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Instationary Balancing of Elastic Rotors With Speed-Dependent System Matrices

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Abstract: *Non-stationary balancing methods collect the measurement data during run-up or run-down processes. During the non-stationary measurement run, the rotor passes a broad speed band in which the system properties can significantly change due to speed-dependent gyroscopic effects and journal bearings. In this article it is shown that the up-to-date non-stationary balancing methods yield good results both with time domain and frequency domain identification although they assume constant system properties within the whole range of rotor speed. This is shown by means of a rotor with journal bearings, whose natural frequencies change up to 10 % within the range of rotor speed passed through during measurement and whose damping coefficients decrease until instability. In spite of these substantial system changes during the measurement, the modal unbalances for the passed frequency range were identified with an error of 3 to 20 %. In fact, the algorithms determine the constant values which overall best approximate the varying true parameters.*
Keywords: *non-stationary, balancing, system identification, transient vibrations, journal bearings.*

Introduction

In the industrial practice, flexible rotors are balanced using the modal or the influence coefficient method. Traditionally, both methods require many time consuming test runs with constant rotational speeds. In order to shorten the balancing procedure, to improve the balancing results and to limit the rotor amplitudes during the test runs, methods have been developed that use non-stationary run-up or run-down processes for unbalance identification (Markert, 1984; Markert, 1988; de Silva, 1991; Seidler and Markert, 1999; Seidler and Markert, 2000). All these methods are based on the assumptions that the rotor system has symmetric system matrices and the system properties are independent of the rotor speed.

However, these assumptions are not valid for rotors in journal bearings or with gyroscopic effects. Both influences are significant in many technical applications. For example, turbines and generators are often supported by journal bearings and balanced under service conditions during assembly or maintenance. Balancing machines for heavy rotors are equipped with journal bearings as well.

Rotor systems with journal bearings and/or significant gyroscopic effects have unsymmetric stiffness and damping matrices. The unsymmetric system matrices lead to an unsymmetric frequency-response function matrix. Consequently one has to distinguish left-hand from right-hand eigenvectors (Nordmann, 1976). For their experimental identification, at least one row and one column of the frequency-response function matrix have to be measured. Experimental modal analysis techniques for rotating machinery considering this property are summarized by Bucher, Ewins and Robb (1996) and Irretier (2000).

Moreover, the journal bearing properties and the gyroscopic effects depend on the rotor speed. Consequently all modal quantities, particularly eigenvalues and eigenvectors, are speed-dependent as well. In the application of non-stationary balancing methods, wide speed ranges are passed in order to collect information about the rotor system in a broad frequency band with a single test run. Therefore, the system parameters, i.e. the system matrices and the modal parameters, can change significantly during the measuring run.

The common non-stationary balancing methods as well as the one applied here neither distinguish left-hand from right-hand eigenvectors nor take into account the speed-dependent change of the system parameters during the non-stationary test runs. Applying these methods it is important to know how they respond to such unconsidered system properties and to which extent they still yield reliable identification results. In the following, the non-stationary balancing method that was described by Seidler and Markert (1999) and successfully tested on a rotor in roller bearings by Seidler and Markert (2000) is first summarized. Then this method is applied to a model rotor in journal bearings, whose unsymmetric system matrices change substantially with the rotor speed during the measuring run. In 2003 Seidler tested the described method successfully on experimental data of a run-up of a turbocharger rotor.

Model for non-stationary balancing

As mentioned above, the unbalance identification method is based on the assumptions that the gyroscopic effects, the antisymmetric parts of the damping and stiffness matrices as well as the speed-dependency of the system parameters are negligible. The system's oscillations in the principal axes y and z are then uncoupled from each other. Furthermore it is assumed that the eigenvectors of the conservative system diagonalize the damping matrix, i.e. $\mathbf{KM}^{-1}\mathbf{B}=\mathbf{BM}^{-1}\mathbf{K}$. In a limited frequency range, the behavior of the real system is described by a discrete model with N degrees of freedom. Then the equations of motion in the z-direction are given by

$$\mathbf{M}\ddot{\mathbf{z}}_W+\mathbf{B}_z\dot{\mathbf{z}}_W+\mathbf{K}_z\mathbf{z}_W=-\mathrm{Re}\{\mathbf{u}[e^{i\varphi(t)}]^{\cdot\cdot}\}\,, \tag{1}$$

where the mass matrix $\mathbf{M}$, the damping matrix $\mathbf{B}_z$ and the stiffness matrix $\mathbf{K}_z$ are real, symmetric and constant. The system excitation results from the unbalances described by the complex vector $\mathbf{u}=[u_1,\ldots,u_N]^T$ rotating with the angle of rotation $\varphi(t)$.

By the modal transformation $\mathbf{z}(t)\approx\mathbf{\Phi}_z\,\mathbf{q}_z(t)$, the equations of motion (1) are decoupled and the number of the unknown system parameters as well as the number of the remaining (modal) degrees of freedom are considerably decreased,

$$\begin{bmatrix}\ddot{q}_{z,1}\\ \ddot{q}_{z,2}\\ \vdots\end{bmatrix}+\begin{bmatrix}2\zeta_{z,1}\omega_{z,1}&&\\&2\zeta_{z,2}\omega_{z,2}&\\&&\ddots\end{bmatrix}\begin{bmatrix}\dot{q}_{z,1}\\ \dot{q}_{z,2}\\ \vdots\end{bmatrix}+\begin{bmatrix}\omega_{z,1}^2&&\\&\omega_{z,2}^2&\\&&\ddots\end{bmatrix}\begin{bmatrix}q_{z,1}\\ q_{z,2}\\ \vdots\end{bmatrix}=\begin{bmatrix}u_{\zeta,z,1}^{(0)}/m_{z,1}^{(0)}&u_{\eta,z,1}^{(0)}/m_{z,1}^{(0)}\\ u_{\zeta,z,2}^{(0)}/m_{z,2}^{(0)}&u_{\eta,z,2}^{(0)}/m_{z,2}^{(0)}\\ \vdots&\vdots\end{bmatrix}\begin{bmatrix}\ddot{\varphi}\sin\varphi+\dot{\varphi}^2\cos\varphi\\ \ddot{\varphi}\cos\varphi-\dot{\varphi}^2\sin\varphi\end{bmatrix}. \tag{2}$$

This reduced set of equations for the rotor deflection in z-direction contains only the modal parameters, i.e. the modal matrix $\mathbf{\Phi}_z$, consisting of the mode shapes $\hat{\mathbf{z}}_m$, the modal masses $m^{(0)}_{z,m}$, the undamped natural frequencies $\omega_{z,m}$, the modal damping ratios $\zeta_{z,m}$ and the modal unbalance components $u^{(0)}_{\zeta,z,m}$ and $u^{(0)}_{\eta,z,m}$ of the M modes taken into account. For calculating the unbalance response in the time domain, the system (2) has to be integrated for a given set of mode shapes, modal masses, natural frequencies, modal damping ratios and modal unbalances.

Alternatively in the frequency domain, the unbalance response in the z-direction is given by

$$\mathbf{Z}_W(\Omega) = \sum_{m=1}^{M} \frac{\hat{\mathbf{z}}_m \hat{\mathbf{z}}_m^T}{m_{z,m}{}^{(0)}(\omega_{z,m}^2 + 2i\,\Omega\,\omega_{z,m}\zeta_{z,m} - \Omega^2)}\,\Omega^2[\mathbf{u}_{\zeta,z}\mathcal{F}\{\cos\varphi\} - \mathbf{u}_{\eta,z}\mathcal{F}\{\sin\varphi\}], \tag{3}$$

where $\mathcal{F}$ denotes the Fourier transform with the frequency variable Ω.

Identification method

Within the unbalance identification process the mode shapes and modal masses are taken from precedent experimental modal analysis or calculations. The rotor rotation angle $\varphi(t)$ is measured simultaneously to the rotor vibration and the varying angular speed $\dot{\varphi}(t)$ and angular acceleration $\ddot{\varphi}(t)$ are computed from the measured $\varphi(t)$ by numerical differentiation and filtering. The unknown parameters (unbalances, natural frequencies and modal damping ratios) are estimated by fitting the model's theoretical response calculated from model equations (2) or (3) to the rotor's vibration response measured during one non-stationary run-up or run-down. This may be done either in the frequency domain or in the time domain.

By including the damping ratios and natural frequencies in the estimation, a nonlinear minimization problem arises that has to be solved by an iterative search algorithm. As shown by Markert (1984), the measurement at a single location during one non-stationary run without trial unbalances contains all information necessary to identify the unbalances, damping ratios and natural frequencies if the mode shapes and the modal masses are known.

For unbalance identification in the frequency domain, the measured frequency response $\tilde{\mathbf{Z}}_W(\Omega_k)$ is calculated by applying the fast Fourier transform (FFT) to the measured time response $\tilde{\mathbf{z}}_W(t)$. The fitting of the theoretical frequency response $\mathbf{Z}_W(\Omega_k)$ to the measured frequency response $\tilde{\mathbf{Z}}_W(\Omega_k)$ is achieved by using the least squares method

$$\min_{\theta} \sum_k \mathbf{E}^{*T}(\Omega_k)\,\mathbf{E}(\Omega_k) = \min_{\theta} \sum_k [\mathbf{Z}_W(\Omega_k) - \tilde{\mathbf{Z}}_W(\Omega_k)]^{*T}[\mathbf{Z}_W(\Omega_k) - \tilde{\mathbf{Z}}_W(\Omega_k)] \tag{4}$$

which minimizes the error vector $\mathbf{E}(\Omega_k)$ containing only the elements related to the locations where measurements are taken. The identification result is the parameter vector θ containing all parameters to be estimated. The minimization can be carried out by standard algorithms, for example provided by MATLAB.

For unbalance identification in the time domain, the optimal estimation for the unknown parameters θ are found by minimizing the error

$$\min_{\theta} \sum_k \mathbf{e}^T(t_k)\,\mathbf{e}(t_k) = \min_{\theta} \sum_k [\mathbf{z}_W(t_k) - \tilde{\mathbf{z}}_W(t_k)]^T[\mathbf{z}_W(t_k) - \tilde{\mathbf{z}}_W(t_k)], \tag{5}$$

where $\tilde{\mathbf{z}}_W(t_k)$ denotes the measured vibration and $\mathbf{z}_W(t_k) = \mathbf{\Phi}_z\,\mathbf{q}_z$ denotes the calculated vibration, both for the same measured course of rotation angle $\varphi(t)$, at the same sample points t_k and only for the positions where measurements are taken. Appropriate algorithms for the model fit are available for this case as well.

Turbocharger Rotor with journal bearings

The balancing method described above assumes symmetric and constant system matrices. As shown by Seidler in 2001, this method yields sufficient accuracy also for rotors in journal bearings, for which these assumptions do not hold. Therefore this unbalancing method is firstly tried to unbalance a small turbocharger rotor. In 2003 a preliminary study on measured run-up and run-down data of small turbocharger rotors (Fig. 1) was done by Seidler. The configuration of the balancing planes of the turbocharger rotor is given in Table 1.

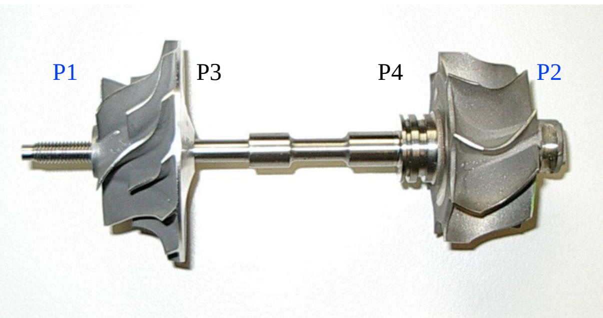

Figure 1. Turbocharger rotor with balancing planes P1 ... P4

Plane P1	compressor wheel, radius=4.95mm
Plane P3	compressor wheel, radius=17mm
Plane P2	turbine wheel, radius=12.5mm
Plane P4	turbine wheel, radius=4.2mm

Table 1. Parameters of the balancing planes

The results of this preliminary study pointed out, that this method is suitable to unbalance rotors in journal bearings.

To enhance the balancing results of this method the exact calculation of the eigenvalues and the eigenvectors is necessary. A first simulation model for the unbalanced rotor in journal bearings from a Garrett GT 15 turbocharger is shown in Fig. 2. This finite element model consists of beam elements with quadratic trial functions for the displacement in the nodes.

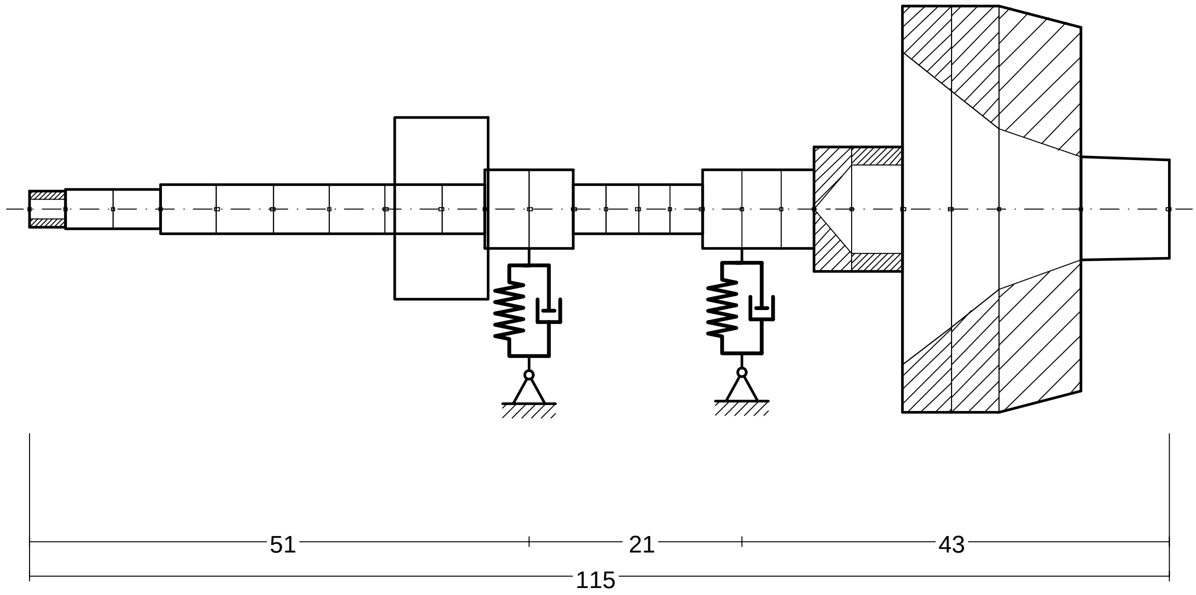

Figure 2. FE modell of the GT 15 Rotor

The modelling of the journal bearings is the most critical point of this modell, because as shown in Fig. 3, varying for example the stiffness values of the journal bearings, the position of the eigenfrequencies is strongly changing. On the other hand, the eigenvectors are nearly not changing.

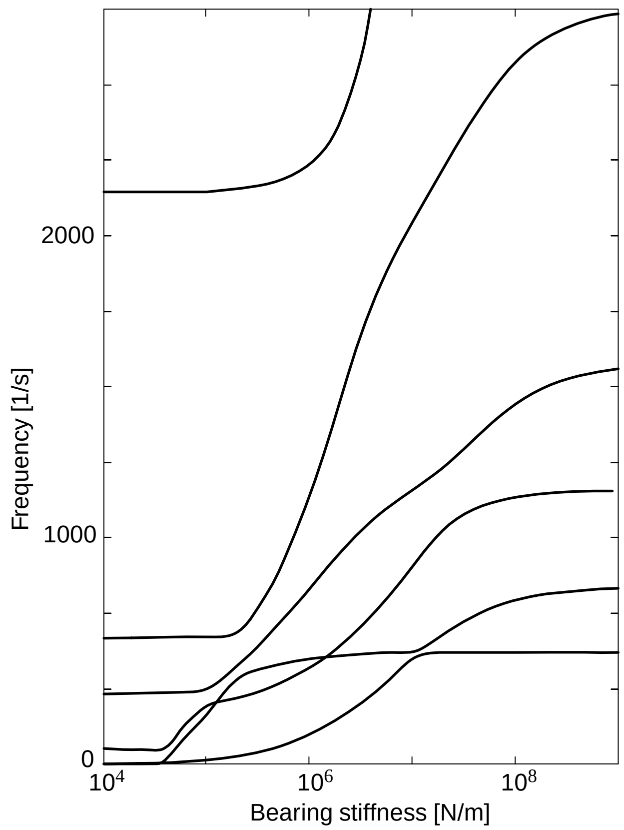

Figure 3. Changing values of eigenfrequencies against bearing stiffness

The stiffness and damping properties of the journal bearings which depend on the Sommerfeld number So and accordingly on the rotor speed were taken from Someya (1988). During a test run steady bearing conditions (static load F_{stat} equal to the rotor mass and viscosity of lubricant η_{oil}) are assumed. Therefore, the Sommerfeld number $So = F_{stat}\psi^2/(LD\eta_{oil}\overline{\Omega})$ depends only on the rotor speed and the bearings are characterized by a single constant parameter, the product $So\,\overline{\Omega}$.

Due to the fact that the system matrices are unsymmetric, the frequency-response function matrix is unsymmetric as well and the left-hand eigenvectors differ from the right-hand eigenvectors. Moreover, the modal quantities change with the rotor speed. With increasing rotor speed, the eigenvalues move along the trajectories, until they reach critical rotor speed, where a pair of eigenvalues moves even into the right half of the complex plane. The rotor system is instable above this limit speed.

The balancing method for rotors with speed-dependent system matrices is still under development, so the balancing method for rotors with constant symmetric system matrices was applied here. Therefore, constant real mode shapes, which are used as both left-hand and right-hand eigenvectors, as well as constant modal masses are required as input. The exact modal data will be obtained from the eigenvalue-problem for different rotor speeds. The column of the frequency-response function matrix relating to a point on the rotor is calculated for one rotor speed. Then the approximations for the natural frequencies, modal damping ratios, modal masses and the real mode shapes are estimated.

Unbalance and parameter identification

For unbalance and parameter identification, only one fast run-up or run-down is needed. The rotor does not have to be accelerated in a particular way; the rotation angle is measured simultaneously, and its actual time history is included in the calculations.

Starting from stationary oscillations at the constant rotational speed $\overline{\Omega}=120\ s^{-1}$, the rotor is accelerated within 1.5 s very quickly to the maximum speed $\overline{\Omega}=2075\ s^{-1}$. Then this final rotor speed is kept constant until a stationary state is regained. During the run-up, the Sommerfeld number So passes from 3 (starting speed) down to 0.17 (final speed).

For unbalance identification the vibration response $z(t)$ at an arbitrary rotor position and the rotation angle $\varphi(t)$ are measured. The vibration response can be measured at a bearing or directly at the shaft. For the measurement of the oscillations of the rotor, only one pickup would be used. However, measurements at additional locations increase the reliability of the results.

Besides the measurement signals the real mode shapes, the modal masses and first approximations of modal damping ratios and natural frequencies are fed into the non-stationary balancing program. The latter two parameter sets are not required to be accurate since their values are only used as starting values for the iteration.

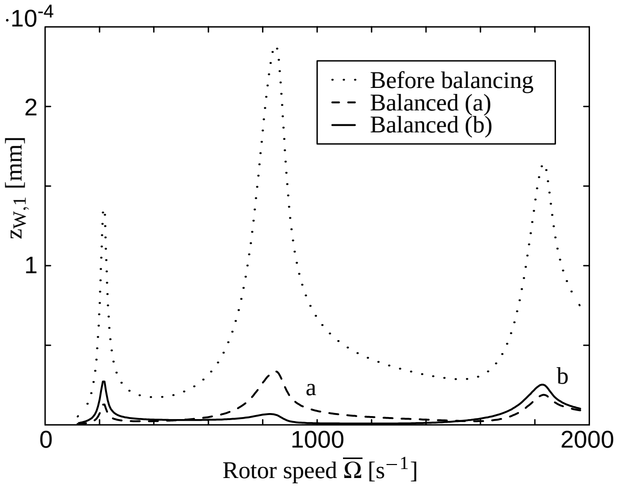

Figure 4. Results of unbalance identification from one fast run-up through the complete speed range 120 – 2075 s^{-1}:
(a) Time domain identification
(b) Frequency domain identification

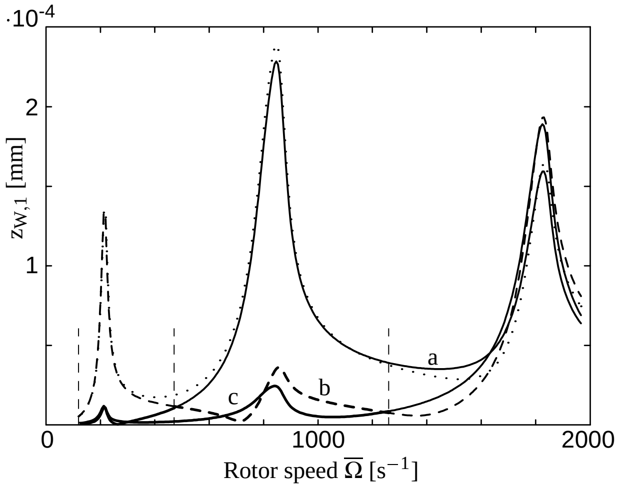

Figure 5. Balancing within limited speed ranges:
· · · Before balancing
(a) Balanced, measuring speed range 120 – 471 s^{-1}
(b) Balanced, measuring speed range 471 – 1260 s^{-1}
(c) Balanced, measuring speed range 120 – 1260 s^{-1}

Former experiments and simulations (Seidler 2001) show that modal parameters and unbalances can be satisfactorily identified only if the corresponding critical speeds are passed through during the measuring run. Therefore, if the measurement run covers a reduced speed range, only the modal parameters corresponding to the eigenfrequencies within this speed range can be identified. However, these parameters are identified very successfully, as the comparison of the stationary rotor amplitudes before and after balancing shows, Fig. 5. If for example only the first resonance is passed in the measuring run, only the first modal unbalance is reduced (line a). The two other resonances remain almost unchanged because the identification is restricted to unbalance vectors orthogonal to the mode shapes of the resonances which were not passed. After balancing the rotor deflection are very small within the rotor speed range passed during the measurement (indicated by vertical lines). The same applies for the two other examples with different rotor speed ranges as illustrated by lines b and c.

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