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A NEW APPROACH TO THE UNDERWATER VEHICLE-MANIPULATOR SYSTEMS KINEMATICS

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Abstract. *This paper addresses the Underwater Vehicle-Manipulator Systems Kinematics (UVMS). Due to the vehicle degrees of freedom such systems are kinematically redundant and need to be solved using some redundancy technique. We present an approach based on introducing kinematically constraints. The approach uses the screw representation of motions and is based on the so-called Davies method to solve the kinematics of closed kinematic chains. We describe the vehicle-manipulator system as an open-loop chain and present a virtual kinematic chain concept. This concept allows to close this chain and so, to apply the Davies method to solve the direct kinematics. The inverse kinematics is obtained using the same approach by introducing extra constraints derived from energy saving requirements, manipulability increasing objectives and obstacle avoiding goals. The paper outlines that the Davies method constitutes a systematic way to express the joint rates of passive joints as functions of the joint rates of the actuated joints in closed kinematic chains and that, combined with the virtual chain concept, it constitutes an useful approach to solve the direct and the inverse kinematics of the UVMS. For the inverse kinematics it is possible to specify extra conditions or introduce additional constrains in order to achieve energy saving, manipulability optimization or obstacle avoidance. The use of the screw representation of the movements results in a computationally efficient coordination algorithm, as was verified in the simulation results.*

Keywords: *UVMS , redundancy , screws, kinematic chains*

1. Introduction

Currently, the use of underwater robotic systems is common to accomplish missions as sea bottom and pipeline survey, cable maintenance, off-shore structures monitoring and maintenance, and collect/release of biological surveys. In this framework, the use of a manipulator mounted on an autonomous underwater vehicle plays an important role. Such systems are commonly named Underwater Vehicle-Manipulator Systems (UVMS).

An UVMS is always kinematically redundant because, due to the degrees of freedom (dofs) provided by the vehicle itself, it possesses more dofs than those required to execute a given task. However, it is not always efficient to use vehicle thrusters to move the manipulator end effector because of the difficulty of controlling the vehicle in hovering. Moreover, due to the different inertia between vehicle and manipulator, movement of the latter is energetically more efficient. On the other hand, reconfiguration of the whole system is required when the manipulator is working at the boundaries of its workspace or close to a kinematic singularity. Thus, motion of the sole manipulator is not always possible or efficient. Also, off-line trajectory planning is not always possible in unstructured environments as in case of underwater autonomous missions.

When a manipulator task has to be performed with an UVMS, the system is usually kept in a confined space (e.g., underwater structure maintenance). The vehicle is then used to ensure station keeping. However, motion of the vehicle can be required for specific purposes, e.g., inspection of a pipeline, reconfiguration of the system, and real-time motion coordination while performing end-effector trajectory tracking.

According to above, a redundancy resolution technique might be useful to achieve system coordination in such a way as to guarantee end-effector tracking accuracy and, at same time, additional control objectives, e.g., energy savings, increase of system manipulability, or obstacles avoidance. For real-time solutions the coordination algorithm must be performed in a computationally efficient way.

The simplest way to obtain the redundancy resolution is to use the pseudoinverse of the Jacobian matrix. With this approach, however, the problem of handling kinematic singularities is not addressed and their avoidance cannot be guaranteed. This problem has been minimized using other techniques based on the use of the pseudoinverse as the Task Priority Redundancy Resolution (Antonelli, 2003). In any way, the use of the pseudoinverse introduces dimensional difficulties (Campos, 2004).

Another approach to redundancy resolution is the augmented Jacobian. In this case, a constraint task is added to the end-effector task to obtain a square Jacobian matrix, which may be inverted. The main drawback of this technique is that algorithmic singularities may arise when the additional task does conflict with the end effector task.

In this paper, we present a new approach to solve the redundancy of UVMS. This approach is based on introducing kinematic constraints to the UVMS movements to obtain a square matrix, which may be inverted, without introducing algorithmic singularities. The additional kinematic constraints are introduced by a virtual kinematic chain that closes the UVMS kinematic chain. The Davies method solves the inverse kinematics of this closed chain. The proposed approach also addresses additional control objectives such as energy savings, increase of system manipulability and obstacles avoidance.

The paper is organized as follows. In the section 2 we describe the kinematics of the UVMS as a manipulator with a mobile base using screws to represent its differential movement. The Davies method, which is used to relate the joint velocities in closed kinematic chains, is presented in section 3 and the virtual kinematic chain concept is introduced in section 4. Then, in sections 5 and 6, we discuss the direct and inverse kinematics of the UVMS. The paper conclusions are presented in section 7.

2. Kinematics of Manipulators with Mobile Base

In this section, we describe the motion of an UVMS. More specifically, we obtain the description of the UVMS manipulator end effector motion with respect to an inertial frame, i.e., the kinematics of manipulators with mobile base. In order to introduce a new approach to solve the inverse kinematics to this class of systems, the model of an UVMS using screws to represent its differential motion is presented in this section. For completeness, the differential kinematics screw representation is shortly described in the sequence.

2.1 Screw representation of differential kinematics

The velocities of the points of a rigid body with respect to an inertial reference frame *O(*X,Y,Z*)* may be represented by a differential rotation $\boldsymbol{\omega}$ about a certain fixed axis and a simultaneous differential translation $\boldsymbol{\tau}$ along the same axis (Mozzi´s theorem). The movement combining rotation and translation is called screw movement or twist. Fig. 1 shows a body "twisting" around an axis instantaneously fixed with respect of an inertial reference frame. This axis is called the screw axis and the rate of the translational velocity and the angular velocity ($h = \|\boldsymbol{\tau}\|/\|\boldsymbol{\omega}\|$) is called the pitch of the screw movement.

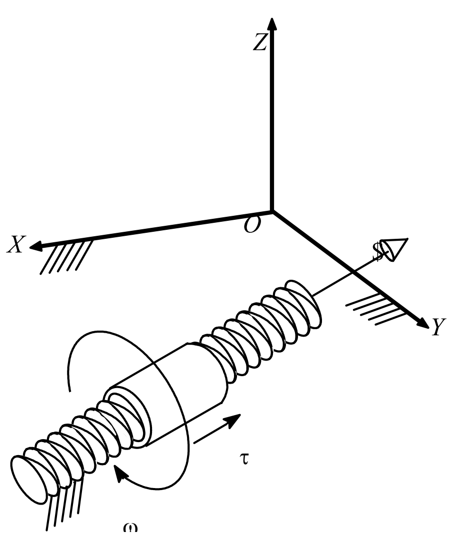

Figure 1 Screw movement or twist.

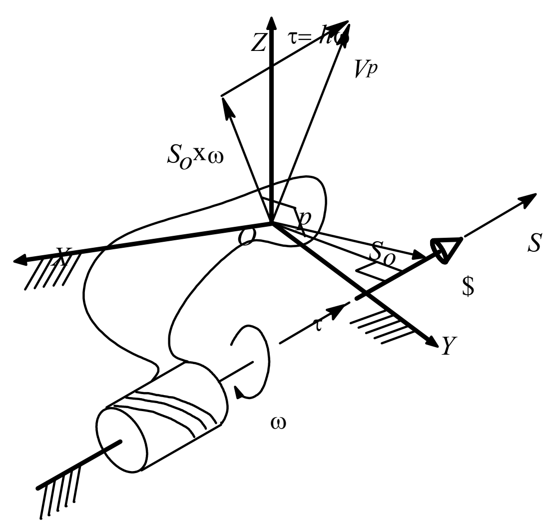

Figure 2 Twist components.

The differential movement of a body with respect to an inertial frame may be expressed by a pair of vectors named screw and given by $\$=(\boldsymbol{\omega};\boldsymbol{V_p})$. The vector $\boldsymbol{\omega}=(L, M, N)$ represents the angular velocity of the body with respect to the inertial frame. The vector $\boldsymbol{V_p}=(P^*, Q^*, R^*)$ represents the linear velocity of a point P attached to the body which is instantaneously coincident with the origin O of the reference frame.

If there are no points of the body instantaneously coinciding with the frame origin O, as in Fig. 1, a fictitious extension may be added to the body such that a point in this extension, named point P, coincides instantaneously with the origin O (see Fig. 2).

The vector $\boldsymbol{V_p}$ consists of two components: a) a velocity parallel to the screw axis represented by $\boldsymbol{\tau}=h\,\boldsymbol{\omega}$; and b) a velocity normal to the screw axis represented by $\boldsymbol{S_O}\times\boldsymbol{\omega}$, where $\boldsymbol{S_O}$ is the position vector of any point at the screw axis.

A twist may be decomposed into its amplitude and its corresponding normalised screw. The twist amplitude Ψ is either the magnitude of the angular velocity of the body, $\|\boldsymbol{\omega}\|$, if the kinematic pair is rotative or helical, or the magnitude of the linear velocity, $\|\boldsymbol{V_p}\|$, if the kinematic pair is prismatic. Consider a twist given by $\$=(\boldsymbol{\omega};\boldsymbol{V_p})^T = (L, M, N; P^*, Q^*, R^*)^T$. The corresponding normalized screw is $\hat{\$}=(L, M, N; P^*, Q^*, R^*)^T$. This normalized screw is a twist in which the magnitude Ψ is factored out, *i.e.*

$$\$ = \hat{\$}\Psi \tag{1}$$

The normalized screw coordinates (Hunt, 2000) may be defined as a pair of vectors, namely, (L,M,N) and (P^*,Q^*,R^*), given by,

$$\hat{\$} = \begin{bmatrix} L \\ M \\ N \\ P^* \\ Q^* \\ R^* \end{bmatrix} = \begin{bmatrix} \boldsymbol{S} \\ \boldsymbol{S_O} \times \boldsymbol{S} + h\boldsymbol{S} \end{bmatrix} \tag{2}$$

where $\boldsymbol{S}$ is the normalized vector parallel to the screw axis. Notice that the vector $(\boldsymbol{S_O}\times\boldsymbol{S})$ determines the moment of a line, the screw axis, around the origin of the reference frame.

The movement between two adjacent links belonging to an n-link kinematic chain may be also represented by a twist. In this case, the twist represents the movement of link i with respect to link $(i$-1).

Often it is useful to represent the differential movement of a body, expressed by a twist \$, in different reference frames. In what follows, a 6 x 6 matrix of transformation T to serve this purpose is presented (see Tsai (1999) for details). Consider two reference frames of interest (X_i, Y_i, Z_i) and (X_j, Y_j, Z_j) as in Fig. 3.

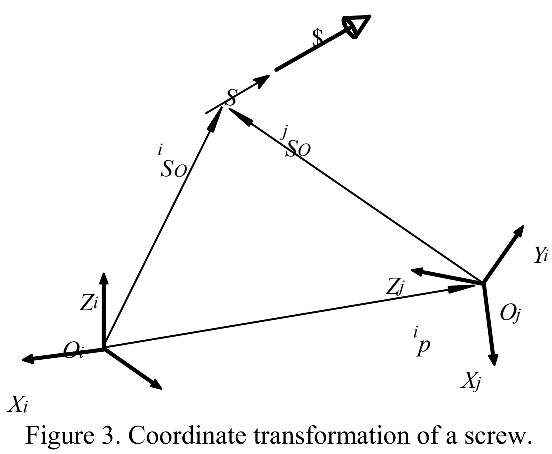

Figure 3. Coordinate transformation of a screw.

The position of origin O_j relative to the (X_i, Y_i, Z_i) frame is given by ${}^ip_j=[p_x,p_y,p_z]^T$ and the orientation of the (X_j, Y_j, Z_j) frame relative to the (X_i, Y_i, Z_i) frame is described by a rotation matrix ${}^iR_j$. A screw represented in the (X_i, Y_i, Z_i) frame is denoted by ${}^i\$$, and the same screw represented in (X_j, Y_j, Z_j) frame is denoted by ${}^j\$$.

The matrix of transformation of screws between the jth frame and the ith frame (${}^i\hat{\$}={}^iT_j\,{}^j\hat{\$}$) is given by

$${}^iT_j = \begin{bmatrix} {}^iR_j & 0 \\ {}^iW_j\,{}^iR_j & {}^iR_j \end{bmatrix} \quad ; \qquad {}^iW_j = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix} \tag{3}$$

where ${}^{i}W_j$ is the 3 x 3 skew-symmetric matrix representing the vector ${}^{i}p$ (expressed in the *i*th frame).

In the sequence, we present the manipulator and the vehicle kinematics in order to describe the UVMS kinematics.

2.2 Serial manipulator kinematics

For a serial manipulator, we may consider the motion of the end effector as being twisted instantaneously about the joint axes of an open-loop chain (Tsai, 1999). These instantaneous twists may be added linearly to give the resulting motion of the end effector. Consider, for example, the manipulator with six rotational joints showed in Fig.4. For this manipulator the differential kinematics may be written as

$$ {}^{B}\$_{E} = \sum_{i=1}^{6} {}^{B}\hat{\$}_{mi}\Psi_{mi} \tag{4} $$

where ${}^{B}\hat{\$}_{mi}$ is the manipulator i-th normalized screw described in the base frame (B-frame in Fig.4), Ψ_{vi} is the correspondent i-th magnitude and ${}^{B}\$_{E}$ is the screw that represents the end effector's motion in the B-frame.

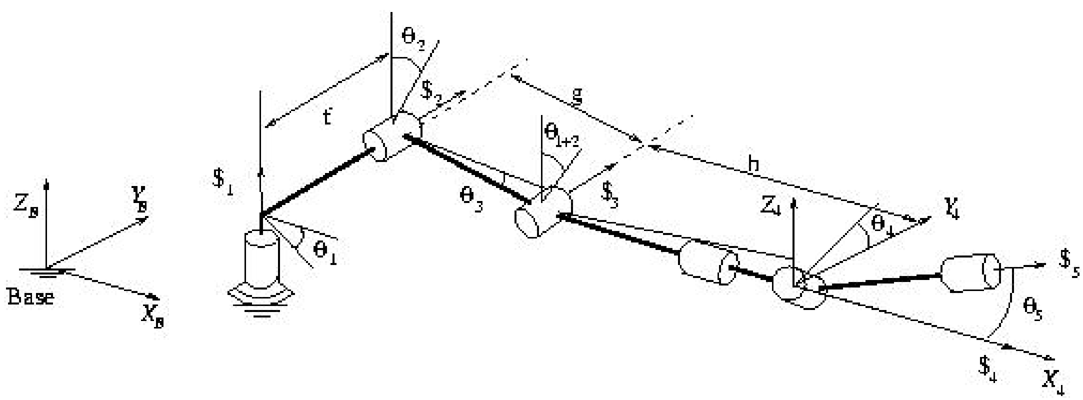

Figure 4 – The serial manipulator.

2.2 Vehicle differential kinematics

We may consider the motion of a rigid body as being twisted instantaneously about several screw axes. These screws form a screw system whose order is defined by the number of linearly independent screws that span the system (Tsai, 1999). A vehicle has, in general, six degrees of freedom and so, its motion may be represented by a screw belonging to a six-th order screw system ($\$_V$). In other words the vehicle motion may be spanned by six independent screws, i.e.,

$$ \$_V = \sum_{i=1}^{6} \hat{\$}_{vi}\Psi_{vi} \tag{5} $$

where $\hat{\$}_{vi}$ is the i-th normalized screw and Ψ_{vi} is the i-th amplitude. It should be outlined that, in case the set of normalized screws $\{\Psi_{vi}, i = 1,6\}$ is linearly independent, Eq.(5) represents the general vehicle motion as well as describes the motion of an open-loop chain with six links and six kinematic pairs (joints) each one with only one degree-of-freedom.

Using Eq.(5) to describe the vehicle's motion, we may choose an open-loop chain with three orthogonal prismatic (P) joints and a spherical (S) one (Fig.5) in order to represent the motion in a Cartesian reference frame. The first joint of this chain is prismatic and allows the motion between the first link and the base along the x-axis. It is named px and its motion is represented by the normalized screw $\hat{\$}_{px}$. The second and the third joints are prismatic, allow the motion between the second and the first, and between the third and the second links, along the y-axis (py) and the z-axis (pz), respectively, and are represented by the normalized screws $\hat{\$}_{py}$ and $\hat{\$}_{pz}$. The spherical joint is instantaneously substituted by three rotational joints in the x, y and z directions (rx, ry, rz) and its motions are represented by the normalized screws $\hat{\$}_{rx}$, $\hat{\$}_{ry}$ and $\hat{\$}_{rz}$. Due to its architecture this chain is often named PPPS.

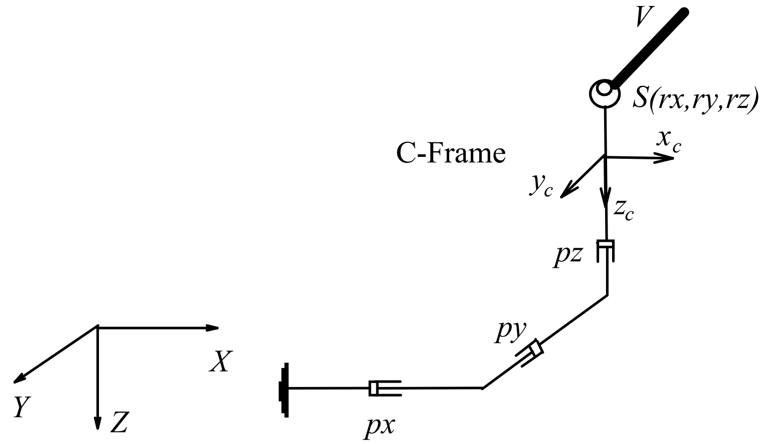

Figure 5 – Vehicle movement represented by an (PPPS) open kinematic chain.

The screws representing the PPPS chain independent motions are expressed in the simplest form if we choose a reference frame attached to the link that connects joints pz and rx (the first rotational joint of the spherical joint), designated as C-frame in Fig. 5, to denote a suitable Cartesian reference frame. Using Eq.(2) we obtain (Campos, 2004):

$$ {}^{C}\hat{\$}_{rx} = \begin{bmatrix} 1\\0\\0\\0\\0\\0 \end{bmatrix};\ {}^{C}\hat{\$}_{ry} = \begin{bmatrix} 0\\1\\0\\0\\0\\0 \end{bmatrix};\ {}^{C}\hat{\$}_{rz} = \begin{bmatrix} 0\\0\\1\\0\\0\\0 \end{bmatrix};\ {}^{C}\hat{\$}_{px} = \begin{bmatrix} 0\\0\\0\\1\\0\\0 \end{bmatrix};\ {}^{C}\hat{\$}_{py} = \begin{bmatrix} 0\\0\\0\\0\\1\\0 \end{bmatrix};\ {}^{C}\hat{\$}_{pz} = \begin{bmatrix} 0\\0\\0\\0\\0\\1 \end{bmatrix} \tag{6} $$

This normalized screws set is even linearly independent and represents the vehicle motion in the C-frame. In practice the vehicle motion is usually described in a frame located at the vehicle gravity center with the axes directed according to its principal inertia directions, here designed by vehicle frame (V-frame). To obtain the vehicle motion description in this V-frame, we may use the matrix of screws transformation between the C-frame and the V-frame given in Eq.(3), i.e., ${}^{V}\$_{V} = {}^{V}T_{C}\,{}^{C}\$_{V}$, where

$$ {}^{C}\$_{V} = \sum_{i=1}^{6} {}^{C}\hat{\$}_{vi}\Psi_{vi} \tag{7} $$

and ${}^{C}\hat{\$}_{v1} = {}^{C}\hat{\$}_{px}$, ${}^{C}\hat{\$}_{v2} = {}^{C}\hat{\$}_{py}$, ${}^{C}\hat{\$}_{v3} = {}^{C}\hat{\$}_{pz}$, ${}^{C}\hat{\$}_{v4} = {}^{C}\hat{\$}_{rx}$, ${}^{C}\hat{\$}_{v5} = {}^{C}\hat{\$}_{ry}$, ${}^{C}\hat{\$}_{v6} = {}^{C}\hat{\$}_{rz}$, , , where the normalized screws $\hat{\$}_{px}$, $\hat{\$}_{py}$, $\hat{\$}_{pz}$, $\hat{\$}_{rx}$ $\hat{\$}_{ry}$ and $\hat{\$}_{rz}$, are given in Eq.(6).

The C-frame is chosen in order to make its origin coincident with the V-frame origin. So, the position vector of one origin with respect to the other is null as well as the matrix W given in Eq.(3). The matrix of screws transformation ${}^{V}T_{C}$ is obtained calculating the rotation matrix between the C and V frames (${}^{V}R_{C}$). To this end it should be observed that the C-frame was chosen parallel to the inertial frame (I-frame) and so, ${}^{C}R_{V} = {}^{I}R_{V}$, where ${}^{I}R_{V}$ is the rotation matrix that gives the orientation of the vehicle in the I-frame commonly measured in the RPY (Roll-Pitch-Yaw) angles (Antonelli, 2003). By transposing ${}^{C}R_{V}$ we obtain ${}^{V}R_{C}$ that is substituted in Eq.(3) to calculate ${}^{V}T_{C}$ and, using Eq.(7) results

$$ {}^{V}\$_{V} = {}^{V}T_{C}\sum_{i=1}^{6} {}^{C}\hat{\$}_{vi}\Psi_{vi} \tag{8} $$

2.4 The UVMS kinematics

In an UVSM the manipulator has a mobile base. So, the manipulator end effector motion with respect to an inertial frame is obtained by adding the manipulator end effector motion with respect to its base to the base motion (Fig.6), i.e.,

$$ {}^{I}\$_{E} = {}^{I}\$_{V} + {}^{I}T_{B}\,{}^{B}\$_{E} \tag{9} $$

where ${}^{I}T_B$ is the matrix of screws transformation between the base manipulator frame and the considered inertial frame.

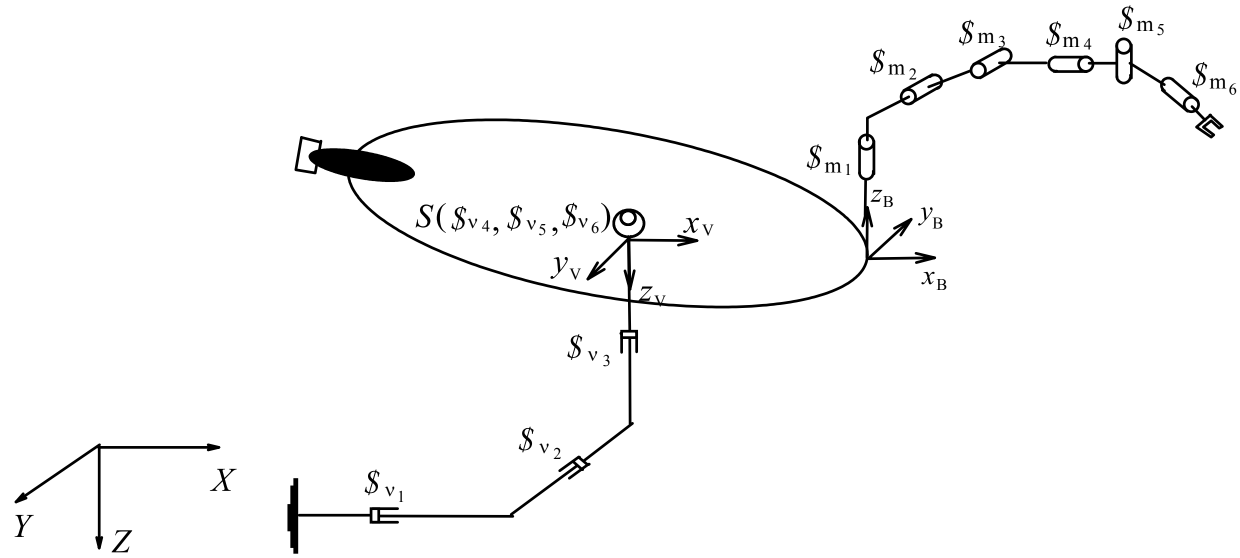

Figure 6 – An Underwater Vehicle-Manipulator System.

Consider an inertial frame instantaneously coincident the vehicle frame (V-frame) defined above. In this case Eq.(9) results

$$ {}^{V}\$_E = \sum_{i=1}^{6} {}^{V}\hat{\$}_{vi}\Psi_{vi} + \sum_{i=1}^{6} {}^{V}\hat{\$}_{mi}\Psi_{mi} \tag{10} $$

where ${}^{V}\hat{\$}_{vi} = {}^{V}T_C\,{}^{C}\hat{\$}_{vi}$, i=1,6, and ${}^{V}\hat{\$}_{mi} = {}^{V}T_B\,{}^{B}T_R\,{}^{R}\hat{\$}_{mi}$.

Eq.(10) may be rewritten as

$$ {}^{V}\$_E = J\Psi \tag{11} $$

where

$$ J = \begin{bmatrix} {}^{V}\hat{\$}_{v1} & {}^{V}\hat{\$}_{v2} & \dots & {}^{V}\$_{v6} & {}^{V}\hat{\$}_{m1} & {}^{V}\hat{\$}_{m2} & \dots & {}^{V}\$_{m6} \end{bmatrix} \tag{12} $$

$$ \Psi = \begin{bmatrix} \Psi_{v1} & \Psi_{v2} & \dots & \Psi_{v6} & \Psi_{m1} & \Psi_{m2} & \dots & \Psi_{m6} \end{bmatrix}^T \tag{13} $$

Eq.(11) expresses the UVMS kinematics as an open-loop chain constituted by the manipulator chain attached to the vehicle chain. It should be observed that the matrix J in Eq.(11) has six rows and twelve columns and outlines the system redundancy. In this paper, we present a new methodology to invert the UVMS kinematics (Eq.(11)) derived from the Davies Method and the virtual kinematic chain concept presented in the sequence.

3. Davies Method

Davies method is a systematic way to relate the joint velocities in closed kinematic chains. It is based on the so-called Kirchhoff-Davies circulation law (Davies, 1981, 2000). Davies solves the differential kinematics of closed chain mechanisms from the well-known Kirchhoff law for electrical circuits. This Kirchhoff-Davies circulation law states that ``The algebraic sum of relative velocities of kinematic pairs along any closed kinematic chain is zero" (Davies,1981).

Using this law, the relationship between the velocities of a closed kinematic chain may be obtained in order to solve its differential kinematics, as is presented in the following example.

Let the planar four-bar mechanism of Fig. 7 be formed by links 1, 2, 3 and 4 and by the rotative joints *A, B, C* and *D*. Let the twist $\$_A$ represent the movement of link 2 in relation to link 1, $\$_B$ represent the movement of link 3 in relation to link 2, $\$_C$ represent the movement of link 4 in relation to link 3 and $\$_D$ represent the movement of link 1 in relation to link 4. The twists $\$_A$, $\$_B$, $\$_C$ and $\$_D$ represent the kinematic pairs A, *B*, *C* and *D,* respectively.

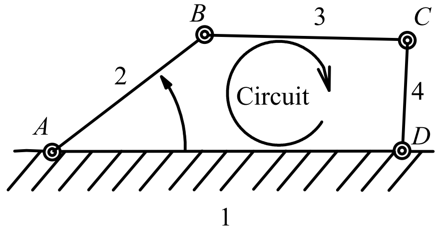

Figure 7 – The planar four-bar mechanism.

Consider that the planar mechanism lies on the xy-plan, so the twists $\$_A$, $\$_B$, $\$_C$ and $\$_D$ have only three components since the linear velocity $\boldsymbol{V_p}$ of any point of the mechanism does not have the R^* component in the z-axis direction. Additionally, the angular velocity $\boldsymbol{\omega}$ of any link of the mechanism does not have the L and M components on the xy-plan. Therefore, for the four-bar mechanism on the xy-plan, the twist components are only N , P^* and Q^* and all the twists are spanned by three independent twists.

The planar four-bar mechanism forms a closed kinematic chain. The movement of link 2 in relation to link 1 is represented by $\$_A$. The movement of link 3 in relation to link 1 is expressed by $\$_A+\$_B$. The movement of link 4 in relation to link 1 is given by $\$_A+\$_B+\$_C$ and the movement of link 1 in relation to itself is $\$_A+\$_B+\$_C+\$_D$.

The kinematic pairs connecting link 1 to itself form a closed kinematic chain and, for this closed chain, the Kirchhoff-Davies circulation law, regarding the circuit direction indicated in Fig. 7, is given by

$$\$_A + \$_B + \$_C + \$_D = \mathbf{0} \tag{14}$$

where **0** is a zero vector which dimension (3x1) corresponds to the dimension of twists *A, B, C* and *D*.

According Eq. (1) this equation may be rewritten as

$$\hat{\$}_A\ \Psi_A + \hat{\$}_B\ \Psi_B + \hat{\$}_C\ \Psi_C + \hat{\$}_D\ \Psi_D = \mathbf{0} \tag{15}$$

where $\hat{\$}_A$ represents the normalized screw of twist $\$_A$ and Ψ_A represents the velocity (angular in this case) magnitude of the twist *A*, similarly for the kinematic pairs *B, C* and *D*.

Equation (15) is referred as the constraint equation of the four bar mechanism and, in matricial form, is given by

$$\begin{bmatrix} \hat{\$}_A & \hat{\$}_B & \hat{\$}_C & \hat{\$}_D \end{bmatrix} \begin{bmatrix} \Psi_A \\ \Psi_B \\ \Psi_C \\ \Psi_D \end{bmatrix} = \mathbf{0} \tag{16}$$

The considered four-bar mechanism is planar and, as explained above, all the twists are spanned by three independent twists. In this case all the normalized screws belong to a third order screw system (Hunt, 2000). Additionally, it may be observed that the four-bar mechanism has only one independent circuit or loop in its closed kinematic chain.

In general, the constraint equation of a mechanism with movements in a *d* order screw system is given by

$$N_{(d\times F_b)}\Psi_{(F_b\times 1)} = 0_{(d\times 1)} \tag{17}$$

where *N* is the network matrix containing the normalized screws which signs depend on the circuit direction, Ψ is the magnitude vector and F_b is the gross degree of freedom, *i.e.* the sum of the degrees of freedom of all mechanism joints ($F_b = \sum f_i$), with f_i being the degree of freedom of the *ith* joint.

Closed kinematic chains, unlike open kinematic chains, contain passive kinematic pairs, in addition to active kinematic pairs. The velocity of an active kinematic pair is given by an external actuator, *e.g.* a servomotor. The velocities of the passive kinematic pairs are functions of the velocities of the active kinematic pairs due to the closure of the kinematic chain.

The use of the constraint equation, Eq. (17), allows to calculate the passive joint velocities as functions of the active joint velocities. To achieve this solution, the constraint equation needs to be rearranged highlighting the actuated and the passive pair velocities. The magnitude vector Ψ is rearranged in *d* secondary or unknown magnitudes Ψ_s and F_N primary or known magnitudes Ψ_p, *i.e.* $\Psi = \begin{bmatrix}\Psi_s & \vdots & \Psi_p\end{bmatrix}^T$. Rearranging the network matrix $[N]_{(d\times F_b)}$ coherently with the magnitude division, we get $[N]_{(d\times F_b)} = \begin{bmatrix}[N_s]_{(d\times d)} & \vdots & [N_p]_{(d\times F_N)}\end{bmatrix}$, where the secondary network sub-matrix N_s

corresponds to the secondary joints and the primary network sub-matrix N_p corresponds to the primary joints. This results in

$$\left[[N_s]_{(d\times d)} \vdots [N_p]_{(d\times F_N)}\right]\begin{bmatrix}[\Psi_s]_{(d\times 1)}\\ \cdots \\ [\Psi_p]_{(F_N\times 1)}\end{bmatrix} = [0]_{(d\times 1)} \tag{17}$$

\

This equation may be rewritten as $N_s\Psi_s = -N_p\Psi_p$ and the joint space kinematics solution is given by

$$\Psi_s = -N_s^{-1}N_p\Psi_p \tag{18}$$

The four-bar mechanism (Fig. 7) is planar (d=3) and has four joints, each one with one degree of freedom (f_i=1). The sum of the degrees of freedom of all mechanism joints results (F_b=4). The net degree of freedom of four bar mechanism is F_N=F_b-d=4-3=1. Consider that A is an actuated (primary) kinematic pair and that B, C and D are non actuated or passive (secondary) pairs. In this case, the velocity magnitude Ψ_A of the pair A is determined by an external actuator and the velocity magnitudes of the passive kinematic pairs, Ψ_B, Ψ_C and Ψ_D, are functions of the magnitude Ψ_A. Rearranging Eq.(14), the network primary sub-matrix results $N_p = \left[\hat{\$}_A\right]$ and the network secondary sub-matrix is $N_s = \left[\hat{\$}_B \quad \hat{\$}_C \quad \hat{\$}_D\right]$. If N_s is invertible, the velocity magnitudes of secondary pairs Ψ_s are calculated by

$$\begin{bmatrix}\Psi_B\\ \Psi_C\\ \Psi_D\end{bmatrix} = -\left[\hat{\$}_B \quad \hat{\$}_C \quad \hat{\$}_D\right]^{-1}\left[\hat{\$}_A\right]\Psi_A \tag{19}$$

which is the kinematic solution for the four bar mechanism.

This outlines that the Davies method constitutes a systematic way to express the joint rates of passive joints as funtions of the joint rates of the actuated joints in closed kinematic chains.

In the next section, we introduce the virtual kinematic chain concept which allows to close open kinematic chains in order to apply the Davies method.

4. The virtual kinematic chain concept

The virtual kinematic chain, virtual chain for short, is essentially a tool to obtain information about the movement of a kinematic chain or to impose movements on a kinematic chain.

In this paper, we use the virtual kinematic chain concept introduced by Campos (2004), who defines a virtual chain as a kinematic chain composed by links (virtual links) and joints (virtual joints) satisfying the following three properties: a) the virtual chain is open; b) it has joints whose normalized screws are linearly independent; and c) it does not change the mobility of the real kinematic chain.

Either to obtain information about the movement of a (real) chain or to change its movement, we apply virtual chains to close open real chains. All the kinematic chains that satisfy the three properties cited above may be used as virtual kinematic chains. An particularly useful virtual chain is the orthogonal PPPS chain presented in (Fig. 5) and described in section 2.2.

In the next two sections, this PPPS chain is used to close the UVMS open-loop chain in order to obtain information about the end effector motion in a Cartesian frame and to impose movements to the UVMS chain.

5. UVMS direct kinematics

The end effector motion description in a Cartesian frame may be obtained closing the UVMS chain by introducing a PPPS virtual chain between the base and the end effector as in Fig. 8.

In this case, regarding the circuit orientation indicated in the figure, the closed-loop chain network matrix is given by

$$N = \left[\hat{\$}_{v1} \quad \ldots \quad \hat{\$}_{v6} \quad \hat{\$}_{m1} \quad \ldots \quad \hat{\$}_{m6} \quad -\hat{\$}_{rz} \quad -\hat{\$}_{ry} \quad -\hat{\$}_{rx} \quad -\hat{\$}_{pz} \quad -\$_{py} \quad -\$_{px}\right] \tag{20}$$

where the superscript V indicating that all the normalized screws are represented in the vehicle frame is omitted for simplicity. The negative terms in Eq.(22) are in accordance with the definitions of the screws that represent the virtual joint motions given in section 2.2. The corresponding magnitude vector is

$$\Psi = \begin{bmatrix}\Psi_{v1} & \ldots & \Psi_{v6} & \Psi_{m1} & \ldots & \Psi_{m6} & \Psi_{rz} & \Psi_{ry} & \Psi_{rx} & \Psi_{pz} & \Psi_{py} & \Psi_{px}\end{bmatrix}^T \tag{21}$$

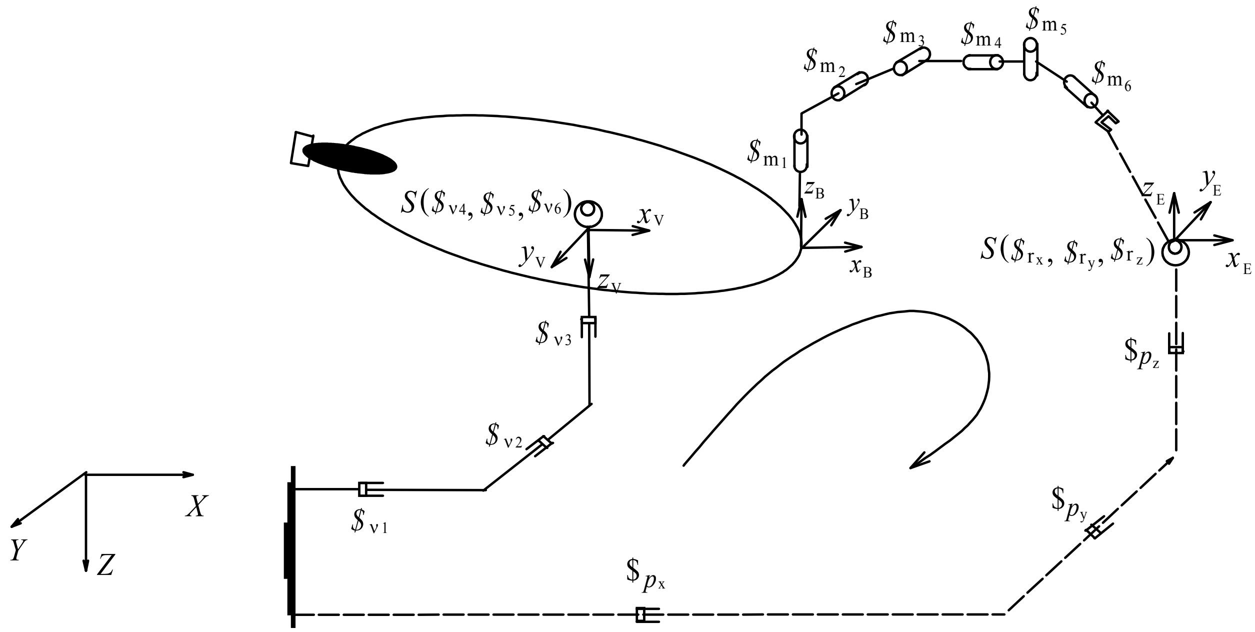

Figure 8 – The Underwater Vehicle-Manipulator System with the PPPS virtual chain.

To solve the direct kinematics, we select the velocities magnitudes corresponding to the operational space (represented by the virtual kinematic pairs) as the components of the primary vector Ψ_p and the magnitudes of the UVMS kinematic pairs as the components of the secondary vector Ψ_s. The corresponding primary and secondary matrices result

$$N_p = \begin{bmatrix}\hat{\$}_{v1} & \hat{\$}_{v2} & \hat{\$}_{v3} & \hat{\$}_{v4} & \hat{\$}_{v5} & \hat{\$}_{v6} & \hat{\$}_{m1} & \hat{\$}_{m2} & \hat{\$}_{m3} & \hat{\$}_{m4} & \hat{\$}_{m5} & \hat{\$}_{m6}\end{bmatrix} \tag{22}$$

$$N_s = \begin{bmatrix}-\hat{\$}_{rz} & -\hat{\$}_{ry} & -\hat{\$}_{rx} & -\hat{\$}_{pz} & -\hat{\$}_{py} & -\hat{\$}_{px}\end{bmatrix} \tag{23}$$

The magnitudes of the secondary kinematic pairs, i.e., the end effector velocities in the Cartesian frame, are calculated using Eq.(18), in which the secondary matrix needs to be inverted. It should be remarked that the secondary matrix has always full rank, as can be observed in the Eq.(6), and so it is always invertible.

6. UVMS inverse kinematics

To obtain the vehicle and the manipulator velocities given the end effector velocities (inverse kinematics), we select the velocity UVMS kinematic pairs magnitudes as the components of the primary magnitude vector Ψ_p and velocity magnitudes corresponding to the operational space (virtual kinematic pairs) as the components of the secondary magnitude vector Ψ_s. In this case, the primary matrix results $N_p = \begin{bmatrix}-\hat{\$}_{rz} & -\hat{\$}_{ry} & -\hat{\$}_{rx} & -\hat{\$}_{pz} & -\hat{\$}_{py} & -\hat{\$}_{px}\end{bmatrix}$ and the secondary matrix results $N_s = \begin{bmatrix}\hat{\$}_{v1} & \hat{\$}_{v2} & \hat{\$}_{v3} & \hat{\$}_{v4} & \hat{\$}_{v5} & \hat{\$}_{v6} & \hat{\$}_{m1} & \hat{\$}_{m2} & \hat{\$}_{m3} & \hat{\$}_{m4} & \hat{\$}_{m5} & \hat{\$}_{m6}\end{bmatrix}$.

Again, to calculate the secondary pairs magnitudes, we need to invert the secondary matrix. By observing the matrix N_s, it becomes clear that this matrix cannot be inverted because it has twelve columns and only six rows. We must specify six supplementary velocity magnitudes and include it in the primary magnitude vector in order to obtain a secondary matrix that could be inverted or, alternatively, we need introduce six additional kinematic constrains.

The velocity magnitudes may be specified considering, for example, energy savings. This energy savings could be achieved by keeping the vehicle aligned with the ocean current and stated as long as the end-effector task is fulfilled with the sole manipulator arm as suggested by Antonelli (2003). Correspondingly, this could be done by specifying the rotation components of the vehicle velocities in order to keep the vehicle aligned with the ocean current and making its linear velocities to be equal zero.

The primary matrix results $N_p = \begin{bmatrix}-\hat{\$}_{rz} & -\hat{\$}_{ry} & -\hat{\$}_{rx} & -\hat{\$}_{pz} & -\hat{\$}_{py} & -\hat{\$}_{px} & \hat{\$}_{v1} & \hat{\$}_{v2} & \hat{\$}_{v3} & \hat{\$}_{v4} & \hat{\$}_{v5} & \hat{\$}_{v6}\end{bmatrix}$ and the secondary matrix is $N_s = \begin{bmatrix}\hat{\$}_{m1} & \hat{\$}_{m2} & \hat{\$}_{m3} & \hat{\$}_{m4} & \hat{\$}_{m5} & \hat{\$}_{m6}\end{bmatrix}$. This secondary matrix could be inverted even it is full rank, i.e. even the kinematic chain is not at a singularity.

This outlines the easiness of obtain the inverse kinematics allowed by the approach proposed in this paper only by choosing adequately the primary and secondary velocity magnitudes. Due to the use of screws to represent the involved motions, this approach allows also choose reference frames in which this representation is simpler and so, the secondary matrix N_s is sparser and, consequently, easier to be inverted. This makes the resulting motion coordination algorithm computationally efficient, as was verified in simulations by Santos and Guenther (2004).

The approach proposed in this paper allows introduce the needed additional kinematic constraints in a simple way using an additional virtual chain. This can be done to increase system manipulability or to achieve obstacle avoidance as discussed in Campos et al. (2004) and implemented in a redundant industrial manipulator in Guenther *et al*. (2004).

7. Conclusions

This paper outlines that the Davies method constitutes a systematic way to express the joint rates of passive (secondary) joints as functions of the joint rates of the actuated (primary) joints in closed kinematic chains.

The presented virtual chain concept may be used to close open-loop chains in order to apply the Davies method.

The resulting approach allows to obtain the direct and the inverse kinematics in a similar way only by selecting adequately the primary and secondary joints. In case of the inverse kinematics it is possible to specify extra conditions or introduce additional constrains in order to achieve energy saving, manipulability optimization or obstacle avoidance.

The use of motion screws representation results in a computationally efficient coordination algorithm, as it was verified in simulation results.

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