Proceedings of the XII International Symposium on Dynamic Problems of Mechanics (DINAME 2007),
Varoto, P. S. and Trindade, M. A. (Editors), ABCM, Ilhabela, SP, Brazil, February 26 - March 2, 2007

Localization of Similar Nonlinear Normal Modes in a Civil and Seismic Engineering Application

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Abstract: A pair of almost identical tall buildings erected very closely is modeled as two inverted pendulums. The small amount of coupling between them provided by the soil on which they are founded is modeled as a nonlinear hardening type of spring, according to our professional judgment. We derive the equations of free vibrations motion of this two-degree-of-freedom nonlinear mathematical model and seek the conditions for the existence of similar nonlinear modes at one-to-one resonance, namely the (almost) symmetrical and (almost) anti-symmetrical ones. Further, a parametric study is carried out to find a certain relationship of the stiffness characteristics of the system leading to a mathematical bifurcation of the anti-symmetrical mode. We find that, in these conditions, we can get the amplitude of free vibrations of one of the two generalized coordinates as large as we desire, compared to the amplitude of vibration of the other one. This phenomenon is a sort of mode localization quite different from the usual localization of linear modes of quasi-periodic systems. We propose that this study may be of interest in Seismic prone regions such as the Latin American Countries of the Pacific Coast and Portugal.

Keywords: nonlinear dynamics, nonlinear modes, mode localization, earthquakes

NOMENCLATURE

K= stiffness ratio, dimensionless
a = stiffness coefficients
c = constant relations between coordinates
n = number of degrees of freedom
x = generalized coordinates

Greek Symbols

Δ = determinant
Δt = time step
δ = elongation of coupling spring.

Subscripts

i relative to a generalized coordinate
r relative to a generalized coordinate
x relative to difference between generalized coordinates

INTRODUCTION

A considerable effort has recently been made by researchers in mechanics to extend to the non-linear case the usual superposition of normal modes technic for dynamic analysis of linear systems. As a natural analogy, Rosenberg (1966) defined a non-linear normal mode of a discrete system as a particular free vibration in which all co-ordinates oscillate in unison. In other words: the motions of all co-ordinates are periodic, of the same period; all co-ordinates reach their extreme values at the same instant of time; and at any instant of time the co-ordinates can be expressed as functions of only one of the others, making it possible to parametrize the oscillations by any of them. This last condition allows for an additional classification of the non-linear normal modes; namely, if the functional relationship is linear the mode is said to be similar, corresponding to straight line trajectories in the configuration space. Otherwise, the mode is called non-similar. Similar modes are detected by imposing matching conditions for the coefficients of respective non-linear terms of the differential equations of motion. A very interesting characteristic of some non-linear systems is the possibility of the number of normal modes to exceed that of the number of degrees of freedom, due to mathematical bifurcations, in contrast to linear systems in which the number of modes must be equal to that of their co-ordinates.

Analytical methods have been proposed to detect non-linear normal modes and applied to certain classes of problems by, among others, Rand (1974), Chi and Rosenberg (1985), Caughey et al. (1990), Caughey and Vakakis (1991), Rand et al. (1992), Vakakis (1991) and Vakakis and Rand (1992). Recently, invariant manifolds concepts were used by Shaw and Pierre (1991, 1993, 1994) to develop a method of obtaining expansions for the non-linear normal modes, with extensive use of symbolic computation. Variations of the algorithm were proposed by Balthazar et al. (1994) for certain systems. A complex normal mode manifold approach to attack cases of internal resonance has also been proposed by Nayfeh and Nayfeh (1994).

In this paper we present some results of our research to determine the non-linear similar normal modes of a two-degree-of-freedom model as inverted pendulums of a pair of almost identical tall buildings. We consider the restoring forces of the model, i.e. those given by the foundations, to be composed of a linear part plus non-linearities of quadratic nature, of both the hardening and the softening kinds. The differential equations of motion are written in a general form to allow the conclusions to be extended to non-structural problems of the same mathematical formulation. We determine in which conditions it is possible for this model to have similar modes and bifurcations. Direct numerical time integration of the equations, without any previous manipulation, using a Runge—Kutta algorithm, is presented to verify the analytical predictions.

We find that, if the bifurcation occurs, we can get the amplitude of free vibrations of one of the two generalized coordinates as large as we desire, compared to the amplitude of vibration of the other one. This phenomenon is a sort of mode localization, quite different from the usual localization of linear modes of quasi-periodic systems. These are structures composed of a series of nearly equal substructures lightly coupled. In such structures, the frequencies are clustered in groups of very close values and the presence of small perturbations in the dynamic characteristics of the substructures make the modes, that are otherwise extended to the whole structure, to get localized, that is, some regions of the structure experience large amplitude vibrations while others remain almost motionless. References on this phenomenon are the works of Brasil and Hawwa (1995), Hawwa and Brasil (1996), Brasil and Mazzilli (1995) and Brasil, Menoita and Balthazar (2000).

Possible practical applications of the material presented in this paper are in support excitation of Civil Engineering structures in seismic prone regions such as the Pacific Coast Latin American Countries and Portugal, both within the influence zone of Brazilian engineering presence.

THE MATHEMATICAL MODEL

As an application, the mathematical model in Figure 1 is analyzed, with wo almost equal cantilever tall buildings of height *h* and constant cross-section, *m* is a part of their masses lumped at their tops. The adopted mathematical model of the structure, shown in Figure 1, consists of two vertical rigid massless bars pinned in their bases where rotational non-linear springs act, coupled by a horizontal flexible massless beam that acts as a non-linear spring. The generalized and normalized co-ordinate x_1 is related to the horizontal displacement of the top of the left column, and x_2 to the horizontal displacement of the top of the right column. Here we are neglecting damping, which we consider in Balthazar and Brasil (1995).

We consider the generalized and normalized restoring force due to each of the rotational springs to have the form -

$$f_i = (a_{01} - a_{2i} x_i) x_i, \qquad i = 1,2, \tag{1}$$

corresponding to a softening type spring to model the nonlinear soil-structure interaction behavior at the foundation of each building, according to our professional experience.

The generalized and normalized restoring force of the coupling flexible massless beam is considered to have the form

$$f_{12} = a_1 \delta_x + a_3 \delta_x^2 \tag{2}$$

where δ_x is the differential displacement of the coupling beam, corresponding to a hardening type of spring, to model the nonlinear interaction between close buildings foundations, according to our professional experience.

The Lagrange differential equations of motion of this model, in a general normalized form, are

$$\ddot{x}_1 + (a_{01} + a_1) x_1 - a_1 x_2 - a_{21} x_1^2 + a_3 \delta_x^2 = 0, \tag{3}$$

$$\ddot{x}_2 + (a_{02} + a_1) x_2 - a_1 x_1 - a_{22} x_2^2 + a_3 \delta_x^2 = 0. \tag{4}$$

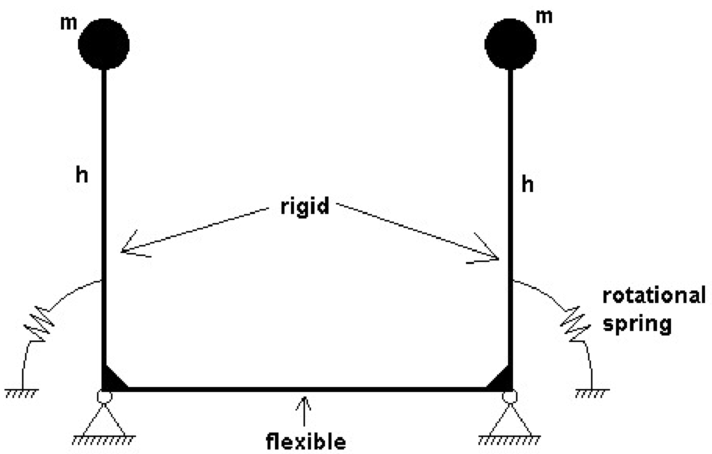

Figure 1 – The 2-DOF mathematical model

SIMILAR NON-LINEAR NORMAL MODES

To detect the existence of similar non-linear modes in a system, with *n* degrees of freedom, the usual procedure is to require, for all times, a linear relationship between the generalized co-ordinates in the form

$$x_i = c_{ir} x_r, \qquad i = 1,...,n, \quad i \neq r, \quad c_{rr} = 1, \tag{5}$$

where c_{ir} are (n — 1) unknown scalar quantities. Substituting the conditions (5) into the equations of motion we obtain n differential equations in x_r . It is evident that they will give the same solution for x_r if and only if all the coefficients of the respective powers of this variable are equal.

In our case we have $n = 2$, so that condition (5) becomes $x_1 = c_{12} x_2$. It is clear that if $a_{01} \neq a_{02}$ similar modes are not possible and we would have non-similar normal modes, which we study in Balthazar and Brasil (1995). If we take these two coefficients to be equal, corresponding to the so-called one-to-one resonance, and balancing the coefficients of the equations, we find that $c_{12} = +1$ and $c_{12} = -1$ are always solutions, corresponding to the symmetric and antisymmetric normal modes, respectively. In what follows, we also make $a_{21} = a_{22}$.

Additional non-linear similar modes are found for certain values of the stiffness ratio

$$K = \left|\frac{a_3}{a_{2i}}\right| \tag{6}$$

For positive values of the determinant quantity

$$\Delta = -\frac{4}{K} + \frac{1}{K^2} \tag{7}$$

corresponding to $K < \frac{1}{4}$, a mathematical bifurcation is found and for each value of this ratio two additional values of c_{12} , in addition to + 1, are possible:

$$c_{12} = (2 + \frac{1}{K \pm \sqrt{\Delta}})/2 \cdot \tag{8}$$

A plot of these conclusions is presented in Figure 2. One finds that the bifurcation springs from the antisymmetric mode, which physically corresponds to the two buildings vibrating in such a way as to be always either approaching or moving away from each other.

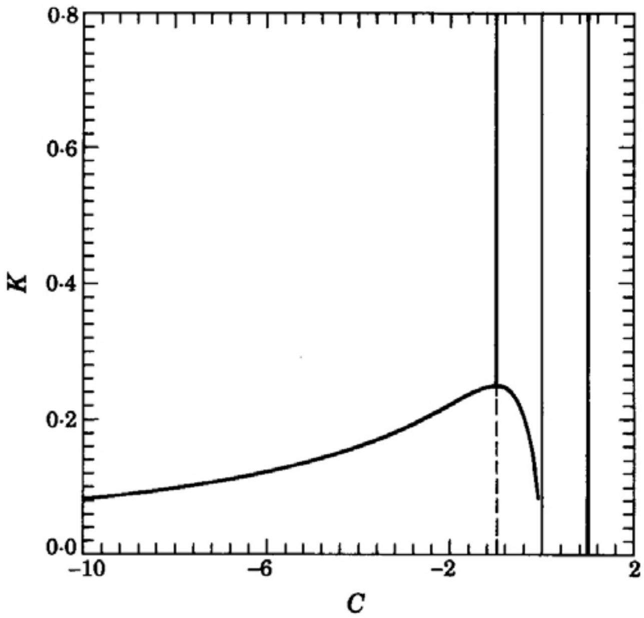

Figure 2 – Values of C_{12} as function of $K = a_3/a_{2i}$

NUMERICAL INTEGRATION

In this section, a Runge—Kutta algorithm is used directly to integrate equations (3) and (4), without any previous manipulation. Structural parameters and initial conditions are chosen in such a way as to have the structure oscillating in a non-linear similar normal mode given by a coefficient c_{12} in one of the branches of the bifurcation shown in Figure 2. The numerical values of the coefficients of the equations used are $a_{01} = a_{02} = a_1 = 1, a_{21} = a_{22} = 0.5$, and $a_3 = 0.1$, so that $K = 0.2$. These values were suggested by a parametric study, in order to force occurrence of certain nonlinear phenomena, and are not to be considered as corresponding to any real buildings. The adopted initial conditions, using equation (8), are $x_1(0) = 0.1, \dot{x}_1(0) = 0, x_2(0) = -0.5236, \dot{x}_2(0) = 0,$ and the integration time step is $\Delta t = 0.05s$.

Time history graphics of the two co-ordinates are superposed in Figure 3, using a solid line for x_1 and a dashed line for x_2. It is very clear that the motions of the co-ordinates are periodic of the same period and that they reach their maxima simultaneously, according to the usual definition of non-linear normal modes.

Plots of the velocities against the displacements for each co-ordinate are superposed in Figure 4, using the same solid and dashed line convention.

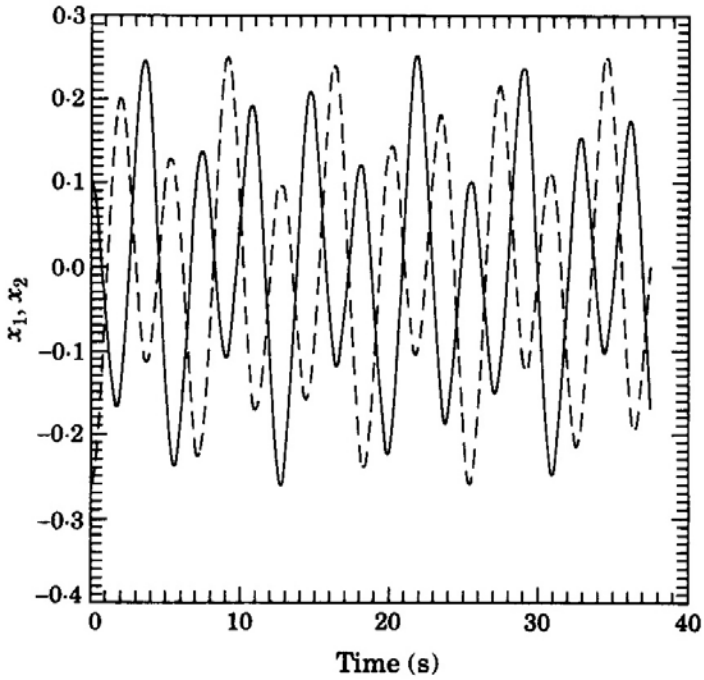

Figure 3 – Time histories of x_1 (_____) and x_2 (- - - -))

MAIN CONCLUSIONS

Non-linear similar normal modes of a two-degree-of-freedom model of a two buildings structure were studied. The restoring forces of the model were considered to be composed of a linear part plus non-linearities of quadratic nature of both the hardening and the softening kinds. The conditions for the model to have similar modes and bifurcations were analytically determined. It was also presented mode localization in the nonlinear mode context. Direct numerical time integration of the equations, without any previous manipulation, using a Runge—Kutta algorithm, results in time histories according to the usual non-linear normal modes definition.

We are presently working on large nonlinear Finite Elements numerical models of the problem, to be presented in further papers, in order to: a) consider the flexibility of the free standing buildings; b) more realistic modeling of the soil-structure interaction at the foundations of each building and the interaction between the buildings.

Possible practical applications of the material presented in this paper is suggested in seismic excitation of Civil Engineering structures. We agree that earthquakes are not frequent nor serious in Brazil, although a new National Code on the subject has recently being proposed by ABNT (2006). But our Civil Engineering has the opportunity to work in all of the Pacific Coast Latin American countries and in Portugal.

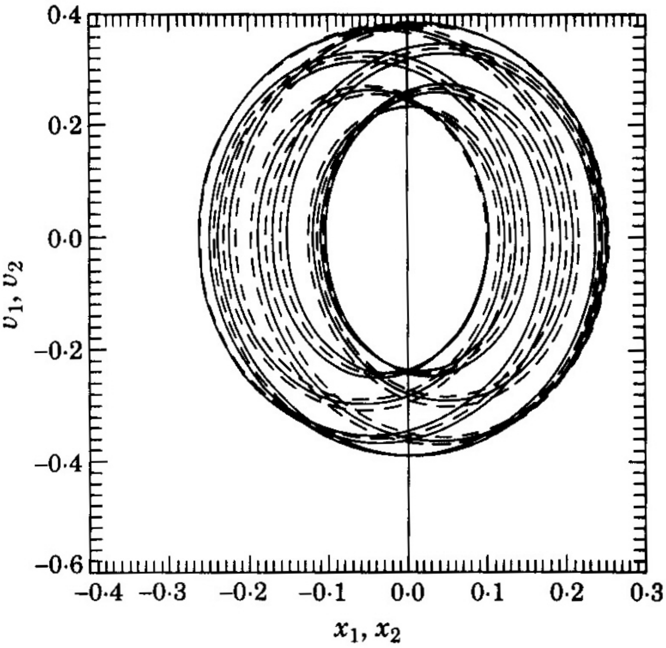

Figure 4 – Phase planes for of x_1 (_____) and x_2 (- - - - -))

ACKNOWLEDGMENTS

The authors acknowledge support by CNPq, Proc. 474005/2004-6, Edital 19/2004 – Universal, and FAPESP, Proc. 05/59316-0.

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