Proceedings of the XI DINAME, 28th February-4th March, 2005 - Ouro Preto - MG - Brazil
Edited by D.A. Rade and V. Steffen Jr.

ACTIVE CONTROL OF A FLEXIBLE ROTOR OPERATING IN TRANSIENT MOTION USING OPTIMAL CONTROL

Ricardo Corrêa Simões
Federal University of Uberlandia
rcsimoes@mecanica.ufu.br

Valder Steffen, Jr.
Federal University of Uberlandia
vstefffen@mecanica.ufu.br

José Francisco Ribeiro
Federal University of Uberlandia
jribeiro@mecanica.ufu.br

Abstract. *Rotating machines are used in various industrial applications involving sophisticated technology and submitted to high performance requirements (aeronautical, petrochemical, energy generation, etc). The attenuation of the vibration level of these machines assures safe and efficient operation, while avoids premature failure. The use of vibration active control techniques in flexible rotors has shown to be more efficient and versatile than the use of passive control techniques. This paper presents a study about active control of rotors by using optimal control techniques. First, the dynamic behavior of the rotor is modeled by using the Finite Element Method (F.E.M.), then the pseudo-modal reduction technique is used to reduce the size of the complete model. Finally a digital linear quadratic regulator is designed to realize the control of the flexible rotor operating in transient motion. Simulation results illustrate the efficiency of the technique developed.*

Keywords*: Rotating Machine, Active Control, Optimal Control, Transient Motion.*

1. Introduction

Rotating machines are used in several types of industrial activities, among which one can find petrochemical, aerospace and energy generation industries. Due to the high level of performance requirements of such services, it is desirable to minimize the vibration level of the rotor. High vibrations can lead the machine to a break, entailing economic losses, inconvenience to users and even losses of human lives. Consequently, the problem of how to control by efficient means the dynamics of rotating machinery has to be considered.

The use of passive vibration controllers is the first alternative that comes to the mind of the rotor designer. A frequent engineering challenge is to modify damping characteristics distributed along the structure, particularly at the bearings. However, there are machines already in service that present vibration levels beyond the acceptable value. Besides, there is also the necessity to design new machines in which only the use of passive vibration absorbers can not reach the design requirements related to the vibration response of the machine, Adams and McClosKey (1990).

The active control acts in the structure to reduce its vibration, by applying a control force to the system. The controller through the values given by the feedback signal determines such a force and the final vibration level desirable for the controlled system. The development of smart materials such as piezoelectric ceramics, electrorheological and magnetorheological fluids, for example, provides a significant advance in the possibilities of applied research devoted to active control. However, still remains the task of turning such techniques applicable and economically affordable for an important range of industrial applications.

The use of active control techniques for vibration reduction in rotordynamics has been the focus of interest of many researchers along the last three decades. Schweitzer and Lange (1976), Burrows et al (1989) propose the use of magnetic actuators for active control of rotating machines. Palazzolo et al. (1989) show the feasibility of using piezoelectric pushers arranged on a plane to control the vibrations of a rotor operating in transient speeds; in another paper Palazzolo et al. (1991) enlarge their research by using piezoelectric pushers arranged on two parallels planes. Malhis et al. (2002) simulate the control of a rotor through piezoelectric pushers by using Fuzzy Logic. Althaus et al. (1993), Ulbrich and Althaus (1989) and Nicoletti and Santos (2001) use hydraulic actuators to provide the control effort to reduce vibrations in rigid rotors. Forte et al. (2002) make experiments in a rotor test rig using a damper containing magnetorheological fluid to provide active control. Bonneau and Frêne (1997) present a study devoted to an active squeeze film damper in such a way that a variable clearance squeeze film damper (feed with electrorheological fluid) or a variable viscosity squeeze film damper is used to control a flexible shaft.

This paper presents a study about active control of rotors by using Optimal Control Techniques. The Linear Quadratic Regulator is described by a set of differential equations as written in the state space form. The optimal proportional feedback gain matrix is computed using the Riccati matrix differential equation as explained in Ramirez

(1994). The dynamic behavior of the rotor is modeled by using the Finite Element Method (F.E.M.), as shown by Lalanne and Ferraris (1997). The pseudo-modal technique is used to reduce the size of the complete model. Finally, the optimal controller gain and the optimal observer gains are calculated for the rotor operating in transient motion and the efficiency of the methodology presented is demonstrated from simulation results.

2. Equations of Motion of the Rotor

The equation that express the dynamic behavior of a rotor in transient motion, modeled by the Finite Element Method, is given in the matrix form by:

$$[M]\{\ddot{d}\}+\left[D_S+\dot{f}\,G\right]\{\dot{d}\}+\left[K_S+\ddot{f}\,K_2\right]\{d\}=F_{ext} \tag{1}$$

where M is the inertia matrix, D_S is the damping matrix of the system, G is the gyroscopic matrix, K_S is the stiffness matrix, K_2 is a stiffness matrix resulting from the transient motion, d is the system nodal displacement and F_{ext} is the vector of external forces. These matrices can be found in Lalanne and Ferraris (1997).

In this contribution the disc elements are assumed as being rigid and have only kinetic energy. Shaft elements are elastic systems exhibiting both kinetic and strain energy. The shaft finite element is shown in Fig.1: the element has 2 nodes; each node has 4 degrees of freedom, namely, 2 displacements (u, w), and 2 rotations (q, y). Then, the nodal displacement vector of the shaft element is given by:

$$d=\{u_1,w_1,q_1,y_1,u_2,w_2,q_2,y_2\}^T \tag{2}$$

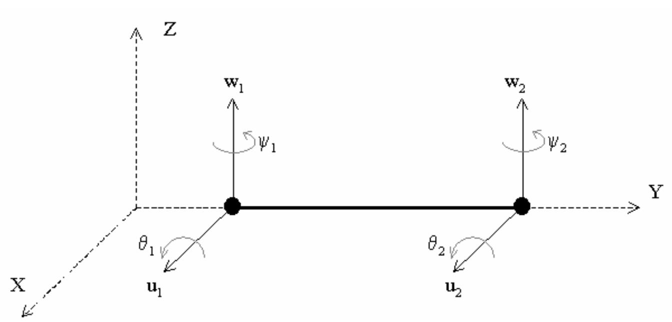


Figure 1. Shaft finite element.

The pseudo-modal technique is used here to reduce the dimension of the finite element model. The goal is to control the first m modes of the system ($m < N$, where N is the total number degrees of freedom). For this purpose, the modal transformation matrix Φ contains the m first modes of the non-gyrocopic associated symmetric system and the reduced modal coordinate vector q is given by:

$$d=\Phi\,q \tag{3}$$

Substituting Eq. (3) into Eq.(1) and multiplying the resulting expression by Φ^T, one has:

$$\Phi^T[M]\Phi\,\ddot{q}+\Phi^T[D_S]\Phi\,\dot{q}+\Phi^T\left[\dot{f}\,G\right]\Phi\,\dot{q}+\Phi^T[K_S]\Phi\,q+\Phi^T\left[\ddot{f}\,K_2\right]\Phi\,q=\Phi^T F_{ext} \tag{4}$$

Thus, the reduced equation of motion for the system (in its modal base) is written in the matrix form as:

$$\left[\tilde{M}\right]\{\ddot{q}\}+\left[\tilde{D}_S+\dot{f}\,\tilde{G}\right]\{\dot{q}\}+\left[\tilde{K}_S+\ddot{f}\,\tilde{K}_2\right]\{q\}=\tilde{F}_{ext} \tag{5}$$

where, $\left[\tilde{M}\right]=\Phi^T[M]\Phi$, $\left[\tilde{D}_S\right]=\Phi^T[D_S]\Phi$, $\left[\tilde{G}\right]=\Phi^T[G]\Phi$, $\left[\tilde{K}_S\right]=\Phi^T[K_S]\Phi$, $\left[\tilde{K}_2\right]=\Phi^T[K_2]\Phi$ and $\tilde{F}_{ext}=\Phi^T F_{ext}$

Equation (5) can be rewritten in the space state form:

$$\{X\}=[A]\{X\}+[B]\{u\} \tag{6-a}$$

$$\{Y\}=[C]\{X\} \tag{6-b}$$

where X is the state vector, Y is the system output vector, u is the vector of excitation and control, A is the dynamic system matrix, B is the input system matrix and C is the output system matrix, as defined below.

$$\{X\} = \{q\ \dot{q}\}^T \quad (7)$$

$$\{u\} = \lfloor\lfloor f_c \rfloor \lfloor f_p \rfloor\rfloor \quad (8)$$

$$[A] = \begin{bmatrix} 0 & I \\ -\tilde{M}^{-1}\left(\tilde{K}_S + \ddot{f}\tilde{K}_2\right) & -\tilde{M}^{-1}\left(\tilde{D}_S + \dot{f}\,\tilde{G}\right) \end{bmatrix} \quad (9)$$

$$[B] = \begin{bmatrix} 0 & 0 \\ -\tilde{M}^{-1} & -\tilde{M}^{-1} \end{bmatrix} \quad (10)$$

where I is the identity matrix, $f_c = \Phi^T F_c$ and $f_p = \Phi^T F_p$. F_c is the control force and F_P is a perturbation force.

3. Control Strategy

Linear control theory (Kwakernaak and Sivan, 1972) has advanced rapidly and became a powerful tool to solve linear feedback control problems. The main ingredients of modern linear control theory involve state space description of the system, control optimization in terms of quadratic performance criteria, and Kalman-Bucy optimal state reconstruction theory. The advantages of modern linear control theory over the classical theory is its applicability to control problems of multi-input-multi-output and time-varying systems, while the classical theory is essentially restricted to single-input single-output and time-invariant systems.
A digital linear quadratic regulator is used in the present contribution to perform the control of the system depicted in Fig. 2.

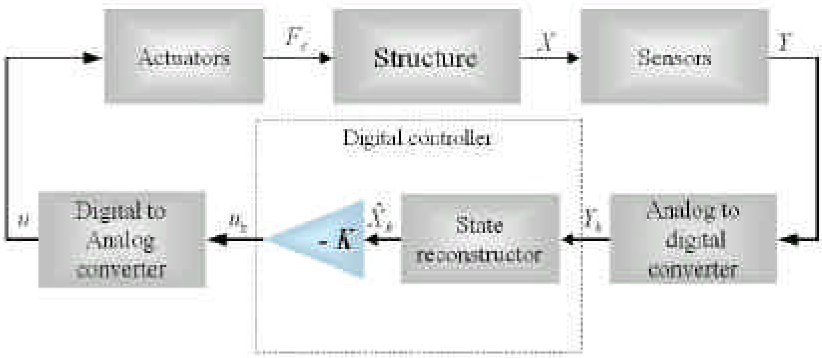


Figure 2. Digital controller components

The continuous matrices A, B and C can be digitalized as follow.

$$[A_d] = e^{[A]\Delta t} \quad (11)$$

$$[B_d] = [A]^{-1}\left[e^{[A]\Delta t} - [I]\right][B] \quad (12)$$

$$[C_d] = C\, e^{[A]\Delta t} \quad (13)$$

where Dt is the discrete time interval. The discrete state space equation of the system is written as:

$$\{X\}_{k+1} = [A_d]\{X\}_k + [B_d]\{u\}_k \quad (14\text{-a})$$

$$\{Y\}_k = [C_d]\{X\}_k \quad (14\text{-b})$$

Given the system described by Eq. (14-a), according to Ogata (1990) the optimal control problem is defined as finding the control input u_c, which minimizes a performance index as given by Eq. (16). The control force is represented by Eq. (15). This means that the gain matrix K is the solution of the optimization problem.

$$\{u_c\}_k = -[K]\cdot\{X\}_k \quad (15)$$

$$J = \sum_{k_{init}=0}^{k_{fin}-1} \left(\{X\}_k^T \cdot [Q] \cdot \{X\}_k + \{u\}_k^T \cdot [R] \cdot \{u\}_k \right) \tag{16}$$

where Q is a positive definite or semidefinite real and symmetric weighting matrix that is related with the relative importance of each state of the system and R is a positive definite real and symmetric matrix that is related with the energy consumption in each actuator to perform the control.

Figure 3 illustrates the optimal control scheme.

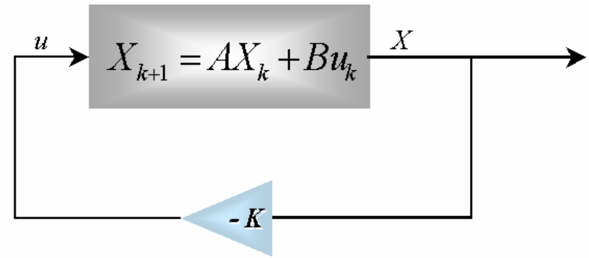


Figure 3. Optimal control scheme.

The matrix K that solves the optimal control problem is given by:

$$[K] = \left([R] - [B_d]^T \cdot [S] \cdot [B_d]\right)^{-1} \cdot [B_d]^T \cdot [S] \cdot [A_d] \tag{17}$$

where S is a positive definite matrix obtained by solving the Riccati equation, Eq. (18):

$$[A_d]^T \cdot [S] \cdot [A_d] - S - \left([A_d]^T \cdot [S] \cdot [B_d]\right) \cdot \left([R] + [B_d]^T \cdot [S] \cdot [B_d]\right)^{-1} \cdot \left([B_d]^T \cdot [S] \cdot [A_d]\right) + [Q] = 0 \tag{18}$$

4. Optimal reconstruction of the modal state

In the present contribution, the control is generated from the modal state of the system. As the modal state of the system cannot be measured directly, it must be reconstructed from measurements and control history, Gaudiller and Der Hagopian (1995), as illustrated in Fig. 4. Consequently, it is necessary to define state observers such that the reconstructed state is optimal in the sense that the mean square reconstruction error is minimized. The problem can be formulated as follows.

$$\{X\}_{k+1} = [A_d]\{X\}_k + [B_d]\{u\}_k + \{w_1\}_k \tag{19-a}$$

$$\{Y\}_k = [C_d]\{X\}_k + \{w_2\}_k \tag{19-b}$$

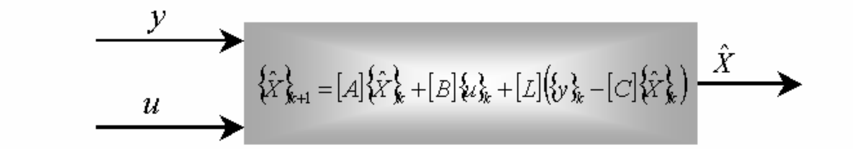


Figure 4. Modal state reconstruction.

where w_1 and w_2 are vectors of stochastic noise related to the state variable and sensor measurements, respectively. The corresponding covariance matrices are computed from the following equations:

$$Q_E = E\left(\{w_1\} \cdot \{w_1\}^T\right) \tag{20-a}$$

$$R_E = E\left(\{w_2\} \cdot \{w_2\}^T\right) \tag{20-b}$$

$$0 = E\left(\{w_1\} \cdot \{w_2\}^T\right) \tag{20-c}$$

where E denotes the expected value operator. The state variables can be estimated by using Eq. (21).

$$\{\hat{X}\}_{k+1} = [A_d]\{\hat{X}\}_k + [B_d]\{u\}_k + [L](\{y\}_k - [C_d]\{\hat{X}\}_k) \quad (21)$$

L is the optimal observer gain matrix, which is obtained by a procedure similar to the one used to obtain the gain matrix of the controller, according to Eq. (22).

$$[L]_k = ([A_d]\cdot[S_E]_k\cdot[C_d]^T)\cdot([R_E]+[C_d]\cdot[S_E]_k\cdot[C_d]^T)^{-1} \quad (22)$$

Similarly, S_E is given by the solution of the Riccati equation:

$$[S_E]_{k+1} = [A_d]\cdot[S_E]_k\cdot[A_d]^T - ([A_d]\cdot[S_E]_K\cdot[C_d]^T)\cdot([R_E]+[C_d]\cdot[S_E]_k\cdot[C_d]^T)^{-1}\cdot([C_d]\cdot[S_E]_k\cdot[A_d]^T)+[Q_E] \quad (23)$$

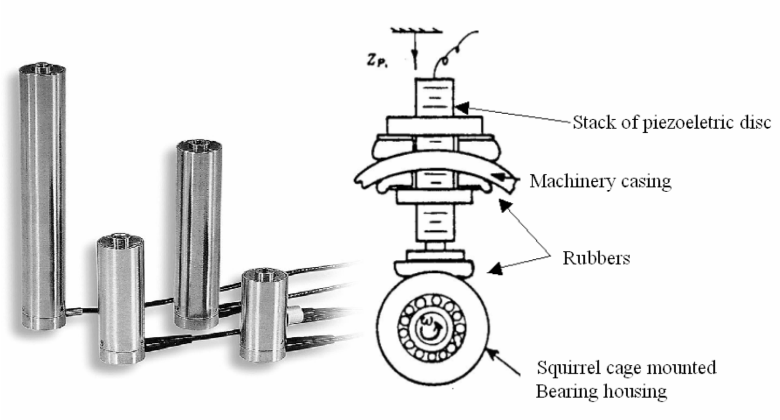
with

$$[S_E]_{k_0} = [S_E]_0 \quad (24)$$

5. Piezoelectric pushers

The piezoelectric pushers appear as an interesting alternative to perform the control of rotating machinery. When the piezoelectric ceramics is submitted to a mechanical strain it generates an electric voltage in its surfaces. This phenomenon is known as the direct piezoelectric effect. Conversely, if an electrical voltage is applied to the ceramics, it results a mechanical strain, corresponding to the inverse piezoelectric effect. Consequently, by bonding a piezoelectric ceramics on the surface of the structure and applying a controlled electrical voltage to the piezo, it is possible to obtain a resulting control force that is introduced to the system.

In certain practical cases, however, due to the inherent necessity of obtaining high stiffness, the piezoelectric actuator is composed by a stack of piezoelectric ceramic discs, mounted one on the top of other and electrically connected in parallel, as shown in Fig. 5-b.



a) b)
Figure 5. Stacks of piezoelectric ceramics.

5. Active Control for the Flexible Rotor

The above methodology is now used to control the vibrations of a flexible rotor whose model is represented by Fig. 6. The characteristics of the rotor-bearing system are presented in the Tab. 1. Fig. 7 shows the rotor Campbell diagram.

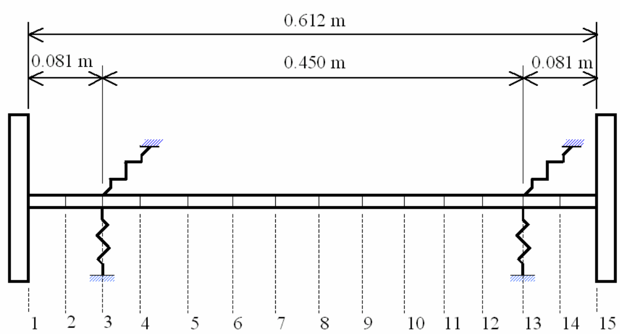


Figure 6. Rotor model

Table 1. Characteristics of the rotor-bearing systems

Rotor	
Disc mass	1.5 Kg
Disc diameter	0.1032 m
Shaft Diameter	0.0252 m
Material	Steel
Bearings	
Bearing mass	1.5 Kg
Stiffness in the x direction	$8*10^6$ N/m
Stiffness in the z direction	$8*10^6$ N/m

5.1. Steady-state case

The simulations are first carried out for the rotor operating in steady state at 10000 rpm, close of its first critical speed. The rotor is excited by an unbalance mass (0.005 Kg) placed in the edge of the disc located in node 15. The signal control is injected at nodes 3 and 13 in the *x* and *z* directions. The sensors are placed at nodes 1 and 15 and measurements are collected in the directions *x* and *z*. The first eight bending modes in each direction are used to obtain the modal finite element model. Using the functions *dlqr* and *dlqe*, from MATLAB®, respectively, one makes the calculation of the gain matrix of the controller and the gain matrix of the observer. Matrices *Q* and *R* are required to compute the controller gain. All states are set with the same importance, $Q = I$, and the energy of all actuator are set in the same way, that is $R = 10^{-5}*I$.

The results for the steady state case are shown in Figs. 8 to 10. Vibration reductions as high as 72%, 53% and 49% are achieved in the nodes 1, 8 and 15 respectively, immediately after starting the control action over the rotor (at time instant equal to 1s).

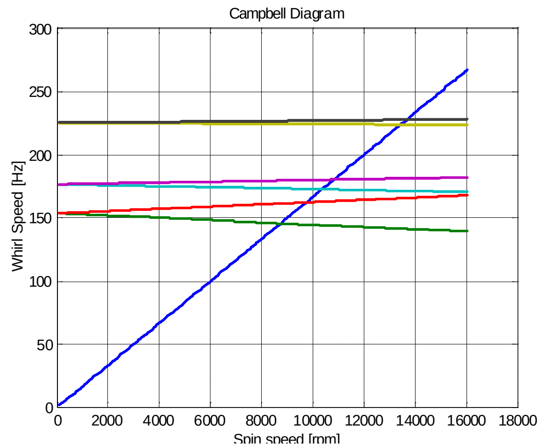


Figure 7. Campbell diagram.

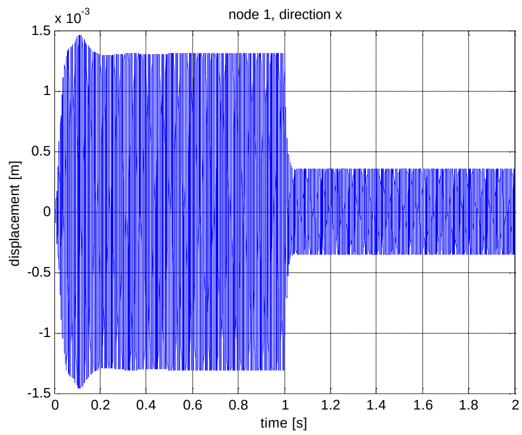


Figure 8. Unbalance response (node 1).

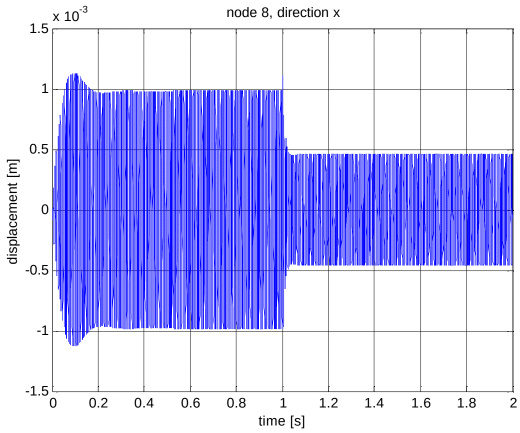


Figure 9. Unbalance response (node 8).

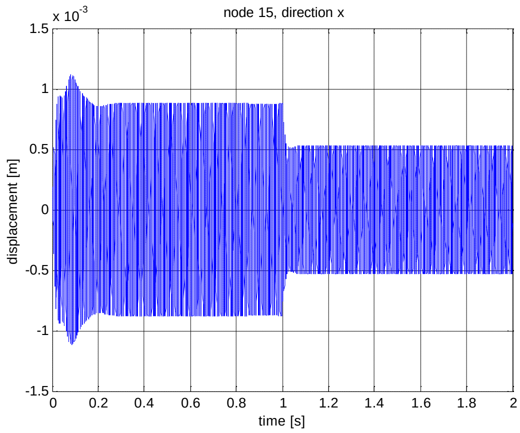


Figure 10. Unbalance response (node 15).

5.2. Transient Case

The interest now is focused in designing a controller for the rotor operating in transient motion. The rotor speed is linearly increased from 0 to 16000 rpm in 15s. Before reaching its operating speed the rotor passes through its first three critical speeds. It is considered that the rotor is submitted to the same excitation force as in the previous case. The control is designed for the rotor operating at 9000 rpm and is used for the rotor running in the range 6400 to 14400 rpm. A different estimator gain is calculated for each 0.75s time interval. All the states and the energy introduced by all the actuators are set with the same importance $Q = I$ and $R = 5*10^{-6}*I$, respectively.

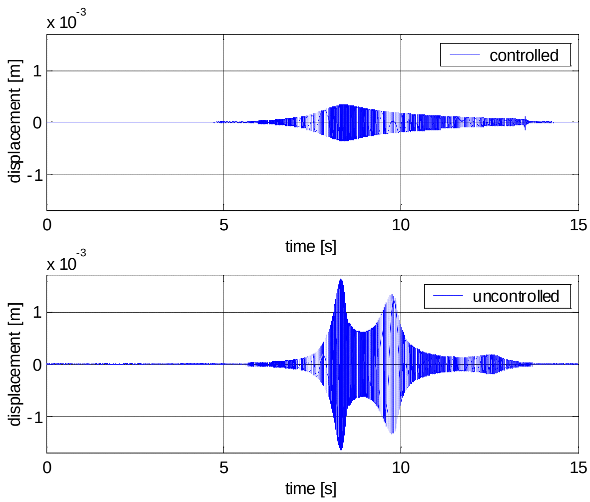


Figure 11. Unbalance response in transient motion (node 1)

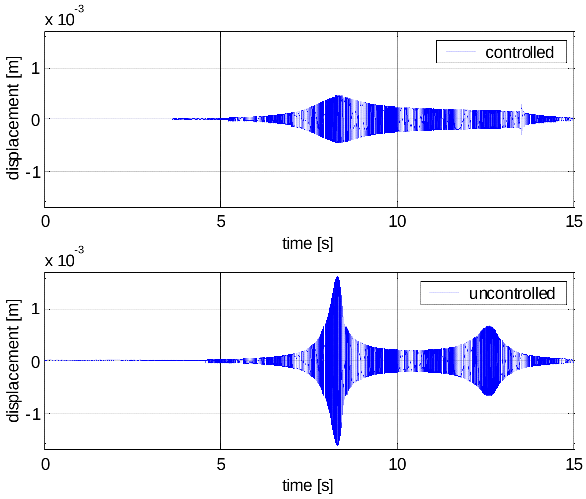


Figure 12. Unbalanced response in transient motion, node 8

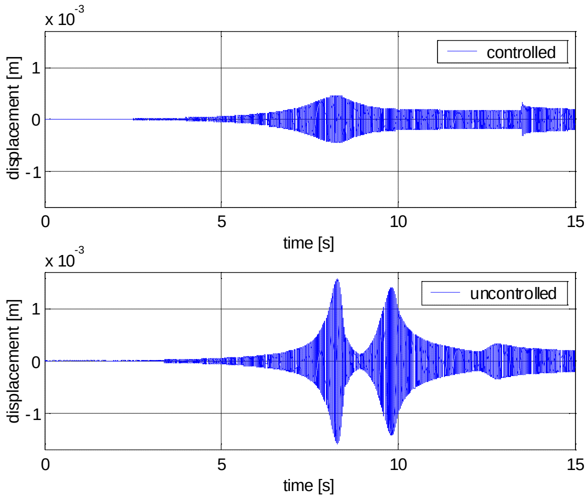


Figure 13. Unbalanced response in transient motion, node 15

Figures 11 to 13 show the unbalance responses for the controlled and uncontrolled rotor in its transient motion. It can be observed that the controller attenuates satisfactorily the vibration level. Vibration reductions as high as 78%, 72% and 71% are achieved in the nodes 1, 8 and 15, respectively, when the rotor passes through its first critical speed. Fig. 14 that shows the electrical voltage at the actuators 1 and 3 represents the control effort. Similar results are obtained for actuators 2 and 4.

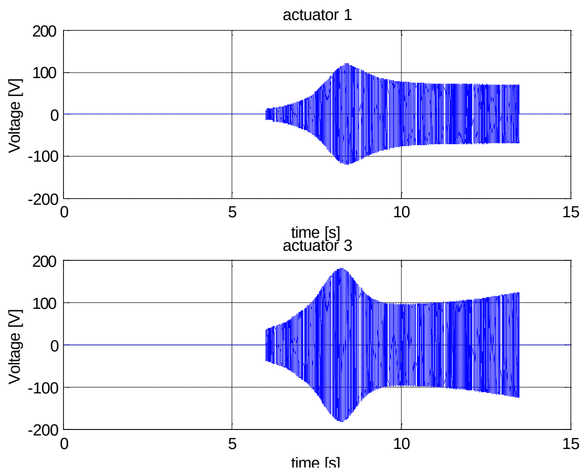


Figure 14. Control effort.

6. Conclusions

An optimal active controller is designed for a rotor excited by an unbalance force for the cases in which the rotor is operating in steady state and transient motion. Results from numerical simulations illustrate the good performance of the methodology proposed. The performance of the LQR controller is intrinsically related to the choice of matrices *Q* and *R*. Next step of this work will be to carry out experimental tests in a rotor test rig equipped with piezoelectric pushers arranged on two planes.

7. References

Adams, M. L., and McClosKey, T. H, 1990, "A Feasibility and Technology Assessment For Implementation of Active Rotor Vibration Control System in Power Plant Rotating Machinery".

Althaus, J., Stelter, P., Feldkamp, B. and Adam, H., 1993, "Aktives Hydraulisches Large Für Eine Schneckenzentrifuge", Schwingungen in Rotierenden Maschinen II, Wiesbaden, Germany, pp.28-36.

Bonneau, O., and Frêne, J., 1997, "Non-Linear Behavior of a Flexible Shaft Partly Supported by a Squeeze Film Damper", Wear 206, pp.244-250.

Burrows, C. R., Sahinkaya, M. N. and Clements, S, (1989), "Active Vibration Control of Flexible Rotors: An Experimental and Theoretical Study", Proc. R. Soc. London. A 422, pp.123-146.

Forte, P., Paternò, M., and Rustighi, E., 2002, "A Magnetorheological Fluid Damper for Rotor Aplications" Proc. of Sixth International Conference on Rotor Dynamic, Sydney, Australia, Vol I, pp.63-70.

Gaudiller, L., and Der Hagopian, J., 1995, "Active Control of Flexible Structures Using a Minimum Number of Components," Journal of Sound and Vibration (1996) 193(3), 713-741.

Kwakernaak, H., and Sivan, R., 1972, "Linear Optimal Control Systems," John Wiley & Sons, New York.

Lalanne, M., and Ferraris, G., 1997, "Rotordynamics Prediction in Engineering," 2nd Edition, John Wiley and Sons, New York.

Malhis, M., Gaudiller, L., and Der Hagopian, J., 2002, "Fuzzy Modal Control of a Flexible Rotor by Piezoeletric Actuators Arranged on a Plane", Proc. of Sixth International Conference on Rotor Dynamic, Sydney, Australia, Vol I, pp.101-108

Nicoletti, R., and Santos., I, F., 2001, " Vibration Reduction of Rotating Systems Using Tilting-Pad Bearings and PD Controlers", Proc. of Nineth International of Dynamic Problems of Mechanics, Florianopólis, Brazil, pp.303-308.

Ogata, K., 1990, "Modern Control Engineering", Prentice-Hall, New York.

Palazzolo, A, B., Lin, R. R., Alexander, R. M., Kascak, A. F., Montague, G. T. 1989, "Piezoeletric Pusher for Active Vibration Control of Rotating Machinery", Transactions of ASME, July, Vol. 111, 298-305.

Palazzolo, A, B., Kascak, A. F., Montague, G. T., and Kiraly. L. J., 1991, "Hybrid Active Vibration Control of Rotorbearing Systems Using Piezoeletric Actuators", Modal Analysis, Modelling, Diagnostics, and Control - Analytical and Experimental, ASME, Vol. 38.

Ramirez, W.F., 1994, "Process Control and Identification", Academic Press, New York

Schweitzer, G., and Lange, R., 1976, "Characteristics of a Magnetic Rotor Bearing for Active Vibration Control", In ImechE Conf. On Vibs. In Rotating Machinery, Cambridge, Paper no. C239/76.

Ulbrich, H., and Althaus, J., 1989, "Actuador Design for Rotor Control", 12th Conference on Vibration and Noise, Montreal, Canada, pp.17-22