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ON NUMERICAL SIMULATIONS OF A NONLINEAR SELF-EXCITED DYNAMIC SYSTEM WITH TWO NON-IDEAL SOURCES

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Abstract. *In the work, the dynamic behavior of self-synchronization and synchronization between two unbalanced identical direct current motors with limited power supported one nonlinear spring (Duffing type) and one damper (Raleigh self-excitation) that are examined by numerical simulations. Furthermore, we study the interaction between on-ideal excitation and self-excitation. In this paper, mathematically we implemented the parametrically excitation of two motors that are switched on/off.*

Keywords: *self-synchronization, non-ideal system, nonlinear system, self-excited system*

1. Introduction

In engineering practice, we can distinguish systems which oscillations are caused by different reasons. The well-known oscillations are:

- Self-excited systems in which, roughly speaking, a constant input produces periodic output;
- Parametrically excited systems characterized by periodically changing in time parameters and
- Systems excited by an external force.

All these systems were comprehensively analyzed in literature separately. However, we can find some papers devoted interaction between different kinds of vibrations, for example: self and external excited vibrations, self and parametric excited vibrations, as well as between self-parametric and external excited vibrations (Warminski et al., 2001; 2002).

We notice that when a forcing function is independent of the system it acts on, and then the problem is called ideal. In such case, the excitation may be formally expressed as a pure function of time. If in a certain model its ideal source is replaced by a non-ideal source, the excitation can be put in the form, where it is a function, which depends on the response of the system. Therefore, non-ideal source cannot be expressed as a pure function of time but rather as an equation that relates the source to the system of equations that describes the model. Hence, non-ideal models always have one additional degree of freedom as compared with similar ideal (Kononenko, 1969); (Balthazar et al., 1997); (Balthazar, et al., 2001, 2003, 2004).

In current literature, the name "self-synchronization" is used for synchronous rotation in the absence of any "direct" kinematics coupling between rotating components. The behavior of the phenomenon of self-synchronization has been studied by a number of authors. Among them we mention the works of (Blekhman, 1998); (Blekman, 2000) and (Dimentberg, 2001). Recently, (Palacios, 2002) and (Palacios, 2003a,b) and (Balthazar et al., 2004), studied self-synchronization of the two identical DC motors, with a limited power supply and with masses attached eccentrically to their rotating shafts supported by a structural frame.

We also remarked that the synchronization phenomenon and accompanying effects of vibration capture and increasing were first discussed in the famous Huygens work about two pendulums. Being attached to an elastic beam the pendulums begin oscillating in phase after a certain period of time. The synchronization phenomenon is caused by the interaction of rotating and oscillating motions of the objects joined together with mechanical links (Bleckman, 1998).

In this paper, we are interested in analyze the influenced of the response of the nonlinear and self-excited system on the two DC motors being an extension of the work of (Brasil, 1999) and (Warminski and Balthazar, 2003).

We organize this work as follows.

In Section 2 we define the mathematical model to be analyze and the derivation of the governing equations. In Section 3 we carried out some numerical simulations results. In Section 4 we present the concluding remarks of this paper. In Section 5 we do some acknowledgements and finally we present the bibliopharfic references used in this work.

2. Mathematcal model and derivation of the governing equations

Let consider nonlinear and self-excited model, which includes two identical direct current (DC) motors with limited power, operating on a structure (Fig. 1).

The excitation of the system is limited by the characteristic of the energy source. Vibration of the system depends on the motion of the motors, and the energy sources motion depends on vibration of the system, as well. Then, coupling of the vibrating oscillator and the two DC motors takes place.

Hence, it is important to analyses what will happen to the motors, as the response of the system changes.

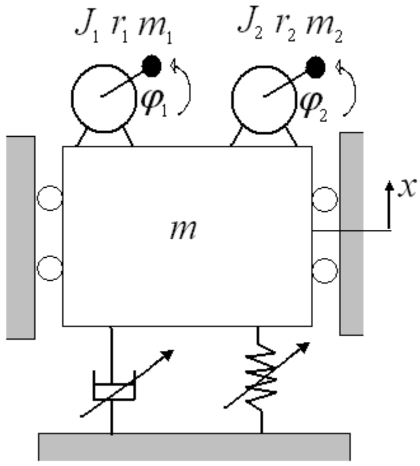

Figure 1. Non-ideal nonlinear and self-excited system model

Taking into account the differential equation of the complete electro-mechanical system presented in (Warminski and Balthazar, 2001) and Fig. 1, we can write:

$$J_s\ddot{\varphi}_s = L_s(\dot{\varphi}_s) - H_s(\dot{\varphi}_s) + m_s r_s \ddot{x}\cos\varphi_s - m_s g r_s \cos\varphi_s\,,\ (s = 1,2)$$

$$m\ddot{x} + (-c_1 + \hat{c}_1\dot{x}^2)\dot{x} + (kx + k_1x^3) = \sum_{j=1}^{2} m_j r_j(\dot{\varphi}_j^2 \sin\varphi_j - \ddot{\varphi}_j \cos\varphi_j) \tag{1}$$

where

x is oscillatory coordinate of vibrating body; φ_s is rotational coordinate of each DC motor; $\dot{\varphi}_s$ is rotational speed of rotors; J_s is a moment of inertia of each motor; m_s is unbalanced mass of each motor; r_s is eccentricity of each unbalanced mass; $L_s(\dot{\varphi}_s)$ is a controlled torque of each DC motor; $H_s(\dot{\varphi}_s)$ is a resistance torque of each DC motor.

In order, to obtain the governing equations of the system in their dimensionless form, we define the non-dimensional time $\tau = \omega t$ and the non-dimensional displacement $X = \dfrac{M}{m_1 r_1}x$, where $\omega = \sqrt{\dfrac{k}{M}}$ is natural frequency of the system and $M = m + m_1 + m_2$.

Hence, we will obtain:

$$\varphi_s'' = \hat{a}_s - \hat{b}_s \varphi_s' + \beta_s R_s X'' \cos\varphi_s \quad (s = 1,2)$$

$$X'' + (-\mu + qX'^2)X' + (X + pX^3) = \sum_{j=1}^{2} R_j (\varphi_j'^2 \sin\varphi_j - \varphi_j'' \cos\varphi_j) \tag{2}$$

The dimensionless parameters in Eq. (2) are defined as:

$$\hat{a}_s = \frac{a_s}{\omega^2 m_1 r_1^2 I_s},\ \hat{b}_s = \frac{b_s}{\omega m_1 r_1^2 I_s},\ R_s = \frac{m_s r_s}{m_1 r_1},\ \beta_s = \frac{m_1}{M I_s},\ \mu = \frac{c_1}{M\omega},\ q = \frac{\hat{c}_1 \omega m_1 r_1^2}{M^3},\ p = \frac{k_1 m_1 r_1^2}{M^2},\ I_s = \frac{J_s}{m_1 r_1^2}.$$

From mathematical point of view was placed two different parametric excitations in the equation of motion in their dimensionless form from second equation of Eq. (2), we will obtain

$$X'' + (-\mu + qX'^2)X' + (1 - \sum_{j=1}^{2} \alpha_j \cos 2\varphi_j)(1 + pX^2)X = \sum_{j=1}^{2} R_j (\varphi_j'^2 \sin\varphi_j - \varphi_j'' \cos\varphi_j) \tag{3}$$

Next, we will present some numerical results.

3. Numerical results

Next, we carried out, a number of numerical simulations, in order to observe the interaction between the two identical non-ideal DC motors and nonlinear and self-excited structural system (as regular and irregular motions). Furthermore, we observe the self-synchronization and synchronization phenomena in pre-resonance, resonance and post-resonance regions.

The dimensionless nonlinear equations (2) were simulated by using the block diagrams of SIMULINK™ that model the non-ideal parametrically self-excited system. To obtain different regimes in the system we varied the torques of each DC motor and the initial rotational of second motor.

Next we will present some numerical simulation results through seven sets of numerical simulation results.

3.1 First set of results:

The first set of numerical results, shown in Fig. 2, illustrates the development of self-synchronization by intervals when the torques of each DC motor are equal approximately $\hat{a}_1$ = 1 and $\hat{a}_2$ = 0.9.

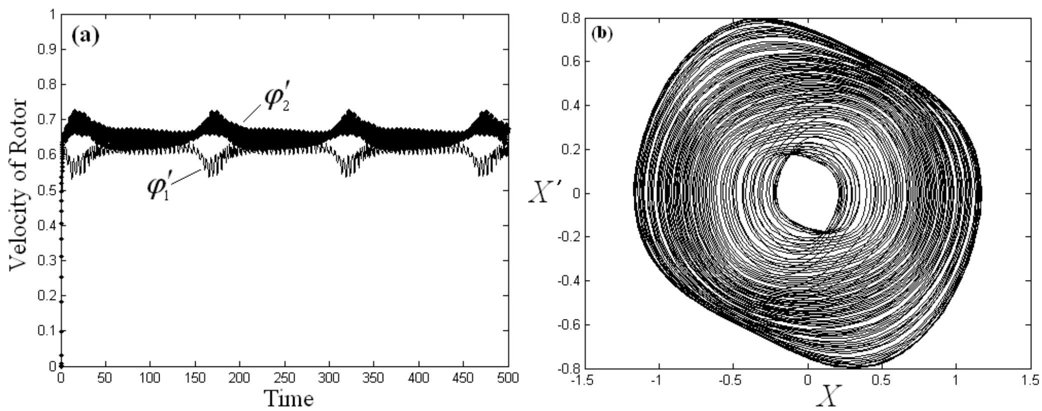

Figure 2. a) Velocity of rotors, b) phase plane for $\hat{a}_1$ = 1 and $\hat{a}_2$ = 0.9: self-synchronization by intervals.

Figure 2(a), shows that the rotors turn in the same direction and arrive it at some average angular velocity in steady state motion where the angular velocities are in phase and anti-phase by intervals (the rotors synchronize phase and anti-phase), the velocities of rotors are below of resonance region (pre-resonance).

Figures 2(b) shows the phase plane of the dynamical behavior of system, in this case, beat phenomenon.

3.2 Second set of results:

The second set of numerical results, shown in Fig. 3, we illustrates the development of self-synchronization when the torques of each DC motor are equal $\hat{a}_1 = 1.0$ and $\hat{a}_2 = 0.93$.

Figure 3(a), shows that the rotors turn in the same direction and arrive it at some synchronous velocity in phase and in steady state motion, the velocities of rotors are below of resonance region (pre-resonance).

Fig. 3(b) shows the dynamical behavior of system on phase plane of periodic motion

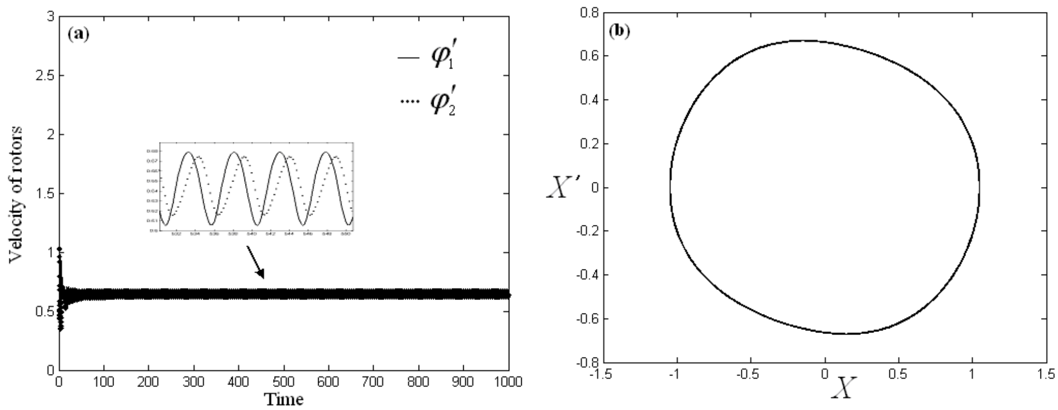

Figure 3. a) Velocity of rotors and b) phase plane for $\hat{a}_1 = 1.0$ and $\hat{a}_2 = 0.93$: self-synchronization.

3.3 Third set of results:

The third set of numerical results, shown in Fig. 4, when the rotors turn in the same direction and arrive it at some synchronous velocity in anti phase by intervals and in steady state motion (see Fig. 4a) for $\hat{a}_1 = 2.0$ and $\hat{a}_2 = 2.005$.

In this case, we have a motion de p-periodic (see Fig 4b).

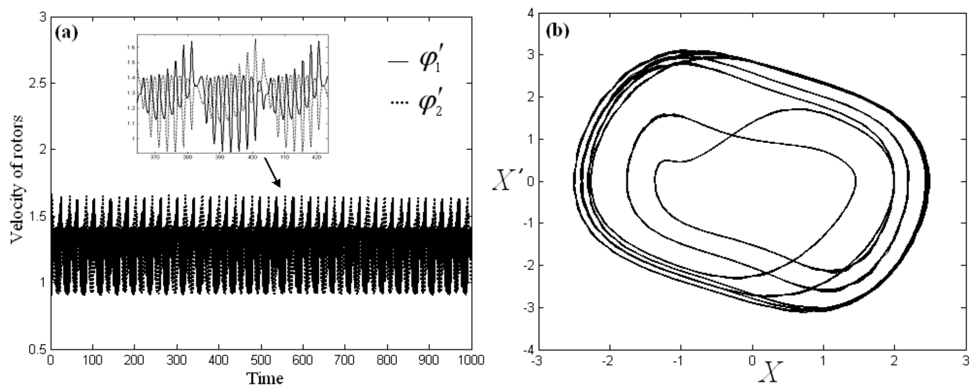

Figure 4. a) Velocity of rotors and b) phase plane for $\hat{a}_1 = 2.0$ and $\hat{a}_2 = 2.005$: absence of self-synchronization.

3.4 Fourth set of results:

The fourth set of numerical results, shown in Fig. 5, we illustrate the absence of self-synchronization when the torques of each DC motor are different $\hat{a}_1 = 3.0$ and $\hat{a}_2 = 2.2$.

Figure 5(a), shows that the rotors turn in the same direction and arrive it at some average angular velocity in steady state motion, the velocities of rotors are in the post resonance region and synchronization anti-phase.

Figures 5(b) shows the dynamical behavior of system on the phase plane of motion p-periodic.

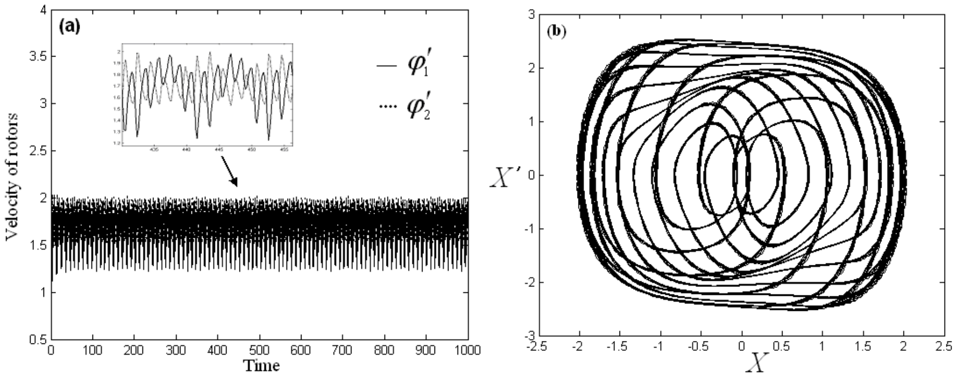

Figure 5. a) Velocity of rotors and b) phase plane for $\hat{a}_1$ = 3.0 and $\hat{a}_2$ = 2.2: absence of self-synchronization.

3.5 Fifth set of results:

The fifth set of numerical results, shown in Fig. 6, we illustrate the presence of chaotic motion on the phase plane when the torques of each DC motor are different $\hat{a}_1$ = 3 and $\hat{a}_2$ for values in the interval [2.3, 2.5].

Here, when the chaotic motion occur in the interval $\hat{a}_2 \in [2.3, 2.5]$ we observe that whose graphs windows has the same qualitative characteristic illustrated as in Fig. 6 corresponding to the torque $\hat{a}_2$ =2.3.

The rotors turn in the same direction and arrive it at some average angular velocity in steady state motion, the velocities of rotors are in the post resonance region.

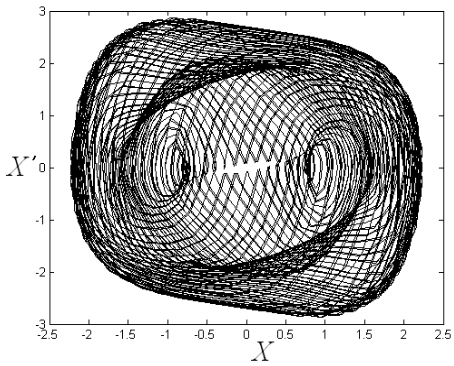

Figure 6. Phase plane plot for $\hat{a}_1$ = 3 and $\hat{a}_2$ $\in$ [2.3, 2.5]: absence of self-synchronization and chaotic motion.

3.6 Sixth set of results:

Figure 7, shows different regimes in the phase plane for the case of a non-ideal parametrically and self excited system when was considered the Eq. (3) in the motion equation and we varied the parameter α_2 of the parametric excitation of the second DC motor. Other parameters are fixed.

For α_2 =1, we see a development chaotic, α_2 =1.5 we see a development periodic with p-period, α_2 =2 we see that is tending to a limit cycle with 1-period, α_2 =3 we see a development periodic with 1-period.

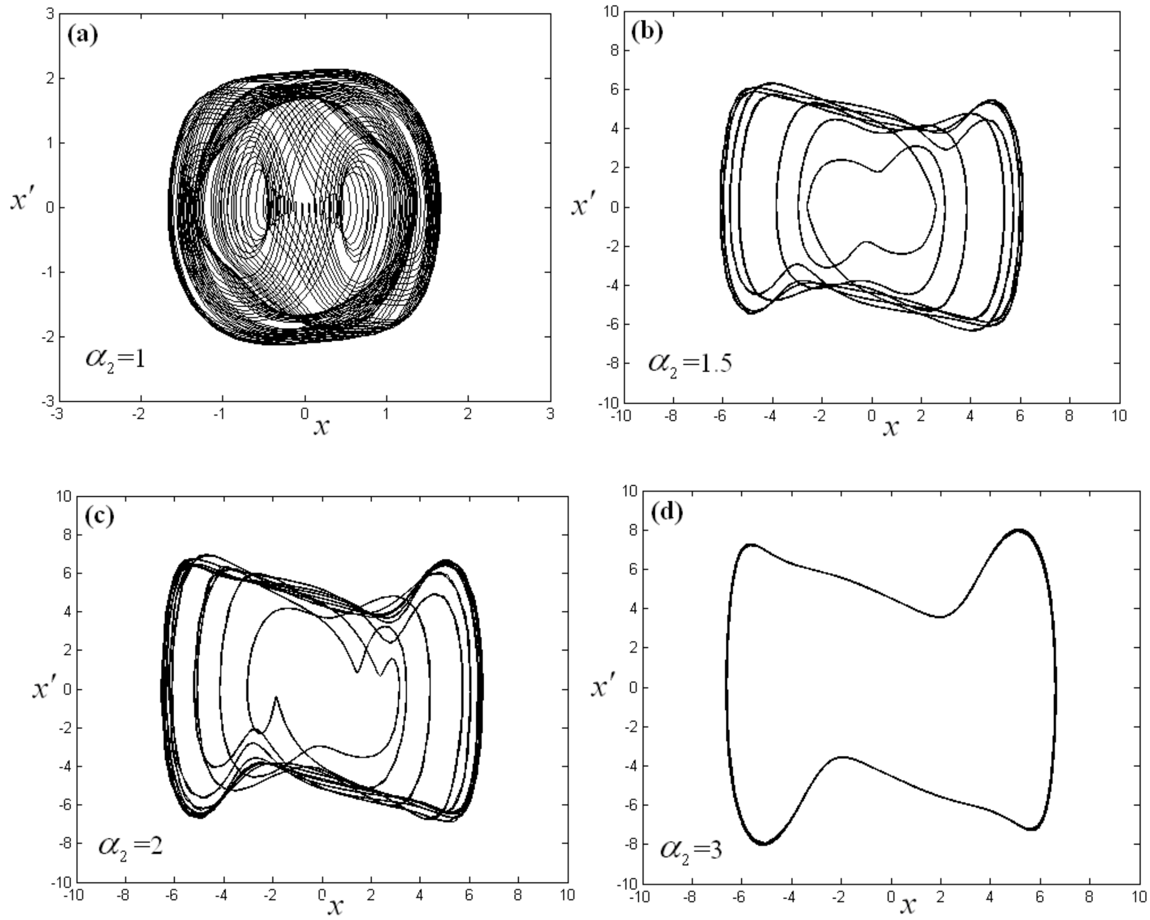

Figure 7. Phase plane for values different of the parameter α_2 of the parametric excitation of second DC motor: (a) α_2 = 1, (b) α_2 = 1.5, (c) α_2 = 2 and (d) α_2 = 3.

3.7 Seventh set of results:

Finally, Figure 8 shows the results of the influence self-excited coefficient q on the cubic nonlinear of the system. In this case, the interaction is examined when the two rotors arrive it at some synchronous velocity, with the cubic nonlinear coefficient fixed p and for values different self-excited coefficient q .

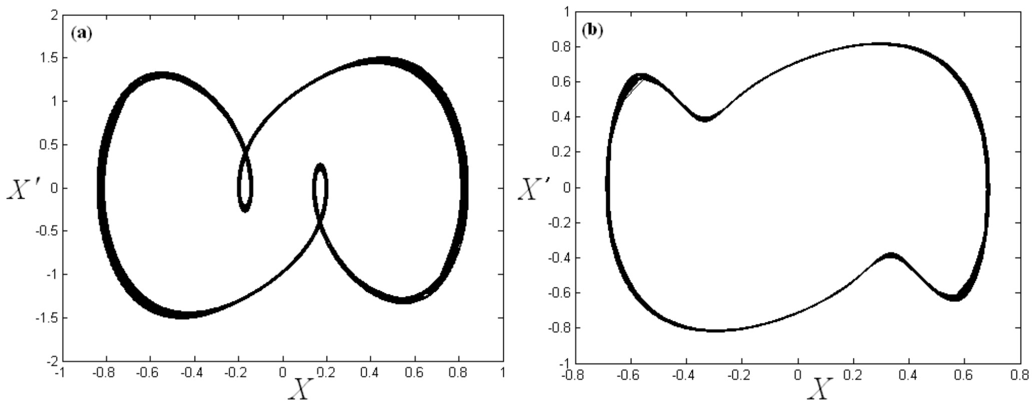

Figure 8. Phase plane for $\hat{a}_1$ = 2.0 and $\hat{a}_2$ =2.01: (a) q =0.05, p =16 and (a) q =4.0, p =16.

4. Conclusions

A particular case of non-linear phenomenon of self-synchronization and synchronization in pre-resonance, resonance and post-resonance regions between the unbalanced dc motors that interacting with the parametrically and self-excited system has been analyzed through numerical simulation.

We remarked that with control parameters (the constants torques) we might control the presence and absence of self-synchronization as synchronization in phase and anti-phase; in this case, we observe motions of regular (periodic, quasi-periodic) and irregular (chaotic). From mathematical point of view, two different parametric excitations were placed in the equation of motion.

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