Proceedings of the XI DINAME, 28th February-4th March, 2005 - Ouro Preto - MG - Brazil
Edited by D.A. Rade and V. Steffen Jr.

TIME-VARYING CONTROLLER FOR CONSTRAINED-LAYER DAMPING BEAM SUBJECTED TO TEMPERATURE VARIATION

Luciano Afonso da Silva
EMBRAER – Empresa Brasileira de Aeronâutica, São José dos Campos, SP, Brazil
Av. Brigadeiro Faria Lima, 2170, Putim, 12227-901
luciano.afonso@embraer.com.br

Valder Steffen, Jr
Faculty of Mechanical Engineering, Federal University of Uberlândia, MG, Brazil
Av. João Naves de Ávila, 2160 - Campus Santa Mônica - Bloco 1M, 38400-902
vsteffen@mecanica.ufu.br

Abstract. *Temperature variations could be a major concern for viscoelastic structures upon active control. It is well known that the material modulus and loss factor vary with temperature. However, investigation of such behavior has not been fully studied in order to improve the performance of controllers. Many applications in industry are at the material level only, i.e., specific materials for specific environments with controllers that are not designed to account for degradation in performance due to temperature shifts. Though this is the natural choice for control, there are situations in which the structure might be subjected to sudden or even slow changes of temperature that lead to changes in the material properties. This paper is devoted to the investigation of a time-varying controller that takes into account the internal modification of the viscoelastic material and uses this information to change the controller gain adaptively as the temperature also changes. This type of controller was applied before for lumped-parameter systems with internal variables. It is now extended to the case in which a finite element model of a constrained-layer damping beam subjected to transverse vibration is concerned. The main focus is on the power requirement as compared to a time-invariant controller. Two cases are tested, namely, time response tracking and regulation. The results show that the time-varying controller is able to simultaneously reduce the vibration and reject the temperature disturbance. The robustness of the controller, when subjected to conditions usually found in real life applications, is also tested and the simulations show a substantial advantage of the time-variant controller.*

Keywords: *temperature, constrained-layer damping, time-varying controller*

1. Introduction

Under- and over prediction of damping can lead to instabilities or extra control input sent to the system, (Banks and Inman, 1992). Problems like these, however, are not exclusively the result of a bad prediction. The result of changes in the properties of certain materials due to temperature variations can also lead to a similar behavior. Such is the case for structures with viscoelastic materials, which under the influence of temperature have their modal properties affected.

Over the past decades, many authors have succeeded in the attempt of finding ways to investigate and correctly describe such temperature influence, but only at the material level. Some authors that have looked more generally into the response of the structure were Lesieutre and Govindswamy (1996), who developed a model compatible with finite element modeling, and provided a more practical model for time response simulations. This model was used to predict the local temperature increase in a viscoelastic material due to self-heating during deformation. It was later extended to construct a nonlinear amplitude-dependent model. Their model is similar to the one proposed by Schapery, which is about a unified theory of the thermodynamical behavior of viscoelastic media based on the study of thermodynamics of irreversible processes (Schapery, 1964). One of the main differences is that his model did not consider motion under dynamic conditions. Lukasiewicz and Xi (1993) extended a nonlinear model to include a temperature dependency based on curves given by Nashif *et. al.* (1995). They assumed that the temperature varied linearly with time, but their model did not account for frequency dependence. However, none of these works investigated any control application.

Recently (Silva, 2003), the method of reduced variables, also known as the time-temperature superposition principle was used to incorporate the temperature dependence in models with viscoelastic material. With the aid of the temperature shift factor it was then possible to have a single variable to simulate the behavior of the material at different temperatures. The lumped parameter model was used for control analysis, and because the system become time-variant, it was necessary to find a controller that could take into account the time variation of the parameters (Silva, 2003). Now, a similar analysis is performed for a constrained-layer damping model. The main idea is to compare the time varying controller with a controller suitable for real-time application. The main focus is on the requirement of both designs in terms of power requirement, the ability to overcome the temperature influence, and the robustness when subjected to conditions usually found in real life applications.

2. Characteristics of the Simulated System

The structure in study is a clamped-free beam with a total length of 0.34 m, Fig. 1. The beam has attached to its bottom two identical piezo patches of length 0.036 m. The first one, located at 0.01 m from the clamped side, is used to input the control signal. The second, located at 0.054 m from the clamped side, is used to input the disturbance signal. The beam is partially covered with a constrained layer damping, also located 0.01 m from the clamped side. The total length of this treatment is 0.08 m. The thickness of the base beam, the constraining layer, the viscoelastic and piezoelectric layers are respectively 3.21 mm, 1.21 mm, 0.127 mm and 0.254 mm. The width of all parts is 25.4 mm. For simulation purposes, the output was sensed at the tip of the beam for all cases that follow.

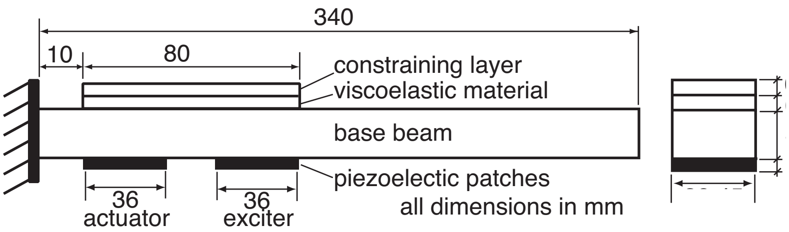

Figure 1. Model used in the simulations.

The physical properties of the structure are given in Table 1, where E is the initial depolarization field and d_{31} is the piezoelectric strain coefficient.

Table 1. Physical properties of the structure

Base beam / CL	VEM (ISD112 – 3M)	PZT
$\rho_b = \rho_c = 2700\,Kg/m^3$	$\rho_v = 1600\,Kg/m^3$	$\rho_p = 7800\,Kg/m^3$
$E_b = E_c = 69\times10^9\ Pa$	$E_v = 14\times10^6\ Pa$ (Relaxed modulus)	$E_p = 66\times10^9\ Pa$
		$d_{31} = -190\times10^{-12}\ m/V$
		$E = 5\times10^5\ V/m$

A full description of the finite element model can be found in Silva (2003). The final format of the matrices, including ϑ internal variables is

$$\begin{bmatrix} M &&&& \\ & \mathrm{I} &&& \\ && a_T C_1^{ev} && \\ &&& \ddots & \\ &&&& a_T C_\vartheta^{ev} \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \dot{q} \\ \dot{q}_1 \\ \vdots \\ \dot{q}_\vartheta \end{Bmatrix} + \begin{bmatrix} 0 & K & -K_1^{vd} & \cdots & -K_\vartheta^{vd} \\ -\mathrm{I} & 0 & 0 & \cdots & 0 \\ 0 & -K_1^{evd} & K_1^{ev} & & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -K_\vartheta^{evd} & 0 & \cdots & K_\vartheta^{ev} \end{bmatrix} \begin{Bmatrix} \dot{q} \\ q \\ q_1 \\ \vdots \\ q_\vartheta \end{Bmatrix} = \begin{Bmatrix} f_p \\ 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} V(t) \tag{1}$$

$$M = M^b + M^c + M^v + M^p \tag{2}$$
$$K = K^b + K^c + K^v + K^p \tag{3}$$

Writing in the short form

$$[\mathbf{M}]\{\dot{\bar{x}}(t)\} + [\mathbf{K}]\{\bar{x}(t)\} = \{\mathbf{F}\}V(t) \tag{4}$$

the state-space formulation can be written as

$$\begin{aligned} \dot{\bar{x}} &= \mathbf{A}\bar{x} + \mathbf{B}V(t) \\ y &= \mathbf{C}\bar{x} + \mathbf{D}V(t) \end{aligned} \tag{5}$$

where $\mathbf{A} = -[\mathbf{M}]^{-1}[\mathbf{K}]$ and $\mathbf{B} = [\mathbf{M}]^{-1}\mathbf{F}$. The $\mathbf{C}$ matrix has entries relating the states with the output, and $\mathbf{D} = 0$.

The so called *temperature shift function,* a_T , is the variable responsible for the introduction of the temperature dependence into the model (Ferry, 1980). With this function, the material behavior at many temperatures can be easily simulated and incorporated into the model.

It becomes possible, with this model, to analyze the behavior of the system at different temperatures, an important task to decide the type of controller to be used. Figure 2 shows the frequency response function at two temperatures. It is easily noticeable that the temperature change causes a much higher variation on damping than on the natural frequencies. Such behavior has the advantage that the change in the time reponse amplitude is a consequence of the changes in the modal damping only. So, if an unknown disturbance is exciting around the first mode at 4.4 ^oC, then the same disturbance will also excite around the first mode at 37.7 ^oC. That was not the case in a previous work (Silva et. al. 2004), in which changing the temperature caused the natural frequency under analysis to shift away from the excitation. This fenomena, the natural frequency shifitting, if left unnoticed will lead to the wrong conclusion that the modal damping variation is the main factor affecting the reponse of the system.

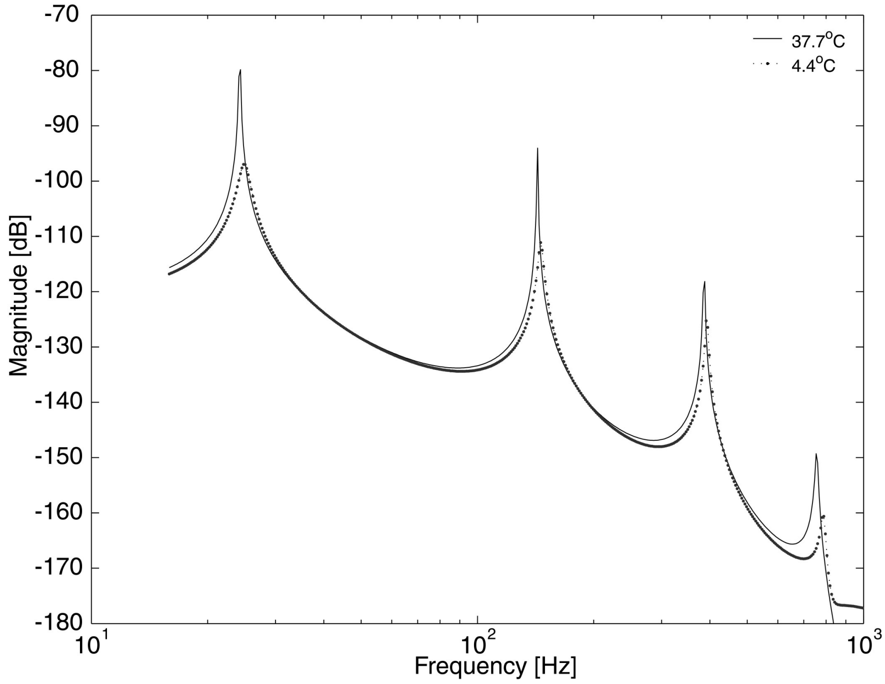

Figure 2. Frequency response function at 4.4 ^oC and 37.7 ^oC.

Figure 3 shows the modal damping variation for the first four modes as the temperature increases from 0 ^oC to 50 ^oC. The first mode, being the one with higher magnitude, will be subjected to control, since the controller, as will be seen, is designed to tackle one mode only. Notice that the modal damping of the mode under analysis always goes down as the temperature goes up. Therefore, the main effect to be investigated in the next sections is the ability of the controller to overcome the loss imparted to the damping of the system.

3. Time-Varying Pole Placement Control (TVPPC)

To access the dynamic variations due to changes on the viscoelastic material, a controller design, based on the pole placement technique, is used. This technique is very useful for time-varying models, since it carries out information about the dynamics of the problem. The drawback is that it might get hard to write down the equations as the order of the model to be controlled increases. In general, the order of the model is kept to two or three; otherwise it would not be straightforward to include the dynamics of the system into the controller parameters.

The advantage is that this method does not require a minimum phase plant, like the model reference adaptive control, which can be adapted for time-varying systems. Another point is the non-requirement for state estimation, since this is an output-feedback based controller. The output alone does not contain enough information about the system, so the plant dynamics must be included in the controller design. To demonstrate the concept, we analyze a second-order system represented by a single mode in the modal domain. For such a low-order model, the solution of the controller parameter is straightforward

The time-varying space-state representation of the plant, in the modal domain, is given by (Silva, 2003 and Meirovitch, 1990)

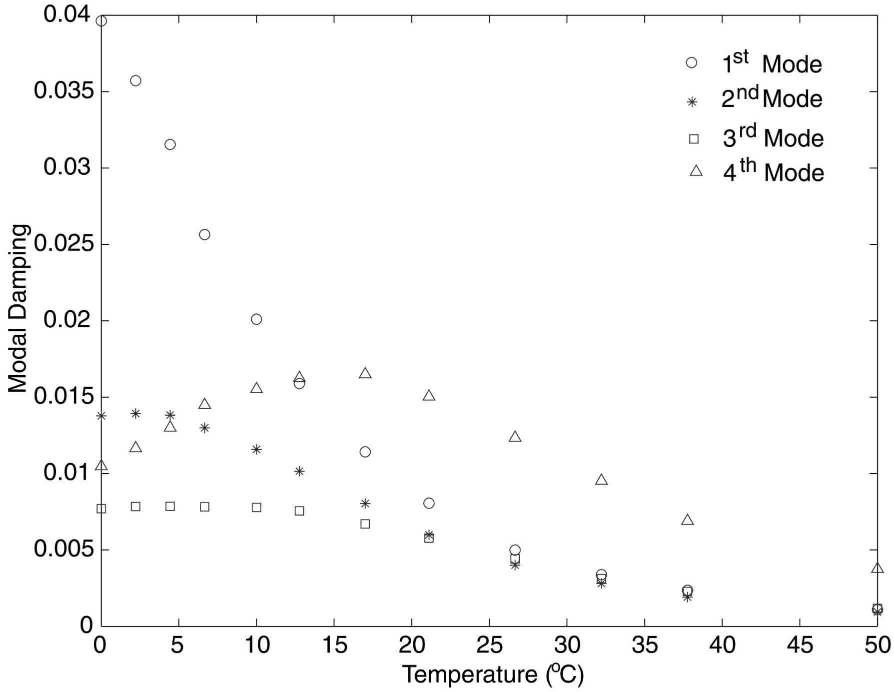

Figure 3. Modal damping versus temperature.

.

$$\dot{x}_p = \begin{bmatrix} a_r(t)\Re & a_i(t)\Im \\ -a_i(t)\Im & a_r(t)\Re \end{bmatrix} x_p + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u_p$$
$$y_p = \begin{bmatrix} c_1 & c_2 \end{bmatrix} x_p, \tag{6}$$

where $\Re$ and $\Im$ are the real and imaginary parts of the targeted eigenvalue, respectively. In (Silva, 2003), the variation of the eigenvalues with temperature was analyzed. As for the lumped-parameter model, we propose to explore this variation by creating time-varying functions $a_r(t)$ and $a_i(t)$ to modify the eigenvalues of the system due to temperature changes. It is assumed that temperature will not affect the coefficients of the input and output matrices, which is a valid assumption when the number of degrees of freedom in the model is much higher than the number of the modes analyzed.

As in (Tsakalis and Ioannou, 1993), we are looking for a control signal such that the state-space realization of plant/controller is illustrated as in Fig. 4. The objective of the TVPPC is to determine a control input u_p such that the closed-loop plant is stable with closed-loop poles equal to the poles of a prescribed monic polynomial representing the desired closed-loop poles.

The control input, Eq. (7), is given by combining the plant input and output through controller parameters θ_i, which are time variant and carries information about the dynamics of the model.

$$u_p = q^T(sI-F)^{-1}\theta_1 u_p + q^T(sI-F)^{-1}\theta_2 u_p + \theta_3 y_p + v = g^T w + v \tag{7}$$

The equations of the controller parameters are the same for any second order system, therefore their development are not shown here again. They are given by

$$\theta_1(t) = a_1(t) - F - A_1 - b_0\theta_3(t)$$
$$\theta_2(t) = \frac{2\dot{a}_1(t) + a_2(t) - a_1^2(t) + A_1a_1(t) + b_0a_1(t)\theta_3(t) - 2b_0\dot{\theta}_3(t) + b_0F\theta_3(t) - A_2}{b_0} \tag{8}$$
$$\theta_3(t) = \frac{\bar{\theta}_{31}(t) + \bar{\theta}_{32}(t)}{b_0(t)a_2(t) - b_1(t)a_1(t) + b_1^2(t)/b_0(t)}$$

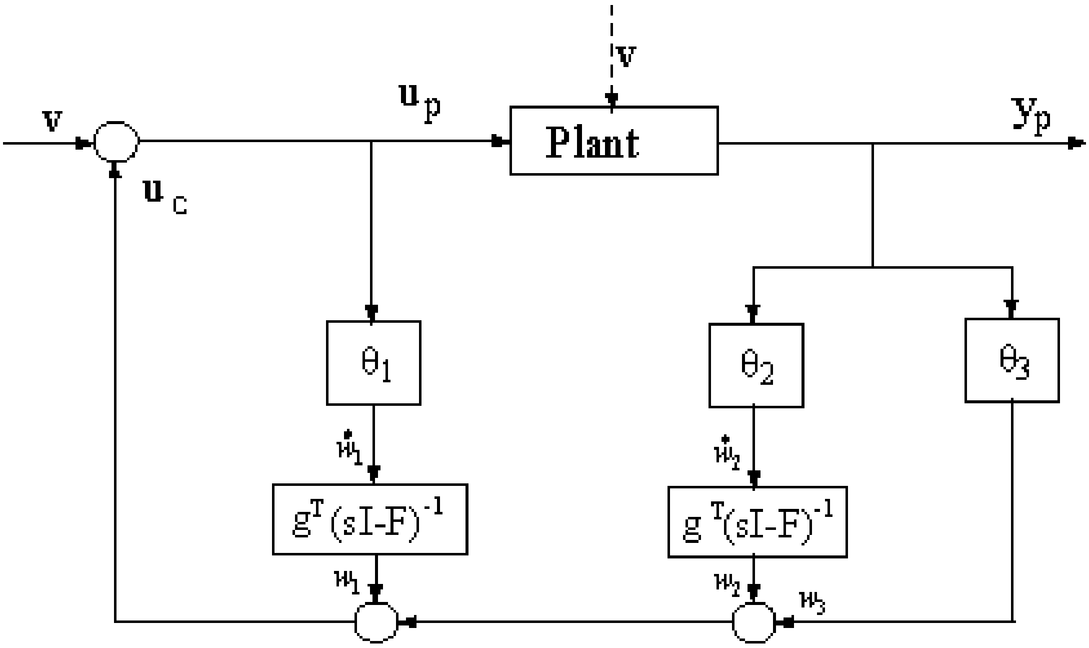

Figure 4. State-space realization of TVPPC; v as a command signal (solid line) or as an unknown disturbance to the plant (dashed line).

with

$$a_1(t) = -2a_r(t)\Re \tag{9.1}$$
$$a_2(t) = [a_r(t)\Re]^2 + [a_i(t)\Im]^2 \tag{9.2}$$
$$b_0(t) = b_1c_1 + b_2c_2 = b_0 \tag{9.3}$$
$$b_1(t) = a_i(t)[b_2c_1 - b_1c_2]\Im - a_r(t)b_0\Re \tag{9.4}$$

and

$$\begin{aligned}\bar{\theta}_{31}(t) &= A_3 + (b_1(t)a_1(t)/b_0 - a_2(t))A_1 - b_1(t)A_2/b_0 - 2b_1(t)\dot{\theta}_3(t) + b_0\dot{\theta}_3a_1(t) - b_0\ddot{\theta}_3(t) + \ddot{a}_1(t) \\ \bar{\theta}_{32}(t) &= b_1(t)a_2(t)/b_0 + a_1(t)a_2(t) - b_1(t)a_1^2(t)/b_0 + (a_1(t) + 2b_1(t)/b_0)\dot{a}_1\end{aligned} \tag{10}$$

The remaining parameters are the coefficients of the third order polynomial, $(s^3 + A_1s^2 + A_2s + A_3)$, that will deliver the desired closed-loop poles. The auxiliary equations are

$$\dot{w}_1 = Fw_1 + \theta_1u_p \tag{11.1}$$
$$\dot{w}_2 = Fw_2 + \theta_2y_p \tag{11.2}$$
$$w_3 = \theta_3y_p \tag{11.3}$$

and $F \ \left(\in R^{(n-1)\times(n-1)}\right)$ is any stable matrix (which will be a scalar in our case), and $g = \begin{bmatrix}q^T & q^T & 1\end{bmatrix}^T$ is a constant vector such that the pair (g^T, F) is observable.

4. Simulation Results

In order to show the performance of the TVPPC design, three sets of simulations were performed and compared with another control design. The controller known as positive position feedback (PPF) (McEver, 1999) was chosen as the baseline for this comparison. Since the temperature influence on the modal damping is the main factor under investigation, a temperature range was chosen so as to verify under what conditions a time-varying controller is better than a time-invariant one.

It has been shown that a time-invariant controller such as PPF can lack its performance and even lead to instability because of the temperature effect (Silva, 2003). There are cases however, in which the temperature range under analysis is not wide enough to show such behavior. In this case, the one considered here, the PPF controller may have a good performance, but not as good as a controller that takes into account the time variation of the system parameters.

4.1. Analysis at constant temperature

The first analysis is made at $4.4^\circ C$, and the system is disturbed at its first natural frequency (24.4 Hz). The goal of the TVPPC was to increase the modal damping to 14.52% by placing the pole at –22.133-150.74i. The parameters of the PPF filter that best approximate the TVPPC performance were: $\omega_f = 34.16\,Hz;\ \ \varsigma_f = 0.0125;\ \ gain = 7.5\times10^4$. The $a_r(t)$and $a_i(t)$ time-varying functions were extract directly from the variation of the pole with temperature, i.e., the real and imaginary parts of the targeted pole is plotted as a function of temperature, or shift factor, and the resulting curve is fitted to a polynomial. In this simulation, a 5th order polynomial was used. These functions are:

$$a_r(t)=\begin{Bmatrix}1.6927\times10^{-5}\\ -6.8060\times10^{-4}\\ 1.0657\times10^{-2}\\ -8.3776\times10^{-2}\\ 3.5628\times10^{-1}\\ -8.5006\times10^{-1}\\ 1.4971\\ 6.9074\times10^{-2}\end{Bmatrix}^T\begin{Bmatrix}a_T^7\\ a_T^6\\ a_T^5\\ a_T^4\\ a_T^3\\ a_T^2\\ a_T\\ 1\end{Bmatrix},\qquad a_i(t)=\begin{Bmatrix}3.0157\times10^{-8}\\ -1.2286\times10^{-6}\\ 2.0110\times10^{-5}\\ -1.7560\times10^{-4}\\ 9.0801\times10^{-4}\\ -2.8866\times10^{-3}\\ 9.0292\times10^{-3}\\ 9.9313\times10^{-1}\end{Bmatrix}^T\begin{Bmatrix}a_T^7\\ a_T^6\\ a_T^5\\ a_T^4\\ a_T^3\\ a_T^2\\ a_T\\ 1\end{Bmatrix} \tag{12}$$

Both techniques gave satisfactory results, Fig. 5, with the TVPPC being able to place the pole at the desired location. The TVPPC controller has a steadier response, while the PPF, being a filter, exhibits its best performance after the system reaches a steady state vibration.

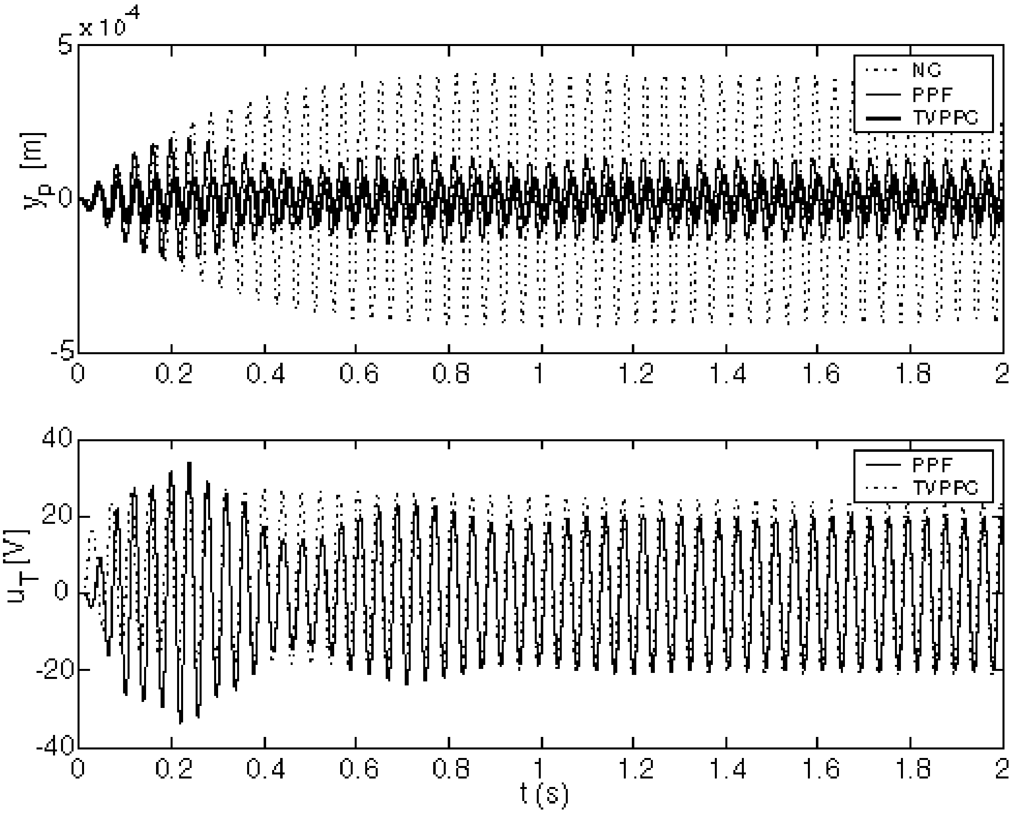

Figure 5. Time response of the system without control and controlled; at $4.4^\circ C$. NC: control off.

An important aspect that might be present during a control task is the delay factor. In a real application, the control is never turned on at time t=0s. Usually, it takes some time to detect the vibration of the system, and only then the controller is automatically turned on. There might be even the case in which the controller is only necessary after the vibration of the system has reached a specified level. If some vibration level is allowed, then this is a good way to save some energy. A category that falls into that is when the controller brain fails for a fraction of seconds. In this case it is necessary to verify if the controller will be able to bring the vibration to an acceptable level after the "failure effect" has ceased.

To simulate these situations, we let the system under the influence of the disturbance and turn the control on after 0.2s (delay simulation). The control is turned off at 0.6s and is back again at 1s (failure effect). This would represent the situation in which some control energy is saved., being the case where the vibration level during that period is acceptable.

As shown in Fig. 6, the TVPPC had a faster response to the control input, being able to bring the vibration level to the desired position. This is done at the cost of a higher control input at the moment the control is turned on. This characteristic is explained by the fact that the TVPPC is forced to push the poles to the desired location.

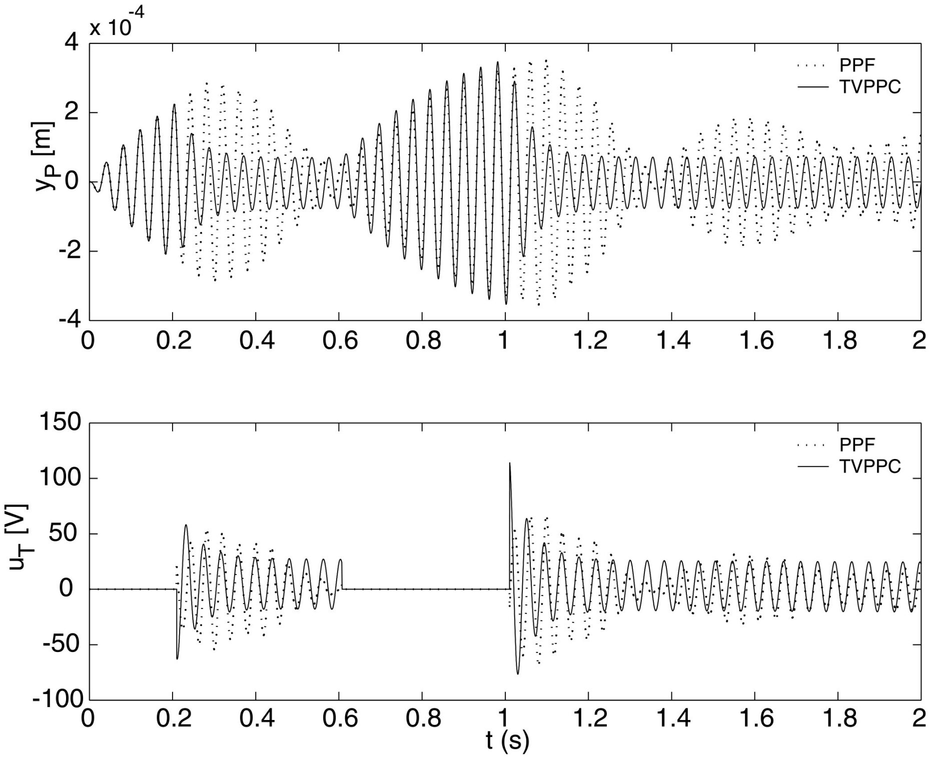

Figure 6. Failure effect / delay simulation of the controller, with saturation, at constant temperature.

4.2. Analysis at variable temperature

In this section, the temperature profile is such that it increases linearly from 4.4^o C at 0.6s to 37.7^o C at 1.2s. The TVPPC performance is still better, Fig. 7. Obviously, different PPF parameters could result in a better performance, although such combination was not found. Despite the amount of reduction imparted by each controller, both were insensitive to the temperature variation. As mentioned before, the good performance of the PPF for this time-variant system is a particular case. Usually, there is a tendency of degradation in the response if the controller is designed to work on an specific condition and the system is subjected to another one (Banks and Inman, 1992, and Silva, 2003).

Again the failure and delay effect are investigated. The result is shown in Fig. 8. Note that the TVPPC, as before, respond faster to the control command, and it brings the structure to the desired vibration level in about 0.1s. We can see that as the vibration level gets higher, due to the temperature variation, the PPF performance degrades after the control is turned on again at 1s. Therefore, if the delay is too long and if under effect of the temperature, only the time-variant controller will lead to an acceptable performance.

Because the temperature is affecting damping in such a way that the system response increases with time, it is reasonable that the control demand at 1s would be much higher than at 0.2s. In practical situations in which the supplied voltage to the piezo patch is limited, that demand may not be achieved. There are two main reasons we may want to limit this voltage: one is due to the capacity of the material, and the other is just the economic factor. The simulation in Fig. 8 is repeated in Fig. 9, but now the voltage supplied to the piezo patch is saturated at 60V, which is about the limit for the PZT with the thickness given in the present work

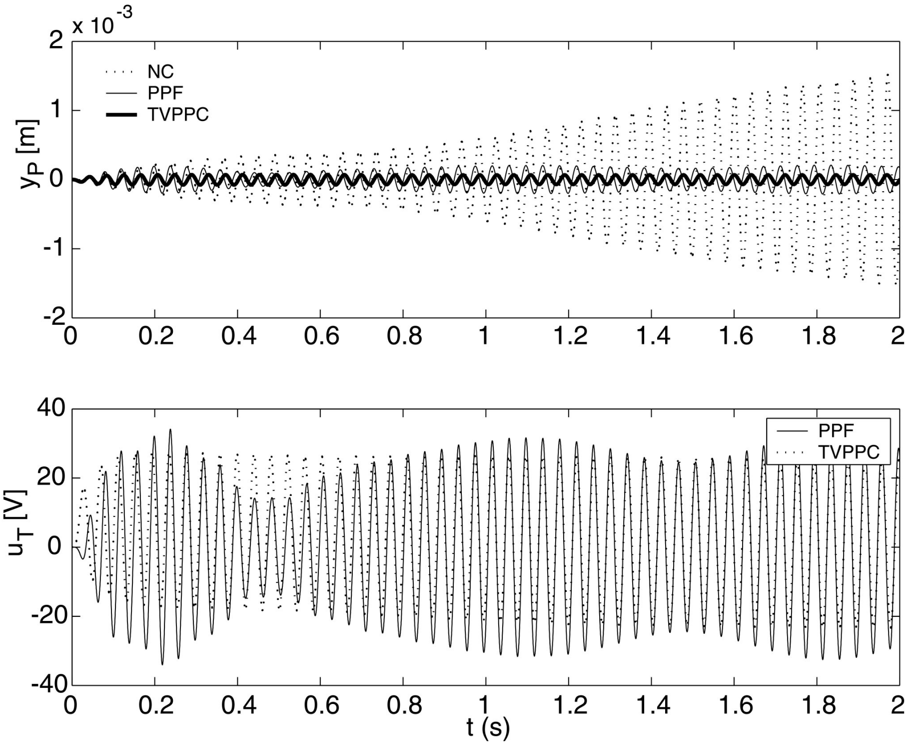

Figure 7. Time response of the system without control and controlled as the temperature goes from $4.4^{\circ}C$ to $37.7^{\circ}C$.

.

Even with the saturation, the TVPPC controller maintained the good performance, being slightly worse when the control input is turned on again at 1s, as compared to the case with no saturation. The PPF, again, was not able to improve the performance. It is clear that, although a time-invariant controller might have a good performance at constant and eventually at variable temperature, this is not the case when there is a delay or a failure effect on the controller.

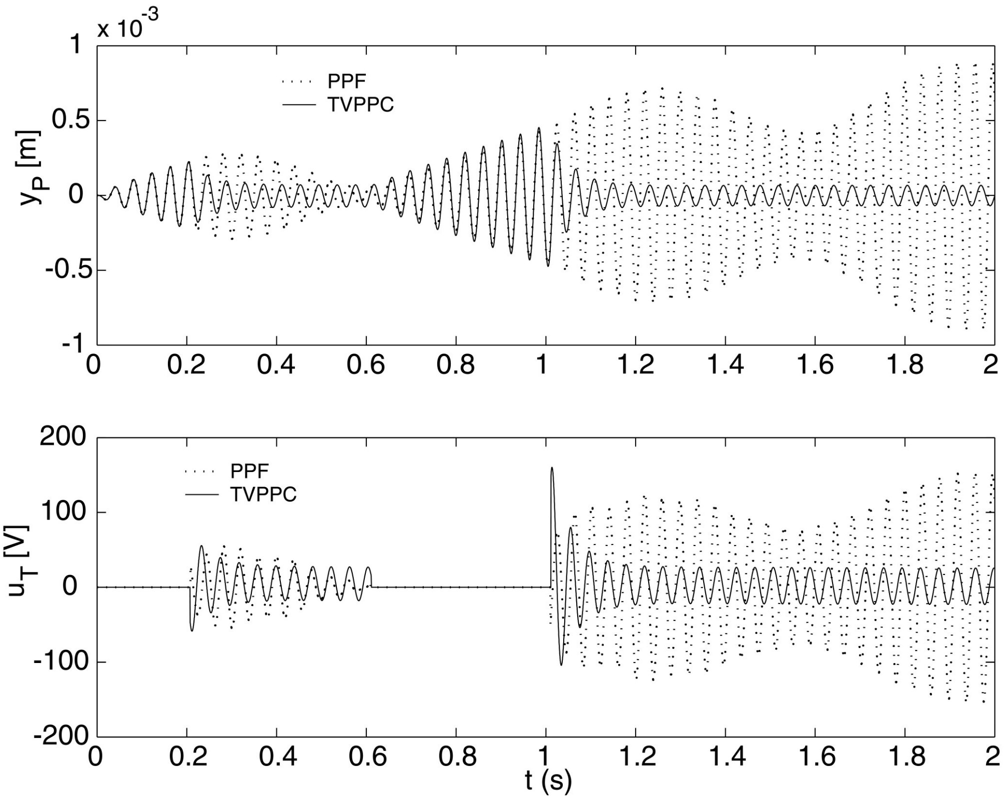

Figure 8. Failure effect / delay simulation of the controller, without saturation, at variable temperature.

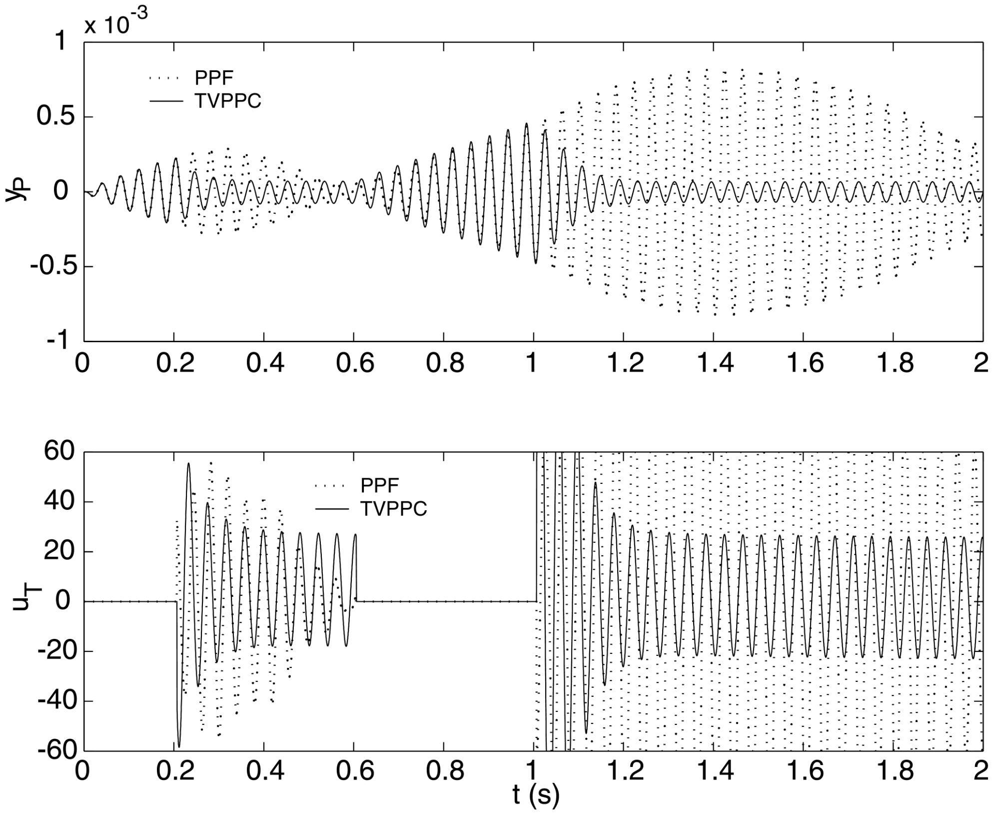

Figure 9. Failure effect / delay simulation of the controller, with saturation, at variable temperature.

5. Conclusion

A control technique designed for time-variant systems was tested on a constrained-layer damping model. The goal was to verify the performance of this controller under the condition where the temperature is affecting the damping of the system. The controller was tested under the circumstances of delay and failure effect of the controller. Since this type of controller has not been tested on these situations, this methodology was compared to a time-variant controller, known as positive position feedback.

The results have shown that:

- Both techniques have similar behavior at constant temperature. If the temperature is varying, the TVPPC is slightly better.
- The TVPPC controller delivered satisfactory results when situations as delay and failure of the controller were tested. It was able to quickly bring the response to the desired level, while the time-invariant controller suffered degradation.
- Also, as a result of saturation on the piezoelectric patch, the TVPPC was able to keep almost the same performance, which did not happen for the PPF controller.

6. Acknowledgements

The first author is thankful to CNPq for his pos-doctoral scholarship. The second author is also thankful to CNPq for his research grant.

7. References

Banks, H. T. and Inman, D. J., 1992, "Significance of modeling internal damping in the control of structures" Journal of Guidance, vol. 15, no. 6, pp. 1509-1512, 1992.

Drake, M. L. and Soovere, J., 1984, "A Design Guide for Damping of Aerospace Structures", AFWAL Vibration Damping Workshop Proceedings, 3.

Ferry, J. D., 1980, Viscoelastic Properties of Polymers, John Wiley & Sons, New York.

Lesieutre, G. A. and Govindswamy, K., "Finite element modeling of frequency-dependent and temperature-dependent dynamic behavior of viscoelastic materials in simple shear", Int. J. Solids Structures, vol. 33, no. 3, pp. 419-432, 1996.

Lukasiewicz, S. and Xia, Z. Q., 1993, “Nonlinear, Damped Vibrations of Sandwich Plates with Time-Dependent Temperature”, Dynamics and Vibration of Time-Varying Systems and Structures, *ASME*, 56, 427-436.
McEver, M.A., 1999, “Optimal Vibration Suppression Using On-line Pole/Zero Identifcation, Masters Thesis”, Virginia Polytechnic Institute & State University.
Meirovitch, L., 1990, Dynamics and Control of Structures, John Wiley & Sons Inc., New York.
Nashif, A. D., Jones, D. I. G. and Henderson, J. P., 1985, Vibration Damping, John Wiley & Sons, New York.
Schapery, R. A., 1964, “Application of Thermodynamics to Thermomechanical, Fracture, and Birefringent Phenomena in Viscoelastic Media”, Journal of Applied Physics, 35 (5), 1451-1465.
Silva, L. A., 2003, “Internal Variable and Temperature Modeling Behavior of Viscoelastic Structures – A Control Analysis”, Virginia Polytechnic Institute and State University, *PhD Thesis*.
Silva, L. A., Austin, E. M. and Inman, D. J., “Time-Varying Controller for Viscoelastic Systems”, 3º Congresso Temático de Dinâmica e Controle da SBMAC, pp. 1040-1061, Ilha Solteira, Sp, Brasil, 2004.
Tsakalis, K. S. and Ioannou, P. A., 1993, Linear Time-Varying Systems: Control and Adaptation, Prentice Hall.

8. Responsibility notice

The authors are the only responsible for the printed material included in this paper.