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KINEMATICAL MODELING AND OPTIMAL DESIGN OF A BIPED ROBOT JOINT PARALLEL LINKAGE

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Abstract.
This paper shows the design and analysis of a parallel three-dimensional linkage, conceived to work as the ankle and hip joints of an anthropometric biped-robot. This kind of mechanism architecture provides low-weight, highly stable assemblies, and allows the use of actuators synergies. On the other hand, the mechanical transmission ratio is not usually favorable, and a non-linear kinematic model has to be derived and solved. The mechanism proposed here is driven by two rotational servo-actuators, and allows the joint to follow a specified angular trajectory determined by the gait pattern. Namely, the joint linkage can generate dorsi/plantar flexion and inversion/eversion of the ankle, and hip flexion/extension and adduction/abduction movements. Several approaches to the direct and inverse kinematical modeling of the linkage are presented and compared, regarding their accuracy and computational cost, where the last performance parameter is closely related to on-line computer implementing of the controller. Strategies to fit current gait angular amplitudes into the linkage workspace, as well as singularity analysis, are discussed. An optimization method was applied to find some geometrical design parameters of the linkage that minimizes a cost function. This function is the sum over a predefined dominion of the relation between the motor input and the joint output torques. The minimization is constrained to a minimum workspace area value and to minimum and maximum values of the design parameters. Several design solutions were generated. The one where the workspace is compatible to the gait amplitude requirements and exhibits the lowest cost function was chosen. An experimental prototype of the ankle joint was built and tested.

Keywords*: parallel linkage, biped robot, optimal design*

1. Introduction

This paper describes the kinematical modeling and design optimization of a parallel linkage used in a biped robot (Menegaldo et al., 2003). Several mechanism designs can be used to generate the joint movements and moments required to produce a human-like gait (Hirai et al., 1998, Duysens et al., 2002). The linkage must transform the rotations and torques generated by a pair of DC servo-motors into robot's ankle movements, namely foot plantar flexion / dorsiflexion and foot inversion / eversion, as well as hip flexion/extension and adduction/abduction. We have chosen a parallel linkage architecture that has a high stiffness, low weight and reasonable transmission ratio between the servo-motors and the end effector, i.e., the foot. The linkage design for ankle and hip, and the overall view of the robot that is being built, is shown in Figures 1 and 2.

If both bars rotate upward, the foot makes a plantar flexion. If they rotate downward, the foot does a dorsiflexion. If one bar rotates up and the other down, the foot may perform an inversion or an eversion. This class of linkages has an important advantage of using simultaneously the torque of two servo-actuators to perform the same movement. On the other hand, the relation between the servos and the joints angles is not trivial, and requires a kinematical model of the linkage.

Here, a functional relationship between the servo angles and the joint angles is derived and some properties of the kinematic and kinetic behaviour of the linkage are discussed, specially the allowable workspace size and shape and the transmission ratio. An optimization problem is formulated, to find the optimal dimensions of the linkage bars regarding the maximization of the transmission ratio, at the same time that a minimal workspace size works as a optimization constraint. The optimized solutions are compared to normal data from experimental gait analysis. The results, although not exhaustively explored, shows that the design parameters obtained allow the construction of a linkage with a feasible workspace for normal gait. On the other hand, the increasing of the workspace size is related to the increase of the torque transmission ratio cost function.

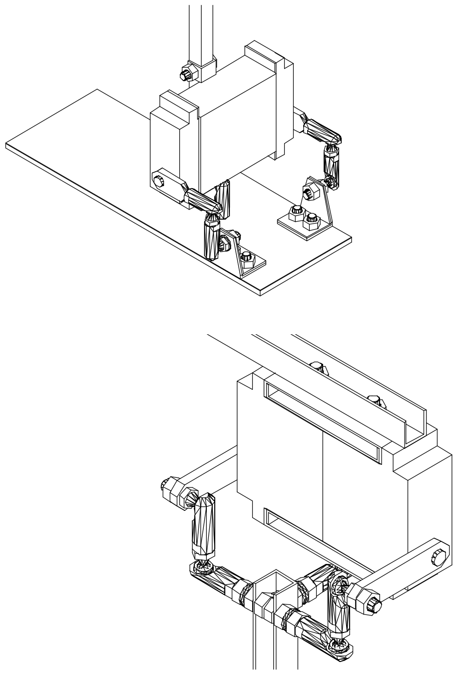


Figure 1: Ankle and Hip linkages

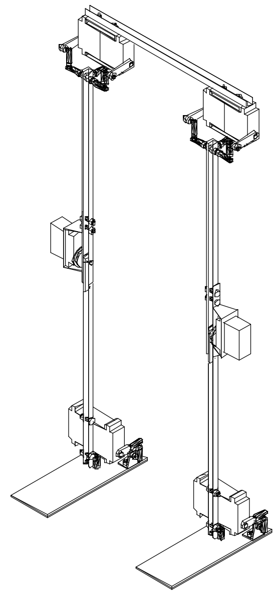


Figure 2: Overall view of the robot

2. Linkage kinematics

The objective associated to the kinematic model is finding the foot angles (inversion/eversion ϕ_1 and ankle plantar flexion/extension ϕ_2) as a function of both servo angles, α_1 and α_2. The ankle reference frame, or System 1 (x_1, y_1, z_1), is fixed to the "shank" shaft, while the System 4 (x_4, y_4, z_4) is fixed to the foot (Figure 3)

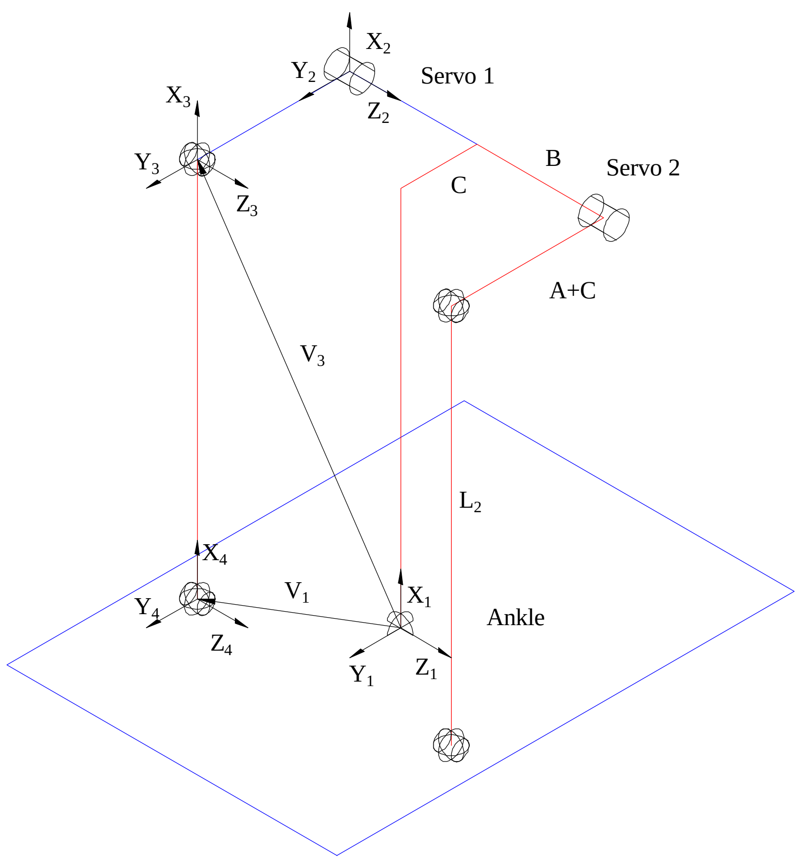


Figure 3: Kinematical model of the linkage used in robot's ankle.

The rotation matrix between both systems can be found considering that the foot flexes the span of the angle ϕ_2 around z_1 and ϕ_1 around y_1. The successive rotations give the transforming matrix:

$$R_1^4 = \begin{bmatrix} \cos(\phi_1)\cdot\cos(\phi_2) & -\cos(\phi_1)\cdot sin(\phi_2) & sin(\phi_1) \\ sin(\phi_2) & \cos(\phi_2) & 0 \\ -sin(\phi_1)\cdot\cos(\phi_2) & sin(\phi_1)\cdot sin(\phi_2) & \cos(\phi_1) \end{bmatrix} \tag{1}$$

The displacement vector v_1, expressed in the foot reference frame, is given by

$$v_1 = R_1^4 \cdot \begin{bmatrix} 0 \\ A \\ -B \end{bmatrix} \tag{2}$$

The coordinates of the System 3 (x_3, y_3, z_3) origin, in the Ankle reference frame, can be expressed by:

$$v_3 = \begin{bmatrix} L_2 \\ 0 \\ -B \end{bmatrix} + R_2^3 \cdot \begin{bmatrix} 0 \\ A \\ 0 \end{bmatrix} \tag{3}$$

where R_2^3 is the rotation matrix between the System 2 and System 3:

$$R_2^3 = \begin{bmatrix} \cos(\alpha_2) & -\sin(\alpha_2) & 0 \\ \sin(\alpha_2) & \cos(\alpha_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The modulus of the difference vector $v_3 - v_1$ is the same as the length of the bar that is located among Systems 3 and 4, namely, L_2. Thus, an equation relating the foot angles ϕ_1 and ϕ_2 with the servo angle α_2 is found doing:

$$\begin{aligned}
&-2\cdot C^2\cdot\cos(a_2)+2\cdot C^2-2\cdot A\cdot L_2\cdot sin(a_2)-2\cdot C\cdot L_2\cdot sin(a_2)-2\cdot A^2\cdot\cos(a_2)\cdot\cos(f_2) \\
&-2\cdot A\cdot C\cdot\cos(a_2)+2\cdot A\cdot C\cdot\cos(f_2)+2\cdot A\cdot C+2\cdot B\cdot L_2\cdot sin(f_1)+L_2{}^2+2\cdot B^2-2\cdot B^2\cdot\cos(f_1) \\
&+2\cdot A\cdot L_2\cdot\cos(f_1)\cdot sin(f_2)+2\cdot A\cdot B\cdot sin(f_1)\cdot sin(f_2)+2\cdot A^2-2\cdot A\cdot C\cdot sin(a_2)\cdot\cos(f_1)\cdot sin(f_2) \\
&-2\cdot C\cdot B\cdot sin(a_2)\cdot sin(f_1)-2\cdot C\cdot A\cdot\cos(a_2)\cdot\cos(f_2)-2\cdot A^2\cdot sin(a_2)\cdot\cos(f_1)\cdot sin(f_2) \\
&-2\cdot A\cdot B\cdot sin(a_2)\cdot sin(f_1)-L_2{}^2=0
\end{aligned} \tag{5}$$

For the opposite kinematic chain and performing the same reasoning, a relationship linking ϕ_1 and ϕ_2 with the other servo angle α_1 is obtained:

$$\begin{aligned}
&-2\cdot C^2\cdot\cos(a_1)+2\cdot C^2-2\cdot A\cdot L_2\cdot sin(a_1)-2\cdot C\cdot L_2\cdot sin(a_1)-2\cdot A^2\cdot\cos(a_1)\cdot\cos(f_2) \\
&-2\cdot A\cdot C\cdot\cos(a_1)+2\cdot A\cdot C\cdot\cos(f_2)+2\cdot A\cdot C-2\cdot B\cdot L_2\cdot sin(f_1)+L_2{}^2+2\cdot B^2-2\cdot B^2\cdot\cos(f_1) \\
&+2\cdot A\cdot L_2\cdot\cos(f_1)\cdot sin(f_2)-2\cdot A\cdot B\cdot sin(f_1)\cdot sin(f_2)+2\cdot A^2-2\cdot A\cdot C\cdot sin(a_1)\cdot\cos(f_1)\cdot sin(f_2) \\
&+2\cdot C\cdot B\cdot sin(a_1)\cdot sin(f_1)-2\cdot C\cdot A\cdot\cos(a_1)\cdot\cos(f_2)-2\cdot A^2\cdot sin(a_1)\cdot\cos(f_1)\cdot sin(f_2) \\
&+2\cdot A\cdot B\cdot sin(a_1)\cdot sin(f_1)-L_2{}^2=0
\end{aligned} \tag{6}$$

The above equations (5) and (6) can be written as:

$$a_1 \cdot sin(\alpha_2) + a_2 \cdot cos(\alpha_2) + a_3 = 0 \qquad (7)$$
$$b_1 \cdot sin(\alpha_1) + b_2 \cdot cos(\alpha_1) + b_3 = 0 \qquad (8)$$
where :

$$a_1 = -2 \cdot A \cdot L_2 - 2 \cdot C \cdot L_2 - 2 \cdot A \cdot C \cdot \cos(f_1) \cdot sin(f_2) - 2 \cdot C \cdot B \cdot sin(f_1) - 2 \cdot A^2 \cdot \cos(f_1) \cdot sin(f_2)$$
$$- 2 \cdot A \cdot B \cdot sin(f_1)$$
$$a_2 = -2 \cdot C^2 - 2 \cdot A^2 \cdot \cos(f_2) - 2 \cdot A \cdot C - 2 \cdot C \cdot A \cdot \cos(f_2)$$
$$a_3 = +2 \cdot C^2 + 2 \cdot A \cdot C \cdot \cos(f_2) + 2 \cdot A \cdot C + 2 \cdot B \cdot L_2 \cdot sin(f_1) + {L_2}^2 + 2 \cdot B^2 - 2 \cdot B^2 \cdot \cos(f_1)$$
$$+ 2 \cdot A \cdot L_2 \cdot \cos(f_1) \cdot sin(f_2) + 2 \cdot A \cdot B \cdot sin(f_1) \cdot sin(f_2) + 2 \cdot A^2 - {L_2}^2$$

$$b_1 = -2 \cdot A \cdot L_2 - 2 \cdot C \cdot L_2 - 2 \cdot A \cdot C \cdot \cos(f_1) \cdot sin(f_2) + 2 \cdot C \cdot B \cdot sin(f_1) - 2 \cdot A^2 \cdot \cos(f_1) \cdot sin(f_2)$$
$$+ 2 \cdot A \cdot B \cdot sin(f_1)$$
$$b_2 = -2 \cdot C^2 - 2 \cdot A^2 \cdot \cos(f_2) - 2 \cdot A \cdot C - 2 \cdot C \cdot A \cdot \cos(f_2)$$
$$b_3 = +2 \cdot C^2 + 2 \cdot A \cdot C \cdot \cos(f_2) + 2 \cdot A \cdot C - 2 \cdot B \cdot L_2 \cdot sin(f_1) + {L_2}^2 + 2 \cdot B^2 - 2 \cdot B^2 \cdot \cos(f_1)$$
$$+ 2 \cdot A \cdot L_2 \cdot \cos(f_1) \cdot sin(f_2) - 2 \cdot A \cdot B \cdot sin(f_1) \cdot sin(f_2) + 2 \cdot A^2 - {L_2}^2$$

The above equations can be solved considering for $e_{1,2,3} = a_{1,2,3}$ or $e_{1,2,3} = b_{1,2,3}$:

$$e_1 \cdot sin(\alpha_{1,2}) + e_2 \cdot cos(\alpha_{1,2}) + e_3 = 0$$

writing:

$$sin(\alpha_{1,2}) = \frac{2 \cdot t}{1+t^2} \quad cos(\alpha_{1,2}) = \frac{1-t^2}{1+t^2} \quad tg\left(\frac{\alpha_{1,2}}{2}\right) = t : \qquad (9)$$

the servo angles $\alpha_{1,2}$ can be calculated solving Equation (9) with the *t* parameter given by:

$$(e_3 - e_2) \cdot t^2 + 2 \cdot e_1 \cdot t + (e_2 + e_3) = 0 \Rightarrow$$
$$t = \frac{-e_1 \pm \sqrt{e_1^2 - (e_3 - e_2) \cdot (e_3 + e_2)}}{(e_3 - e_2)} \qquad (10)$$

Only one of the solutions of the above equation is physically realizable. The solution (7) - (10) of equations (5) and (6) is suitable for control proposes, as the foot angle is controlled through the servo angles. On the other hand, if the servo angles are known and the foot angles must be found, e.g., for generating a computer graphical animation, or to provide information for a hierarchical controller, both equations (5) and (6) shall be solved simultaneously, as an algebraic non-linear system. The system was solved with minimal-squares (*Matlab* fsolve function) and Newton methods, and the results are very similar. A third alternative solution, that gives a fast answer appropriate for real-time applications, uses interpolating polynomials, as, for example:

$$\phi_1 = a_1 + a_2 \cdot \alpha_1 + a_3 \cdot \alpha_2 + a_4 \cdot {\alpha_1}^2 + a_5 \cdot {\alpha_2}^2 + a_6 \cdot \alpha_2 \cdot \alpha_1$$
$$\phi_2 = b_1 + b_2 \cdot \alpha_1 + b_3 \cdot \alpha_2 + b_4 \cdot {\alpha_1}^2 + b_5 \cdot {\alpha_2}^2 + b_6 \cdot \alpha_2 \cdot \alpha_1 \qquad (11)$$

Through varying the foot angles $-15^o \le \phi_1 \le 15^o \quad and \quad -15^o \le \phi_2 \le 15^o$, with 0.5^o step-size, the servo $\alpha_{1,2}$ angles are found by solving the equations (7) to (10). Collecting the series of *n* solutions in matrix-form:

$$\underbrace{\begin{bmatrix} 1 & \alpha_1(1,1) & \alpha_2(1,2) & \alpha_1(1,1)^2 & \alpha_2(1,2)^2 & \alpha_2(1,2)\cdot\alpha_1(1,1) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & \alpha_1(n,1) & \alpha_2(n,2) & \alpha_1(n,1)^2 & \alpha_2(n,2)^2 & \alpha_2(n,2)\cdot\alpha_1(n,1) \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \phi_1(1) \\ . \\ . \\ . \\ . \\ \phi_1(n) \end{bmatrix}}_{B} \qquad (12)$$

and solving the non-square linear system trough:

$$x = A^{+}B \qquad (13)$$

where + denotes the pseudo-inverse matrix.

The same procedure is used to find the *b* coefficients to fit the flexion/extension angle as a function of the servo angles, i.e., the second of the equations (11). The three solutions were implemented and the results were very similar. As expected, the third method gives much faster results when compared to *matlab* fsolve and Newton methods. The same analysis is performed to find the kinematical model of the hip. In this case, the “foot” is considered as the “pelvis” of the robot.

4. Workspace analysis

The proposed linkage must achieve the joint angle limits usually observed in normal gait. Nevertheless, the dimensions of the linkage constitutive elements strongly influence the shape and the size of the workspace. One of the constraints is that the solution of equations (5) and (6) must have only real parts. Varying the ankle angles in the interval for:

$$-90^o \le \phi_1 \le 90^o \ \textit{and} \ -90^o \le \phi_2 \le 90^o, \quad \text{with } 1^o \, step-size$$

one can find the domain region where both α_1 and α_2 are real. Figure 4 was generated with the dimensions of a linkage prototype: A=55.3 mm, B=68.4 mm and L2=34.2 mm.

On the other hand, the spherical joints used to assembly the mechanism (Termicom S. A., São Paulo, Brazil) allows only a limited amount of angular displacement, namely $\pm 21^{\circ}$. The angular displacements generated in the four spherical joints were calculated, spanning robot joints angular limits:

$$-25^o \le \phi_1 \le 25^o \ \textit{and} \ -25^o \le \phi_2 \le 25^o \quad \text{with } 0.5^o \, step-size$$

If the values of the angles of the four joints were inside the allowable limits, the solution was considered as valid. In Figure 4, the white region corresponds to the solutions obtained from the first test (only real solutions), and the darker from the second (spherical joint limits).

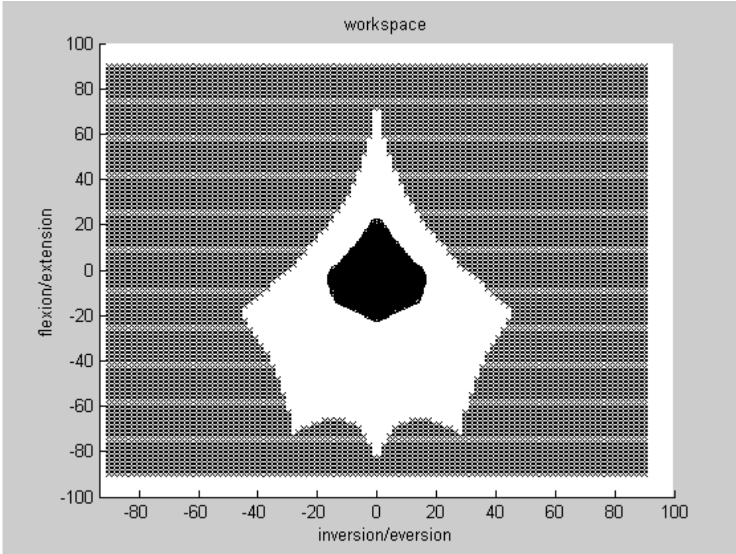


Figure 4: Workspace for a particular set of linkage dimensions. The lightest region corresponds to the real solutions of the linkage kinematic model, and the dark the workspace where the spherical joints are inside their angular displacement limits.

5. Linkage optimal design

The objective of the linkage optimal design is to find the best arrangement of A, B, C and L2 parameters such that the torque is maximally transmitted between the servos and the foot. These parameters must not exceed maximum mechanical construction limits. The optimization problem can be formulated as minimizing the cost function η:

$$\eta = \frac{\left(\sum_{i=1}^{n} \lambda_i\right)}{n} \tag{14}$$

If the solution of the kinematical model is real and the relative angles of the bars linked to the spherical joints do not exceed the constructive constraints (see Sec. 4 above) the parameter λ is computed for this particular point:

$$\lambda_i = \frac{\sqrt{T_{M1_i}{}^2 + T_{M2_i}{}^2}}{\sqrt{T_a^t \cdot T_a}} \tag{15}$$

n is the number of points of the discretized domain for ϕ_1 and ϕ_2 which λ_i was evaluated. T_{M1} and T_{M2} are the torques delivered by the servo-actuators and Ta is a vector of the torques applied in foot or pelvis Ta = [Tϕ_1, Tϕ_2].

The limits for this domain were fixed inside the interval [-25^o, 25^o]. To evaluate λ, Equations (5) and (6) must be differentiated in relation to time (the first equation is referred as eq_1 and the second eq_2).

$$\begin{aligned}
\frac{deq_1}{dt} = 0 \Rightarrow \frac{deq_1}{d\phi_1}\cdot\frac{d\phi_1}{dt} + \frac{deq_1}{d\phi_2}\cdot\frac{d\phi_2}{dt} + \frac{deq_1}{d\alpha_1}\cdot\frac{d\alpha_1}{dt} + \frac{deq_1}{d\alpha_2}\cdot\frac{d\alpha_2}{dt} = 0 \\
\frac{deq_2}{dt} = 0 \Rightarrow \frac{deq_2}{d\phi_1}\cdot\frac{d\phi_1}{dt} + \frac{deq_2}{d\phi_2}\cdot\frac{d\phi_2}{dt} + \frac{deq_2}{d\alpha_1}\cdot\frac{d\alpha_1}{dt} + \frac{deq_2}{d\alpha_2}\cdot\frac{d\alpha_2}{dt} = 0
\end{aligned} \tag{16}$$

By defining the Jacobian matrixes Jx anb Jq and the vectors $\dot{\phi} = [\dot{\phi}_1 \quad \dot{\phi}_2]^T$ and $\dot{\alpha} = [\dot{\alpha}_1 \quad \dot{\alpha}_2]^T$

$$J_x = \begin{bmatrix} \frac{deq_1}{d\phi_1} & \frac{deq_1}{d\phi_2} \\ \frac{deq_2}{d\phi_1} & \frac{deq_2}{d\phi_2} \end{bmatrix} \qquad J_q = \begin{bmatrix} \frac{deq_1}{d\alpha_1} & \frac{deq_1}{d\alpha_2} \\ \frac{deq_2}{d\alpha_1} & \frac{deq_2}{d\alpha_2} \end{bmatrix} \tag{17}$$

the following relation is found:

$$J_x \dot{\phi} = J_q \dot{\alpha} \quad \rightarrow \quad \dot{\phi} = \left(J_x^{-1} J_q \right) \dot{\alpha} \tag{18}$$

It can be shown (Craig, 1986) that for the torques, the above equation can be written as:

$$\begin{bmatrix} T_{M1} \\ T_{M2} \end{bmatrix} = \left(J_x^{-1} J_q \right)^T \begin{bmatrix} T_a \\ T_a \end{bmatrix} \tag{19}$$

For all tests, unitary torque values were considered for Ta. In addition, the minimization of Equation (14), evaluated from Eqs. (14) and (19), is constrained to minimum and maximum values of the design parameters as well as of the Workspace (W):

$$\begin{aligned} &\min_{A,C,L_2} \quad \eta \\ &\text{such that:} \quad W \geq W_{min} \\ &\qquad\qquad\quad L_{2\,min} \leq L_2 \leq L_{2\,max} \\ &\qquad\qquad\quad C_{min} \leq C \leq C_{max} \\ &\qquad\qquad\quad A_{max} \leq A \leq A_{max} \end{aligned}$$

The Workspace (W) was evaluated as ratio between the total number of points of the discretized domain (m) and the number of valid points, n.

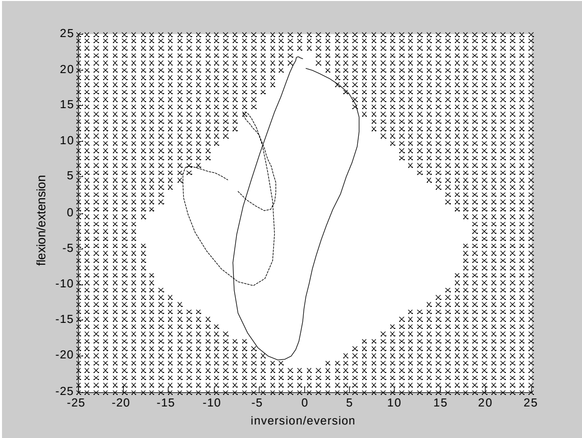
6. Results

Three solutions has been investigated for the linkage optimal design. In the first, the dimensional parameters of a linkage prototype (Menegaldo et al., 2003) were used to find reference values for W and η. In the second and third approach, the first one was used as the initial guess for the optimization routine (*Matlab* fmincon). In the second case, the workspace W was fixed in W=0.3 and in the third, W=0.25. The optimized parameters as well as the values of the objective function η and the workspace W are shown in Table 1. The B parameter was fixed in 68.4 mm for all simulations.

Table 1: Linkage bar dimensions, workspace size and cost function of the optimization problem. Parameters for case 1were found heuristically, and used to build the first linkage prototype.

Solution	**A (mm)**	**C (mm)**	**L2 (mm)**	**W**	**η**
1	60	0	34	0.226	0.659
2	60	3	62	0.25	0.691
3	60	11.2	70	0.3	0.784

In order to verify if the obtained linkage designs are able to reach the joint angles usually observed in normal human gait, real data from gait lab analysis (from Normative Database of AACD – Associação de Assistência à Criança Defeituosa, São Paulo, Brazil) were plotted over the workspace diagram of the solutions. The workspaces obtained for solutions 2 and 3 (white area), superposed to the real gait trajectories are shown in Figure 5.



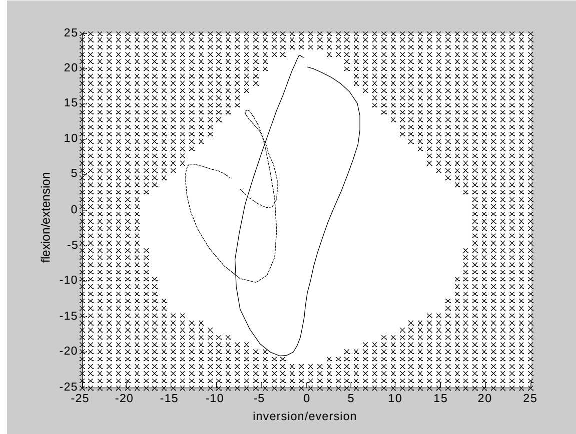


Figure 5: Workspace and gait trajectories for Solution 2 (left) and Solution 3 (right). The continuous line is the hip angular displacements in one gait cycle, and the dashed line represents ankle angular displacements.

7. Discussion

The value of the objective function for the built prototype (Solution 1) has shown that the torque transmission ratio relationship for this particular solution is good, when compared to the optimized solutions. However, Solution 1 is not able to reach all the workspace points required for normal gait. The optimization approach has shown that the workspace size can be controlled, by defining the constraint W_{min}. However, the increasing of the workspace size has led to a less-efficient solution from the torque transmission ratio point of view, as can be observed in Table 1.

Solution 2 is acceptable, although gait trajectory trespass the allowable limits of linkage operation in some few regions. If this small reduction in joint amplitude is acceptable for some particular robot application, the geometry obtained from Solution 2 should be used. On the other hand, if gait trajectory must be followed in all the span of the real gait data used, Solution 3 should be the most appropriate choice.

The cost function η is an estimator of the overall transmission ratio of the linkage. Actually, this ratio varies along the workspace, but the estimator is an indicative design index. A prototype with design dimensions obtained from both solutions are being built to be used in a robot. The authors expect that experimental tests in real robot may show if the design optimization procedure, described in this paper, can lead to adequate robot joint designs, from both transmission ratio and workspace points of view.

8. Acknowledgements

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10. Responsibility notice

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