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Modelling an overhung rotor with clearance and unbalance – Karhunen–Loève Decomposition

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Abstract: Rotating machines are very important on the productive processes and each day the processes demand machines to operate for longer periods, with greater loads and at higher speeds. Bearing is the component that supports all the energy of loads and impacts. This works aims to study some effects of unbalance and clearance on the bearings of an overhung rotor. A rotor-bearing system is modeled as a continuous system. The discretization of the continuous system is conducted by the Galerkin method. The system is approximated by Finite Element Method and reduced using the Normal Modes and also the Karhunen–Loève Decomposition. The forces acting on the bearings of an overhung rotor and the Fast Fourier Transform (FFT) of the transient response are computed. Numerical results are qualitatively compared with real cases of machines showing good agreement.

Keywords: overhung rotor, non-linear dynamics, reduced model, Karhunen–Loève Decomposition

NOMENCLATURE

A = area of the transversal section, m^2
a, b = coefficients, dimensionless
c = clearance, m
$[C]$ = damping matrix, $N.s/m$
d = damping coefficient, $N.s/m$
d_h = housing damping N/m
E = Young Modulus Pa
e_{per} = specific residual unbalance, $kg.m/kg$
f = transversal force per unit of length, N/m
$\mathbf{F}$ = force vector, N
g = acceleration of gravity, m/s^2
G = balance quality grade, mm/s
$[G]$ = gyroscope matrix, $N.s/m$
I = inertia moment of the transversal section, m^4
I_r = rotor inertia moment, $kg.m^2$
k_h = housing stiffness, N/m
$[K]$ = stiffness matrix, N/m
L = length, m
$[M]$ = mass matrix, kg
M_f = bending moment, N
M_r = rotor mass, kg
N = number of elements used in the approximation
P = axial force, N
Q = shear force, N
$[R]$ = correlation matrix, m^2
u = transversal displacement, m
$\dot{u}$ = derivative of u, m/s
U = unbalance, $kg.m$
$[U]$ = dynamic response, m
x_{B3} = position of the displacement at bearing number 3, m
x_{B4} = position of the displacement at bearing number 4, m
$\mathbf{X}$ = displacement vector, m

Greek Symbols

Ω = speed of shaft rotation, rad/s
ρ = density, kg/m^3
ϕ = trial function

Subscripts

1 = relative to the direction 1
2 = relative to the direction 2
per = relative to permissible unbalance
h = relative to the bearing housing
r = relative to the rotor

INTRODUCTION

Each day the quest for efficiency demand machines to operate for longer periods, with greater loads and at higher speeds. Bearing is the component that supports all the energy of loads and impacts.

Unbalance, misalignment and clearance/looseness are responsible for almost 90% of rotating machines vibration problems. Bearing is the component that stands the effects of these vibrations. The effect of clearance and unbalance on the dynamics of a rotor is investigated in many recent papers. Concerning internal radial roller bearing clearance, Harsha (2005a and 2005b), Sinou and Thouverez (2004) and Tiwari et al. (2000) investigate this problem. All of them consider the nonlinearities caused by hertzian contact and develop lumped parameter system models. Karlberg and Aidanpää (2003 and 2004) and Vakakis and Azeez (1999) consider clearance between bearing and housing and clearance between bearing and shaft, respectively. All of these articles center their discussions around chaotic motion, Poincaré maps, bifurcation diagrams and Lyapunov exponents. In this paper, on the other hand, it is used the frequency domain to characterize some of the problems. It is acknowledge in this paper that the most used tool to identify the causes of rotating machine problems through vibration analysis is the Fast Fourier Transform (FFT).

The great limitation of lumped parameter system models is that there is no scheme to compute the error of the model besides the difficulty to choose the parameters. They certainly help to understand qualitatively the problems, but can not describe the effects quantitatively since there is no scheme of approximation. Continuous models do not have this limitations; when discretization is made with finite elements the properties that appear are material properties and there is an intrinsic scheme of approximation. On the other hand, finite element rotor analysis usually results in large dimensionality problems what turns the transient solution very time-consuming. It might include many insignificant modes because of the widely spread eigenspectrum. Vakakis and Azeez (1999) used the Normal Modes and Karhunen–Loève Decomposition (KLD) to reduce the problem.

KLD consists in detecting spatially coherent modes in the dynamics of a statio-temporally varying system. It is a powerful and elegant way of obtain an approximate description of reduced dimension of a process. KLD is first found in the literature as PCA (Principal Component Analysis) as a tool to signal analysis. Later it was extended for image processing and, then, to diverse applications in engineering: turbulence, control in chemical engineering, oceanography, etc (Holmes, 1996). In mechanical engineering the first applications were in turbulent flows, Lumley, 1970.

This technique aims to obtain the dominant characteristics of a signal (for example a dynamic response) based on experimental data or numerical data. It uses statistics to form a basis to project the dynamics that inherit most of its coherence.

The Group of Vibration Analysis of PUC-Rio is working with KLD applied to structural dynamics and non–linear systems for some time, Sampaio (2001), Wolter (2001), Wolter (2002). Recently some works have been published by Sampaio, in 2004, 2005 and 2006.

In this paper a continuous model of an unbalanced overhung rotor with clearance is simulated. Springs and dashpots symmetrically distributed on the shaft perimeter will introduce the non–linearities. A initial discretization is made with the Finite Element Method to compute the modes, according to a given prescription, that will be used to project the dynamics and to reduce the continuous model.

CASE STUDIED

Blower - nominal rotation speed 1185 *RPM*

It is considered the case of the blower showed in Figure 1:

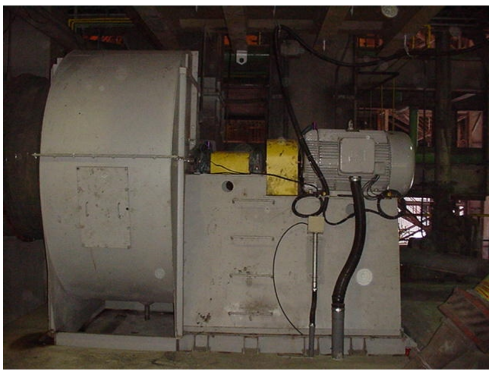
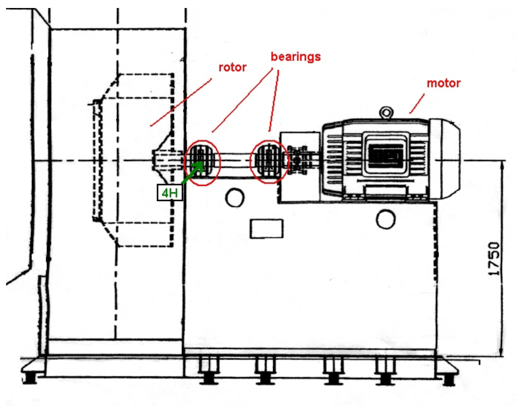
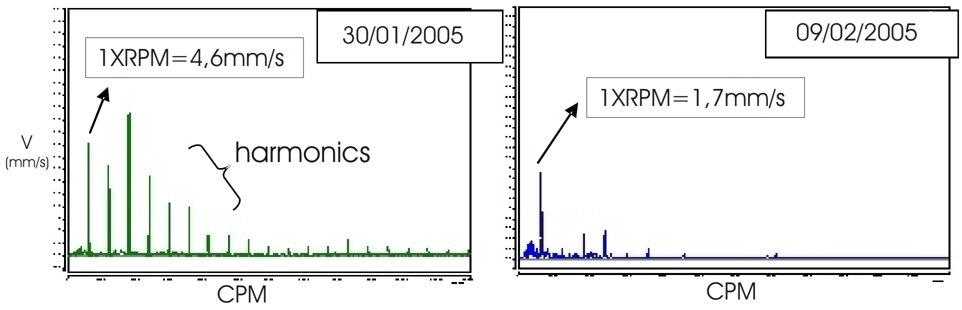



Figura 1 – Blower

The vibration was measured with an accelerometer at the position 4H (horizontal), Figure 1. The velocity is obtained by integration. Figure 2 shows the frequency spectrum.



(a) (b)

Figura 2 – Frequency spectrum of velocity. (a) January 30th 2005, (b) February 9th 2005

A piece of metal got stuck on a rotor blade at January 30th 2005 thus raising the unbalance. The effect of clearance is register in the frequency spectrum, Figure 2a. One can see many harmonics (2X, 3X,..).

After the removal of the piece of metal from the blade and the performance of field balancing, the harmonics almost disappeared, Figure 2b. The residual unbalance accomplished was 1,7 mm/s 0-Peak.

Unbalance x Bearing Life

To determine the rotor unbalance one uses the balance quality grade (G), an index derived from accumulated practical experience with a large number of different rotors, see ISO 1940/1.

In general the permissible residual unbalance (U_{per} $[kg \times m]$) for rotors is proportional to the rotor mass ($U_{per} \sim M_r$). So permissible specific residual unbalance is defined as: $e_{per} = U_{per}/M_r$. The experience shows that permissible specific residual unbalance varies inversely with the speed of the rotor. ($U_{per} \sim 1/\Omega$). Finally:

$$G = e_{per}\Omega 1000 \ \ [mm/s] \tag{1}$$

Where Ω is the rotation speed in rad/s and e_{per}=$[kg \times m/kg]$.

In the numerical analysis performed in the following sections G will be chosen and the unbalance is calculated:

$$U = \frac{GM_r}{\Omega 1000} \ \ [kg \times m] \tag{2}$$

Mass unbalance in a rotating system often produces excessive synchronous forces that reduce the life span of various mechanical elements.

For example, consider a machine with balance quality grade $G = 6,3$ and speed of 2000 RPM, with spherical roller bearings (SKF 6917). The bearing life can be calculated with or without the extra load due to unbalance.

Considering a load of 1757 Newtons the bearing life will be 21,4 years. The centrifugal force due to unbalance for this case is 132 Newtons. The additional load due to unbalance reduces bearing life by 27% (15,6 years). This calculation is not accounting for excessive clearance and shock. Depending on the machine characteristics and speed this reduction can be smaller or bigger.

WEAK FORMULATION

Figure 3 shows the scheme of a continuous rotor bearing system.

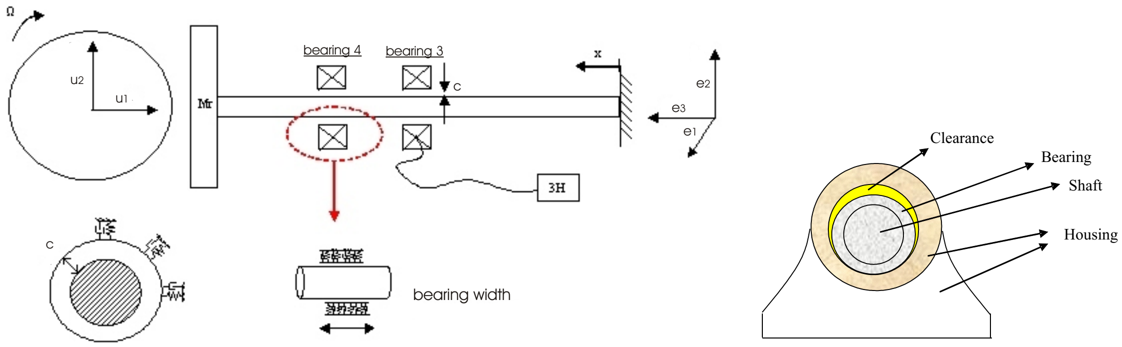


Figura 3 – Rotor system considered

The right diagram in Figure 3 illustrates the clearance considered between bearing and housing. Harmful clearance happens if there is wear of the parts or manufacture/assembly problems. In many machines, due to shaft thermal expansion, one bearing must have freedom to move in the axial direction, so there should be some clearance between bearing and housing. Bearing is the component that absorbs all the energy from impacts and loads having, as a consequence, its life reduced.

The two directions are coupled due to gyroscopic forces. One assumes that rotary inertia has very small effect on the dynamics for the system parameters under consideration.

The Euler-Bernoulli beam model is used (Meirovitch, 1997). Damping is added to the beam's dynamics: $d\dfrac{\partial u}{\partial t}$, where d is the damping coefficient. The problem gets more complicated when the gyroscopic moments are considered: $\Omega I_r \dfrac{\partial^2 u}{\partial x \partial t}$. Now there is coupling between the two directions.

The problem is written in the weak formulation (after the approximation):

$$\boxed{\begin{array}{c}
\ddot{a}_i \int_0^L m\phi_i\phi_j dx + \ddot{a}_i M_r \phi_i(L)\phi_j(L) + a_i \int_0^L EI\phi_i''\phi_j'' dx + \dot{b}_i \Omega I_r \phi_i'(L)\phi_j'(L) + \\
+\dot{a}_i \int_0^L d_1 \phi_i\phi_j dx = P_1(t)\phi_j(x_{B3}) + P_1(t)\phi_j(x_{B4}) + Q_1(t)\phi_j(L) \\
\ddot{b}_i \int_0^L m\phi_i\phi_j dx + \ddot{b}_i M_r \phi_i(L)\phi_j(L) + b_i \int_0^L EI\phi_i''\phi_j'' dx - \dot{a}_i \Omega I_r \phi_i'(L)\phi_j'(L) + \\
+\dot{b}_i \int_0^L d_2 \phi_i\phi_j dx = P_2(t)\phi_j(x_{B3}) + P_2(t)\phi_j(x_{B4}) + Q_2(t)\phi_j(L) \\
a_i(0) = 0 \quad , \quad \dot{a}_i(0) = 0 \quad ; \quad b_i(0) = 0 \qquad \dot{b}_i(0) = 0
\end{array}} \tag{3}$$

In the weak formulation the boundary conditions are incorporated in the equation. At $x = 0$ the shaft is clamped (essential condition) and in $x = L$ there is a mass attached (natural condition). The parameters are:

$u_1 \rightarrow$ displacement at e_1 direction	$u_2 \rightarrow$ displacement at e_2 direction	$\phi \rightarrow$ vibration modes
$E \rightarrow$ elasticity module	$I \rightarrow$ shaft inertia momentum	$m \rightarrow$ density times area section (ρA)
$M_r \rightarrow$ rotor mass	$I_r \rightarrow$ rotor inertia momentum	$L \rightarrow$ shaft length
$\Omega \rightarrow$ rotation speed	d_1 and $d_2 \rightarrow$ damping coefficients	x_{B3} and $x_{B4} \rightarrow$ bearing positions

And:

$$u_1(x,t) = \sum_{i=1}^{N} a_i(t)\phi_i(x) \quad ; \quad u_2(x,t) = \sum_{i=1}^{N} b_i(t)\phi_i(x) \tag{4}$$

N is the number elements of the base used in the approximation. ϕ is called trial function. The system is projected in the basis composed by the trial functions.

The external forces acting on the system are: 1. centrifugal forces due to unbalance; 2. force of the rotor weight due to gravity; and 3. forces due to impact. The non–linearity of the system comes from the forces due to impact. Unbalance forces ($U\Omega^2 cos(\Omega t)$), rotor weight ($M_r g$) and impact forces ($P_{1,2}$) are written as:

$$\begin{array}{cc}
Q_1 = U\Omega^2 cos(\Omega t) & Q_2 = U\Omega^2 sen(\Omega t) - M_r g \\
P_1 = -\xi\left(k_h \dfrac{(r-c)u_1}{r} + d_h \dfrac{(\dot{u}_1 u_1 + \dot{u}_2 u_2)u_1}{r^2}\right) & P_2 = -\xi\left(k_h \dfrac{(r-c)u_2}{r} + d_h \dfrac{(\dot{u}_1 u_1 + \dot{u}_2 u_2)u_2}{r^2}\right)
\end{array} \tag{5}$$

Where:

$r = \sqrt{u_1^2 + u_2^2}$	k_h and d_h = housing stiffness and damping, respectively	U = unbalance
c = clearance	$\xi = 1$ if r $\geq c$, and $\xi = 0$ if $r < c$	g = acceleration of gravity

APPROXIMATION

To find an approximation, the system is discretized by the Galerkin method. In this method the error of the approximation is orthogonal to the projection space. When a finite number of terms are used to approximate the solution for u_1 and u_2 The weak formulation is transformed into a system of ordinary differential equations that can be written as:

$$[M]\ddot{\mathbf{X}} + [C+G]\dot{\mathbf{X}} + [K]\mathbf{X} = \mathbf{F} \tag{6}$$

Where $[M]$ is a symmetric positive definite matrix; $[K]$ is a symmetric positive semi-definite matrix; $[C]$ is a symmetric positive semi-definite matrix; $[G]$ is an skew-symmetric matrix; $\mathbf{X}$ is the displacement vector; and $\mathbf{F}$ is the force vector.

With the system of ordinary differential equation in hands one can calculate an approximation using numerical computation and analyze the convergence. For the approximation scheme, the permissible error is defined and then an approximation is computed with N elements of the basis. The relative error is calculated (with respect to the approximation computed with $N/2$ elements of the basis, for instance). If it is bigger than the permissible error, the number of elements of the basis is increased and another approximation is computed. If the error is smaller than the permissible error than one gets an approximation. The error considered in the calculations is the mean square error computed considering the displacements and their first and second derivatives.

Finite Element Method - FEM

In FEM the trial functions (interpolation functions) of each element are described in local coordinates. The global matrices are calculated by assembling the element matrices.

The natural frequencies and normal vibration modes of the system can be calculated after the computation of the global matrices $[M]$ and $[K]$. The following eigenvalue problem must be solved:

$$(-\omega_i^2[M] + [K])\phi_i = \mathbf{0} \tag{7}$$

Where ω_i is the i–th natural frequency and ϕ_i is the i–th vibration mode.

The FEM is an effective method and it is largely used. But, depending on the problem, it might result in huge matrices. Besides, performing a non–linear problem can be very demanding computationally.

Using the Normal Modes

In this method the trial functions, ϕ_i, are the vibration modes of the associated conservative system.

In many cases, the modes of the system are not known a priori. So they must be computed with another approximation, FEM for example.

By using the Normal Modes, one gets a good representation a linear problem because it generates matrix not so big. But the method is not very effective when one deals with non-linear dynamics.

Karhunen–Loève Decomposition

In Karhunen–Loève Decomposition (KLD), the trial functions are the KL basis obtained from the correlation matrix $[R]$, which is symmetric. It generates the eigenvalues and the orthogonal eigenvectors which are the proper orthogonal modes (POM), also called empirical modes. The proper orthogonal values (POV) are the eigenvalues of matrix $[R]$. Matrix $[V]$ is formed as follows:

The dynamic response computed through different simulations are gather in matrix $[U]$, Eq. (8):

$$[U] = [\mathbf{u}_{p1}\ \ \mathbf{u}_{p2}\ \ \dots\ \ \mathbf{u}_{pn}] = \begin{bmatrix} \mathbf{u}_{p1}(t_1) & \mathbf{u}_{p2}(t_1) & \dots & \mathbf{u}_{pn}(t_1) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \mathbf{u}_{p1}(t_m) & \mathbf{u}_{p2}(t_m) & \dots & \mathbf{u}_{pn}(t_m) \end{bmatrix} \tag{8}$$

$\mathbf{u}_{p1}$ means displacement at position 1. Matrix $[U]$ has dimensions $m \times dn$: m time instants at n spatial points. Because of the two directions, u_1 and u_2, $d = 2$. Using the hypothesis of stationarity and ergodicity, the variation of the field with respect to the mean value is:

$$[V] = [U] - \frac{1}{m} \begin{bmatrix} \sum_{i=1}^{m} \mathbf{u}_{p1}(t_i) & \sum_{i=1}^{m} \mathbf{u}_{p2}(t_i) & \dots & \sum_{i=1}^{m} \mathbf{u}_{pn}(t_i) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \sum_{i=1}^{m} \mathbf{u}_{p1}(t_i) & \sum_{i=1}^{m} \mathbf{u}_{p2}(t_i) & \dots & \sum_{i=1}^{m} \mathbf{u}_{pn}(t_i) \end{bmatrix} \tag{9}$$

Then matrix $[R]$ can be computed, Eq. (10):

$$[R] = \frac{1}{m}[V]^T[V] \tag{10}$$

Matrix $[R]$ has dimensions $dn \times dn$.

The KLD is the method that generates the best basis to represent a dynamic problem. It is the best in the sense that it is necessary less elements to represent a dynamics.

NUMERICAL SIMULATION

The computer used to perform the simulations was a desktop: 3,2 GHz processor speed and 2 GB RAM.

The dimensions and the material properties used in the simulation are the same as the blower, Figure 1, except for system damping and bearing stiffness. The system damping was chosen big on purpose, it is due, unfortunately, to the lack of computer processing availability. The parameters values used for the standard simulation are:

Shaft length, $L = 3053\ mm$	Shaft diameter, $D_s = 110\ mm$	Rotor mass, $M_r = 100\ kg$
Elasticity modulus, $E = 193\ GPa$	Density, $\rho = 8000\ kg/m^3$	Damping, $d_1 = d_2 = 1000\ Ns/m^2$
Balance quality grade, $G = 6\ mm/s$	Clearance, $c = 69\ \mu m$	Bearing stiffness, $k_h = 0,1$ GN/m
Bearing 3 position, $x_{B3} = 1,692\ m$	Bearing 4 position, $x_{B4} = 2,302\ m$	Bearing width, $B_w = 58\ mm$
Rotation speed, $\Omega = 124\ rd/s = 1185\ RPM$		

Finite Element

The program *Flexpde* was used to compute the response by Finite Element Methods. For a precision specified of $< 0,5\%$ in the 40th vibration mode it was necessary 320 elements (these modes will be used to project the dynamics). The dynamic response was also calculated by FEM. Figure 4 shows the convergence analysis:

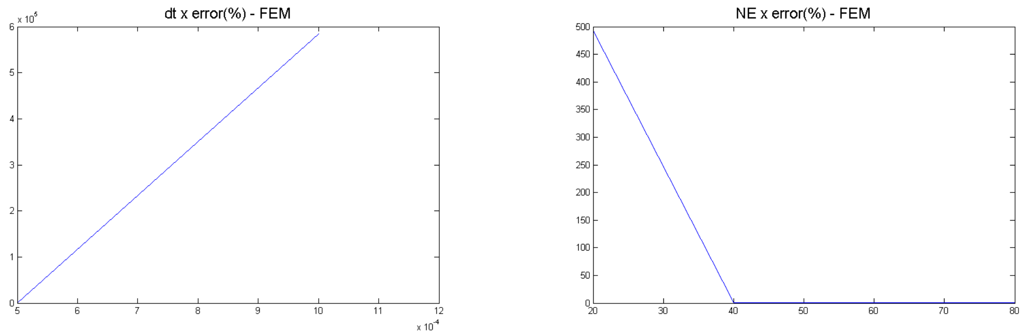


Figura 4 – Convergence of the approximation, Left: varying dt; Right: varying NE

For a precision of $< 0,5\%$ it is necessary to use $\Delta t = 0,0005s$ and 40 finite elements. One point that should be remarked is that 40 finite elements means 120 interpolation functions.

Using the Normal Modes

The normal modes computed by FEM of a clamped beam with a mass on its extremity are used to reduce the system, so an approximation of the dynamics can be calculated. In order to select the proper time steps and the number of normal modes used to generate the dynamic response.

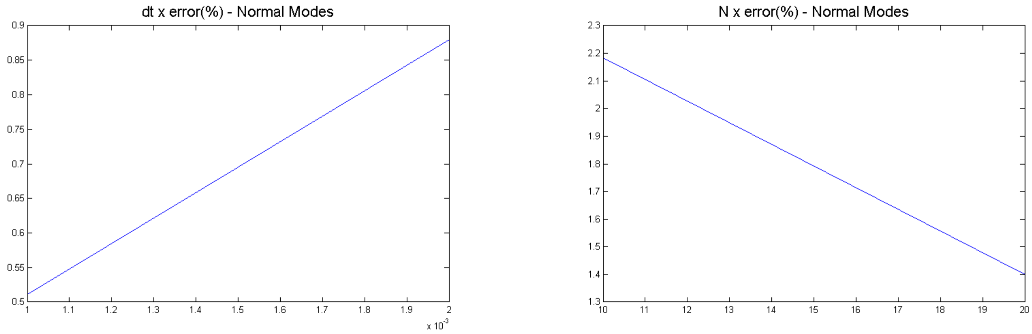


Figura 5 – Convergence of the approximation, Left: varying dt; Right: varying N

In this work the subroutine used to solve the system of ODEs was *ode45* from MATLAB. It is based on Runge-Kuta method of 4th and 5th order.

Figure 5 shows the percent error in the dynamic response for different number of modes and for different time steps. The error decreases increasing the number of modes and decreasing dt, respectively. For a precision specified of $< 1,5\%$ it is necessary $\Delta t = 0,001$ and $N = 10$.

Since there are two directions, u_1 and u_2, it is necessary $2 \times 10 = 20$ modes to represent the problem. The time spent to compute the dynamic response depends on the number of modes used in the approximation. For $N = 10$, the average time spent in a simulation is 20 minutes.

Karhunen–Loève - KL

The KL basis can be used to reduce even more the system. To construct the correlation matrix $[R]$, 10 simulations were computed, varying G uniformly from 1 to 10 (1, 2, .., 10). This range of G was chosen because this is a reasonable unbalance for this kind of machine to operate, where $G = 10$ is a big unbalance. So, the KL basis is valid only for an specific range: 1 <G< 10. To compute the eigenvalues of matrix $[R]$ the time spent was about 45 minutes.

With the empirical modes in hands one can reconstruct the dynamics.

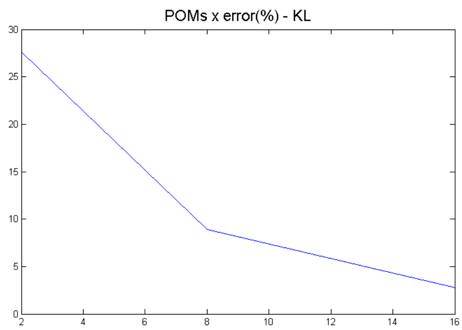


Figura 6 – Convergence of approximation for KLD

Figure 6 shows the convergence comparing the dynamic response reconstructed by KL basis and the dynamic response computed by FEM. For a precision specified of $< 3\%$, the number of empirical modes needed is 8, which means 16 elements of the basis.

Figure 7 compares the convergence of the approximation using the Normal Modes and using KL basis with the response computed by FEM as a reference.

Overhung Rotor – KL

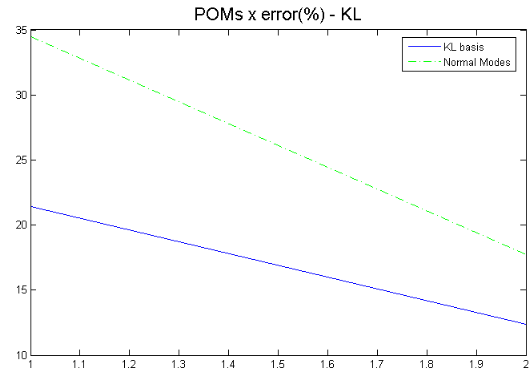


Figura 7 – Convergence Normal Modes x KLD

It is clear that KL basis can approximate better the dynamic response. Even though this is true, the computation time required to integrate the system of ODEs when one uses KL basis is greater then when one uses the normal modes. That is why Figure 7 only shows the approximation for the first two modes. Using two modes, the percent error is still high – $17,7\%$ using two normal modes and $12,4\%$ using two POMs – comparing with the response computed by FEM. Figure 8 shows the shape of the first two normal modes and the first two POMs.

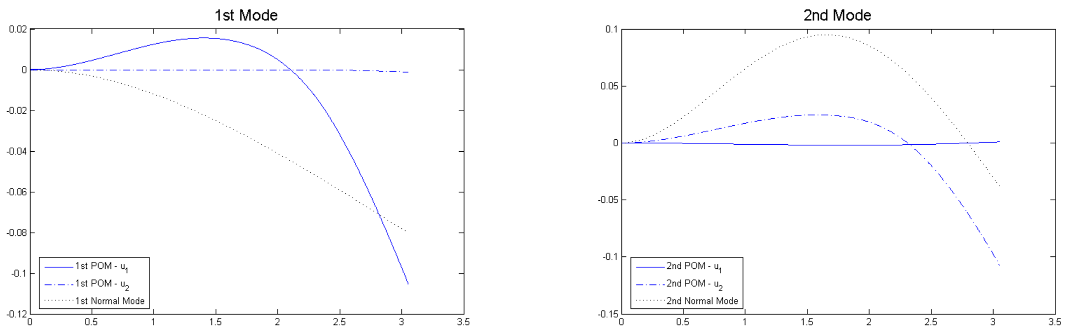


Figura 8 – First two modes

To represent the problem:

- FEM $\longrightarrow$ 120 elements of the basis (40 finite elements);
- Normal Modes $\longrightarrow$ 20 elements of the basis (normal modes);
- KLD $\longrightarrow$ 16 elements of the basis (empirical modes).

KLD is the best method to represent this non–linear problem in the sense that one needs less elements of the basis.

OTHER SIMULATIONS

The dynamic response of the shaft was computed so the problem can be studied. Figure 9 shows the shape of the shaft when the machine is in operation. Observe that the load zone of bearing number four is at the bottom part of the housing while the load zone of bearing number three is at the top.

Figure 10 shows the dynamic response, u_1 and u_2 at bearing number three, and the orbit of u_2 at bearing number three, all computed by FEM. The results were analyzed after the system stabilization.

Figure 11 shows two FFT curves for $G = 0,5$ and $G = 6$ [mm/s]. One can observe multiples of fundamental frequency (2X, 3X,...) on the spectrum for high levels of unbalance. If the unbalance is small, $G = 0,5$ for example, one can barely see the harmonics. If the unbalance is big, $G = 6$ for example, the harmonics are evident.

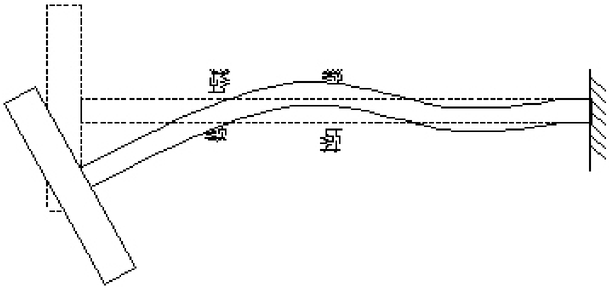


Figura 9 – Shape of the shaft under operation condition

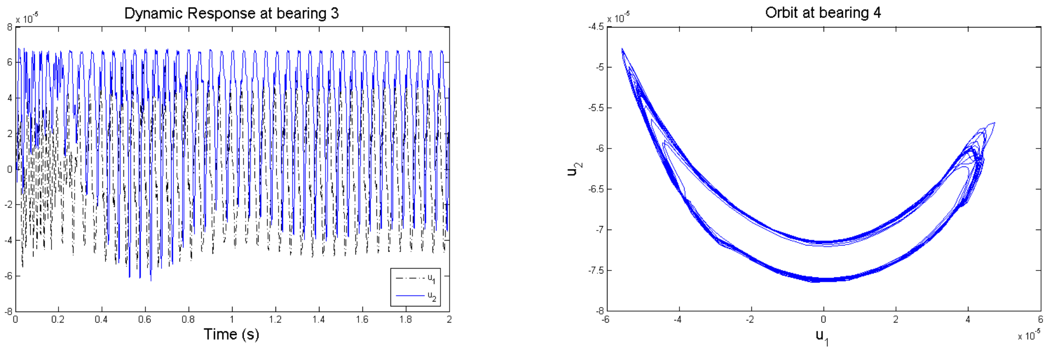


Figura 10 – Left: dynamic response at bearing number 3 - u_1(blue) and u_2(green); Right: orbit at bearing number 3 after the transient

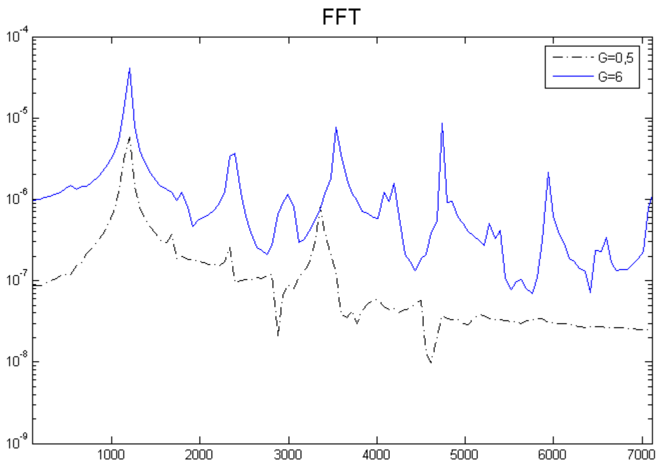


Figura 11 – FFT of the displacement of bearing number 3: $G = 0,5$ (blue) and $G = 6$ (magenta)

This result shows good qualitative agreement with the experimental result, Figure 2.

Due to lack of space, other results, as the forces acting on the bearings when other parameters vary, are not presented in this paper.

CONCLUDING REMARKS

A rotor-bearing system model was proposed in this work. Numerical results showed good qualitative agreement with real rotating machine behavior. The model can thus help the understanding of the behavior of a rotor system.

Concerning the bases chosen to represent this non-linear problem, Karhunen–Loève Decomposition (KLD) generated the most reduced model. To compute the vibration modes by the Finite Element Method (FEM) one needs at least 120 elements of the basis (40 finite elements) to achieve a prescriptive precision. By using the Assumed Modes to calculate the dynamic response one needs at least 20 elements of the basis (normal modes). By using KLD one needs at least 16 elements of the basis (empirical modes).

Since the problem is strongly non-linear, it seems that to get a better description of the dynamics one should search for a smart base that takes into account the non-linearities (the normal modes do not). KLD aims to describe the observed phenomenon in a reduced dimension and it is capable to capture the most interesting features of the dynamics.

A rotor-bearing system is very complex. Several other aspects of the rotor system should be taken into account in future works, such as: a better description of the damping, the influence of lubricant and the control of the dynamics.

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RESPONSIBILITY NOTES

The authors are the only responsible for the printed material included in this paper.