7\times10^{-4}\,Nms;\ \ I = 12.5\times10^{-4}\,Kgm^2;\ \ \rho = 4\times10^{-4}\,m$$
$$\chi = 7.5\times10^{-4}\,Ns/m;\ \ k = 1.5\times10^{-7}\,Ns/m$$

so that the natural frequency is $\omega = 72.86\ rad/s^2$.

FREQUENCY DOMAIN

Using these numerical values, we start a numerical simulation at a known solution to the left at the resonance peak. The power setting is increased in steps by increasing the value of the constant η_1 in equation ($\eta_2 = 0{,}002$). For each of these values we allow the system to reach a steady-state regime and obtain the amplitudes values of the generalized coordinates of the structure, which are plotted against the resulting speed of the motor in Fig. 2. One can observe that a large amount of energy is necessary for the speed of the motor to reach values in the region to the right of the resonance peak. This is the Sommerfeld effect of getting stuck in resonance, that is, one may not have enough power to reach higher speed regimes with low energy consumption as most of this energy is applied to vibrate the structure and not to accelerate the shaft. Another finding is that no stable solutions are obtained inside a considerable band of speeds to the right of the resonance regions. We can note the Jump phenomenon on left and on right of the two unstable points T and R, as predict by the analytical solution. Note the bending of the curve given by Fig. 2.

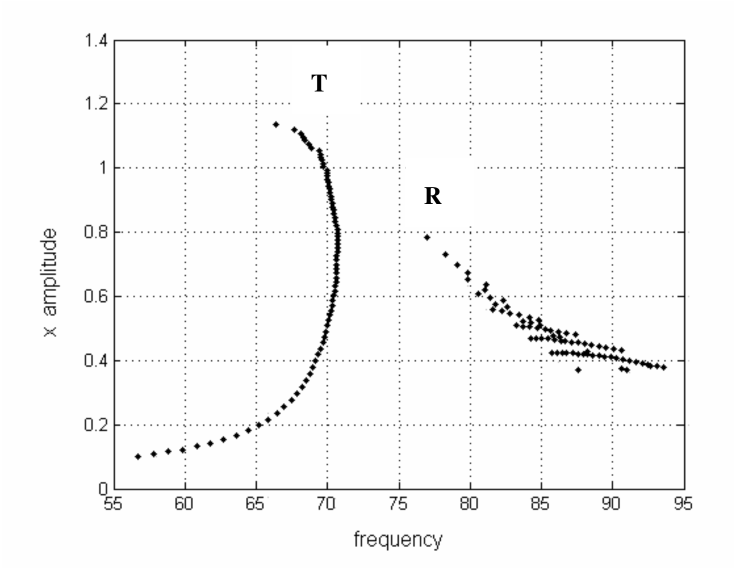

Figure 2 Frequency Response. T and R unstable Equilibrium Points(Somerfeld Effect)

Note that if one increases the value of eccentricity ρ of the disk to ρ =0.01 it is possible to see an increase of amplitude of oscillating during passage to resonance (Fig. 3). If one increases the mass of the system its natural frequency is affected and the frequency response curve will shift to a lower value (Fig. 3). Besides , a significant change occurs in the inclination of the curve of response due to damping. The jumping phenomena are still observed.

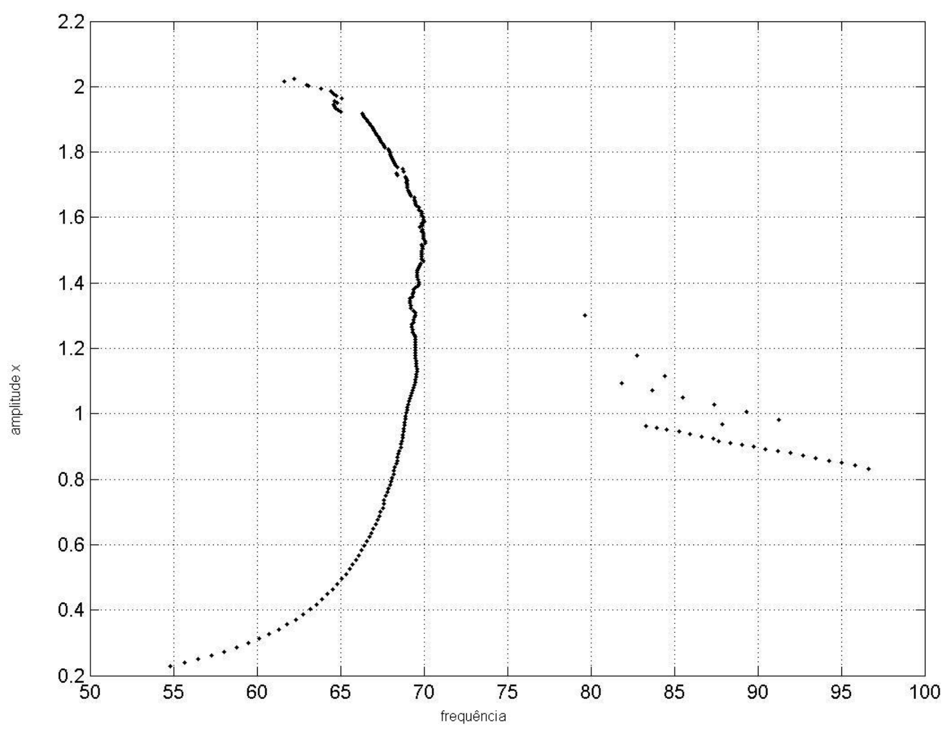

Figure 3 Frequency Responses. Influence of the eccentricity and mass on the system

TIME DOMAIN

Next, we show that the topology changes when taking into account suitable values of the parameter η_1 . This fact is related to greater or less interaction depending of the value of driving torque, that is, the dimensionless values of $\eta_1 = 0.0025; 0.0027; 0.0030$ that we set to investigate the passage through resonance. We consider dimensionless fixed value $\eta_2 = 0.0027$ (See Fig. 4). Fig. 4a illustrates a case when angular velocity is below resonance with $\eta_1 = 0.0025$, Fig. 4b illustrates a case when angular velocity is captured in the resonance region with $\eta_1 = 0.0027$ and Fig. 4c illustrates a case of passage through resonance region and increase of the transverse oscillations amplitudes of the shaft (Sommerfeld effect) with $\eta_1 = 0.0030$.

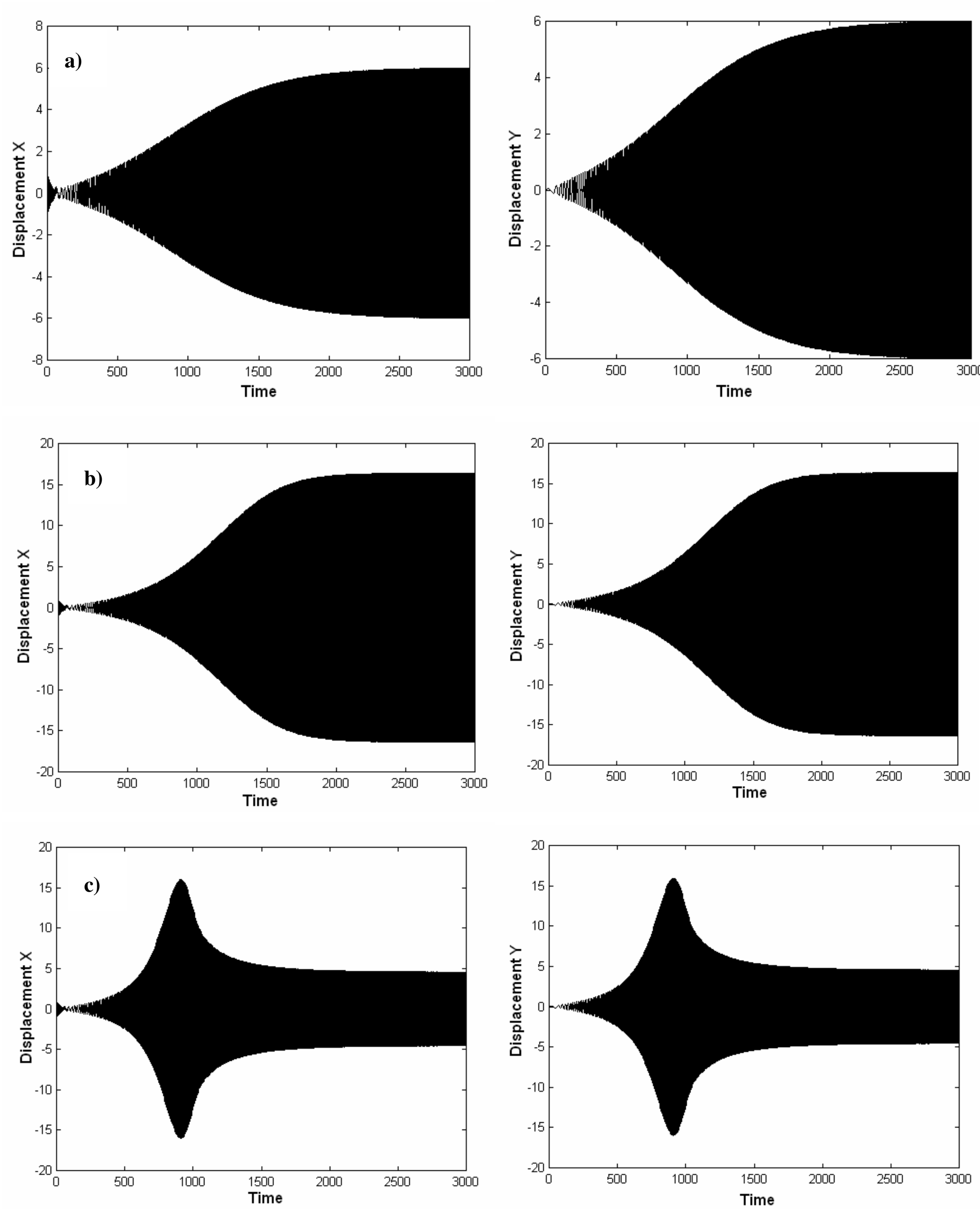

Figure 4. Time history of the shaft transverse oscillations a) $\eta_1 = 0.0025$**, b)** $\eta_1 = 0.0027$**, c)** $\eta_1 = 0.0030$

CONCLUSIONS

The studied symmetric rotor with an unbalanced disk driven by a limited power supply has an unstable region due to the characteristics of the energy source. It depends also on the eccentricity of the disk. Good agreements between both numerical and analytical solutions (via an average method) are found. The problem is also examined by numerical simulations with suitable values of the parameters. We believe to have clearly illustrated some problems associated with driving a system through resonance.

Extension to the study of transverse non-ideal vibrations of a weightless shaft with asymmetric springs (Iwatsubo et al, 1972) in each direction of the shaft cross section and carrying two disks through resonance will be present in future publications by the authors. A technique of Control in order to facilitate the passage through resonance as used by (Yamanaka and Murakami, 1989) by using a gradient based optimization technique or optimal control by using nonlinear programming technique has recently being presented by (Rafikov and Balthazar, 2005).

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