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MODELLING AND SIMULATION OF MECHANICAL FAULT IN ROTATING MACHINERY USING MATLAB® AND SIMULINK

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***Abstract**: It is common pratice nowadays with critical machinery to measure, either on-line or off-line, the vibration response at a number of suitable locations an to assess the state of health of the equipament based on observed changes in one or more ot the reponse parameters. Early diagnosis of potentially damaging faults or defects can lead to substantial cost savings. A model-based approach to the detection of mechanical faults in rotating machinery using Matlab® and Simulink is studied in this paper. For certain types of faults, for example, an increase in mass unbalance and changes in stiffness and damping, the Finite Elements Methods (F. E. M.) is utilized for developing and evaluating using computer simulation.*

***Keywords**: Modeling, Fault Simulation, Rotating Machinery, Matlab and Simulink.*

1. Introduction

Rotating machines are essential components in most of today's manufacturing and production industries. Because it is usually not practical or economical to the use redundant systems, real-time monitoring, fault detection and diagnostics for rotating machinery equipment is required. Common rotating machinery faults include self-excited vibration, due to system instability, and, more often, vibration due to some externally applied load, such as craked or bent shafts, mass unbalance or parameters variation of journal hydrodynamic bearings (stiffness and damping).

In this paper, model-based techniques are developed for fault detection of faults in rotating machines. Model-based approach involves the establishment of a suitable process model, either mathematical or signal-based (Steffen, 1981; Oliveira, 1999 and Tadeo, 2003), which can estimate and predict process parameters and variables. Isermann (1994) described the main principles involved in model-based procedures an outlined the importance for realistic modeling of faults. He concluded that more than one method of FDI (Fault Detection in Isolation) should be utilized, in order to best reach an accurate diagnosis.

Natke and Cempel (1991) used the definition that a fault will alter the dynamic behavior of a system, to construct a model to detect changes in this dynamic behavior and thus identify faults. Various physical parameters and model sensitivity to fault size are used to detect and locate faults.

Model-based approaches which use statistical analysis (Chen, 2001) and artificial intelligence techniques (Eduardo, 2003) or in the use of hybrid models for analysis of the dynamic behavior (Brito, 2002) play an ever-increasing role in the diagnosis of faults in dynamic systems, particularly in those systems where information may be scarce and estimates need to be made. The context of that work interferes in relation to the model-based techniques.

The aim of this paper was to contribute for the development and application of programs computational (Matlab® and Simulink) capable to proceed to the dynamic behaviour of rotating machinery, with emphasis in the modellling and simulation of mechanical fault, with views to the monitoring of its operation and to its predictive maintenance. The good results of the example show the viability of further studies in this area.

2. Finite Element Modeling of Rotor-Bearing Systems

Rotor-bearing systems are modeled as an assemblage of rigid disks, shaft elements of distributed mass and stiffness, discrete bearings and flexible supports. A finite element scheme based on Timoshenko beam theory is used to produce the equations of the shaft elements (Craig and Roy, 1981). The formulation accounts for the effects of rotary inertia and gyroscopic moments. The elements equations the equations of motion of rigid disk mounted on the shaft gyroscopic (Minh, 1981; LaLanne, 1998).

For the simulation of rotor-bearing system, the classic finite element, the linear beam element, has been employed, with two deflections (y and z) and two rotations (ψ e φ) in the *y*- and *z*-axes, respectively, per node:

$$q = \{ y_1 \quad y_2 \quad \psi_2 \quad z_1 \quad z_2 \quad \varphi_2 \}^T \tag{2.1}$$

The mechanical system considered is a rotor with a flexible shaft coupled to a fixed end electric motor. Between the motor and the disc, there is a bearing of mass m_1 fixed elastically. The Figure 2.1 show schematic model of the rotor-bearing system

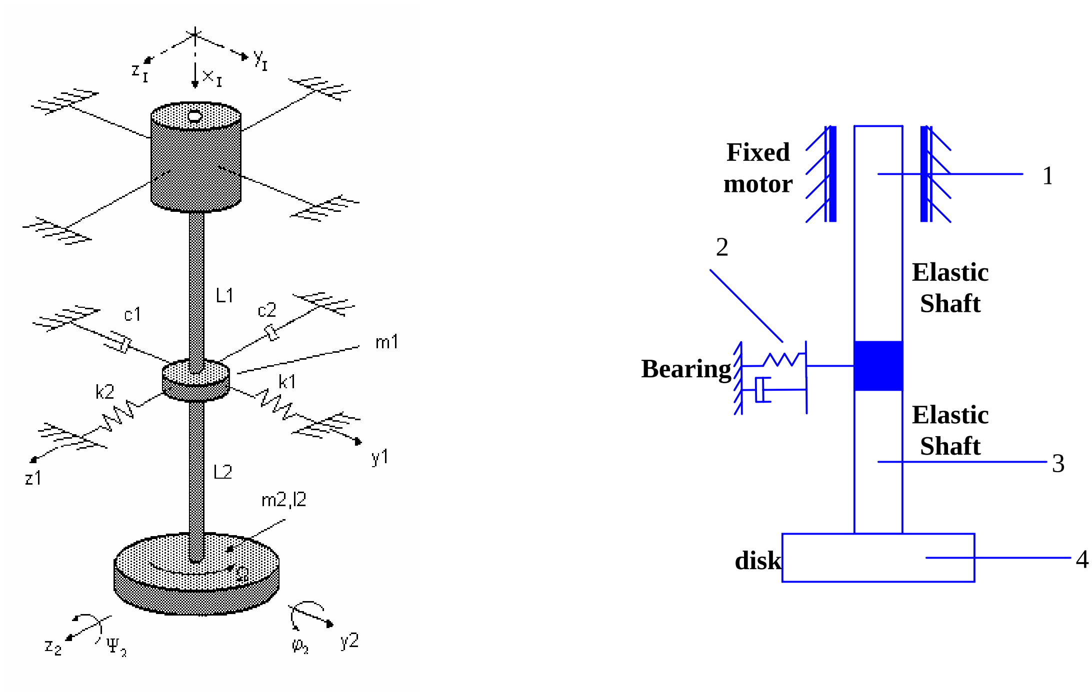

Figure 2.1. Schematic model of the rotor-bearing system

2.1 Equations of motion of elements

The use of the Lagrange equations leads to the determination of the element equations of motion.,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U}{\partial q_i}=Q_i \tag{2.2}$$

Here $i\left(1\le i\le N\right)$ is the number of degrees freedom; q_i are generalized independent coordinates; Q_i are generalized forces, and "$\cdot$" denotes differentiation with respect to time t.

2.1.1 Finite Element of Rigid Disk

A typical rigid disk can be modeled as four-degree-of-freedom rigid body of small thickness. The governing equations of motion of a thin rigid disk follow form the kinetic energy expression of the disk and the unbalance forces due to the mass center eccentricities of the disk and the use of Lagrange equations. Thus,

$$\left(M_T^d+M_R^d\right)*q^{\ddot{d}}-\Omega*G^d*q^{\dot{d}}=Q^d \tag{2.3}$$

Expanding the equation (2.3),

$$\begin{bmatrix} m_2 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & I_2 \end{bmatrix}\begin{Bmatrix} \ddot{y}_2 \\ \ddot{\psi}_2 \\ \ddot{z}_2 \\ \ddot{\varphi}_2 \end{Bmatrix}-\Omega\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{2p} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -I_{2p} & 0 \end{bmatrix}\begin{Bmatrix} \dot{y}_2 \\ \dot{\psi}_2 \\ \dot{z}_2 \\ \dot{\varphi}_2 \end{Bmatrix}= \tag{2.4}$$

$$\begin{Bmatrix} 0 \\ \Omega^2 m_2 e^y \\ 0 \\ \Omega^2 m_2 e^z \end{Bmatrix}\cos(\Omega t)+\begin{Bmatrix} 0 \\ -\Omega^2 m_2 e^y \\ 0 \\ \Omega^2 m_2 e^z \end{Bmatrix}\text{sen}(\Omega t)$$

where, M_T^d is the disk translation mass matrix; M_R^d is the disk rotational mass matrix; G^d gyroscopic matrix of rigid body; $\ddot{q}^d$ is vector of acceleration; $\dot{q}^d$ is vector of velocities; Q^d is vector of forces and moments; m_2 is the disk translational mass; I_2 is the polar mass moment of inertia; e_y and e_z are the mass center eccentricities of the disk in *y* and *z* directions.

2.1.2 Shafts elements of length L_1 and L_2

The shaft is represented as beam with a circular cross-section, and is characterized by strain and kinetic energies. The resulting equation of motion associated with each element is given by

$$K^{shaft1}q^d + K^{shaft3}q^d = F(t) \tag{2.5}$$

where: K^{shaft1} is the stiffness of shaft (element 1-Figure 1) that involve the length L_1 , *E* Young's modulus of the material and *I* moment of inertia. K^{shaft3} is the stiffness of shaft (element 3-Figure 1) that involve the length L_2 , *E* Young's modulus of the material and *I* moment of inertia.

Expanding (2.5),

$$\begin{bmatrix} \frac{12EI}{L_1^3}+\frac{12EI}{L_2^3} & -\frac{12EI}{L_1^3} & \frac{6EIL_2}{L_1^3} & 0 & 0 & 0 \\ -\frac{12EI}{L_1^3} & \frac{12EI}{L_1^3} & \frac{-6EIL_2}{L_1^3} & 0 & 0 & 0 \\ \frac{6EIL_2}{L_1^3} & \frac{-6EIL_2}{L_1^3} & \frac{8EIL_2^2}{L_1^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{12EI}{L_1^3}+\frac{12EI}{L_2^3} & -\frac{12EI}{L_1^3} & \frac{-6EIL_2}{L_1^3} \\ 0 & 0 & 0 & -\frac{12EI}{L_1^3} & \frac{12EI}{L_1^3} & \frac{6EIL_2}{L_1^3} \\ 0 & 0 & 0 & -\frac{6EIL_2}{L_1^3} & \frac{6EIL_2}{L_1^3} & \frac{8EIL_2^2}{L_1^3} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ \psi_2 \\ z_1 \\ z_2 \\ \varphi_2 \end{Bmatrix} = \begin{Bmatrix} F_{y1} \\ F_{y2} \\ F_{\psi_2} \\ F_{z_1} \\ F_{z_2} \\ F_{\varphi_2} \end{Bmatrix} \tag{2.6}$$

2.1.3 Bearing model

The virtual work done by the forces due to the bearing acting on the shaft (LaLanne, 1998). The representation schematic of model of bearing journal is illustrated in Figure 2.2,

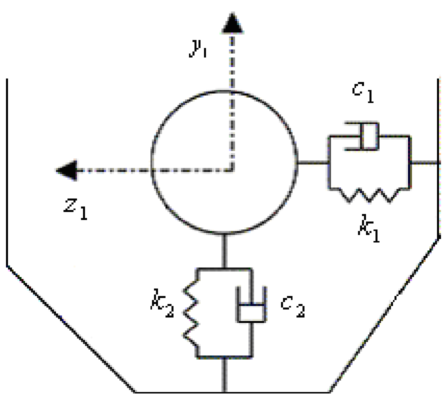

Figure 2.2 – Model of bearing journal

In modeling a rotor bearing system a finite element node is normally chosen at a bearing location, Fig.1. Therefore, the equation of motion of such a rotor element that is in contact with the bearing is given by:

$$M^{m2}\ddot{q}^{m2} + C^{m2}\dot{q}^{m2} + K^{m2}q^{m2} = F^{m2} \tag{2.7}$$

where: M^{m2} : is the bearing journal mass matrix; C^{m2} : is the bearing journal damping matrix; K^{m2} : is the bearing journal stiffness matrix; $\ddot{q}^{m2}$: is vector of acceleration; q^{m2} : is vector of displacement; F^{m2} : forces in bearing journal. Expanding (2.7)

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_1 \end{bmatrix}\begin{Bmatrix} \ddot{y}_1 \\ \ddot{z}_1 \end{Bmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{y}_1 \\ \dot{z}_1 \end{Bmatrix} + \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}\begin{Bmatrix} y_1 \\ z_1 \end{Bmatrix} = \begin{Bmatrix} F_{y1} \\ F_{z1} \end{Bmatrix} \tag{2.8}$$

2.2 Assembly and system equations

In the previous sections the element equations are formulated for a typical element. The equations are then assembled to construct the global equation which describes the dynamics of the rotor-bearing motion are of the form,

$$M\ddot{q}(t) + P\dot{q}(t) + Qq(t) = Su(t) + Hn(t) \tag{2.9}$$

where: *M, P, and Q* are the mass, damping and stiffness matrices of the system. The order of matrices is *n x n*, where $n = 4(n_e + 1) + 2n_b$, with n_e =number of elements, n_b number of bearings. *S and H* are input matrices for the stochastic forces, unbalance forces. The vectors *u(t)* and *n(t)* representing the stochastic(white noise process) and harmonic forces. The vectors $\ddot{q}(t), \dot{q}(t), q(t)$ are acceleration, velocities and displacement.

The matrices have the following internal structure (Eduardo, 2003)

$$M = diag\{m_1 \quad m_2 \quad I_2 \quad m_1 \quad m_2 \quad I_2\} \tag{2.10}$$

$$P = \begin{bmatrix} C_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Omega I_{2P} \\ 0 & 0 & 0 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Omega I_{2P} & 0 & 0 & 0 \end{bmatrix} \tag{2.11}$$

$$Q = \begin{bmatrix} K_1 + 6(W+Y) & -6W & 3WL_2 & 0 & 0 & 0 \\ -6W & 6W & -3WL_2 & 0 & 0 & 0 \\ 3WL_2 & -3WL_2 & 4WL_2^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_2 + 6(W+Y) & -6W & -3WL_2 \\ 0 & 0 & 0 & -6W & 6W & 3WL_2 \\ 0 & 0 & 0 & -3WL_2 & 3WL_2 & 4WL_2^2 \end{bmatrix} \tag{2.12}$$

In the Equation (2.12):

$$W = \frac{2EI}{L_1^3}; \qquad Y = \frac{2EI}{L_2^3} \tag{2.13}$$

$$H = \begin{bmatrix} 0 & 0 \\ m_2 e\Omega^2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & m_2 e\Omega^2 \\ 0 & 0 \end{bmatrix} \tag{2.14}$$

In the table 2.1 shows the descriptions and values used in the simulations.

Table 2.1 –Descriptions and values parameters

Parameters	**Descriptions**	**Values**	**Units**
m_1	Mass of bearing journal	15	kg
m_2	Mass of disk	10	kg
I_2	Moment of inertia transverse of disk	0.25	kg.m^2
I_{2polar}	Moment of inertia polar of disk	0.50	kg.m^2
L_1=L_2	Length between: motor – bearing journal and – bearing journal - disk	0.4	m
K_1	Stiffness in the direction Y_I	90.000	N/m
K_2	Stiffness in the direction Z_I	120.000	N/m
c_1	Damping in the direction Y_I	30.000	kg/s
c_2	Damping in the direction Z_I	37.500	kg/s
Ω	Rotation of rotor	60	rad/s
e	Mass center eccentricity of the disk	0,00001	m

The input matrix *S* has the following internal structure in (2.9):

$$S^T = \{ s_1 \quad s_2 \quad s_3 \quad s_4 \quad s_5 \quad s_6 \} \tag{2.15}$$

In which s_i (i= 1,...6) values constants.

Table 2.2 – Values of input matrix

Variables	**Values**
s_1	1,2
s_2	1,0
s_3	1,0
s_4	1,2
s_5	1,0
s_6	1,0

2.3 Representation of state space equation

The state equation can be obtained,

$$\dot{x}(t) = Ax(t) + Bu(t) + En(t) \tag{2.16}$$

Consider the state vector $x(t)$,

$$x(t) = \begin{Bmatrix} q(t) \\ \cdots \\ \dot{q}(t) \end{Bmatrix}, \tag{2.17}$$

With,

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}Q & -M^{-1}P \end{bmatrix} \tag{2.18}$$

$$B = \begin{bmatrix} 0 \\ M^{-1}S \end{bmatrix} = \begin{bmatrix} 0_{(n,p)} \\ B_{1(n,p)} \end{bmatrix}; \qquad E = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix} \tag{2.19}$$

where *A* is input matrix; *B* and *E* are output matrix.

3. Numerical Simulation

In the numerical simulation have been conducted using variation of the physical parameters (matrices M , P and Q - Equation 2.9). Faults in the rotor can be detected by monitoring the variation of these physical parameters. The data obtained by the software Matlab & Simulink.

To simulate a failure the value of the parameter was varied in the range 10 - 90%. The system was simulated and the signals calculated. In the Table 2.3 show the conditions and physical parameters fault.

Table 2.3 – Conditions and physical parameters fault

Conditions	**Parameter fault**
1	Stiffness k_1
2	Damping c_1
3	Unbalanced mass
4	Stiffness k_2
5	Damping c_2

The Figure 3.1 show the methodology in modeling and simulation of mechanical fault in rotating machinery using MATLAB® and SIMULINK.

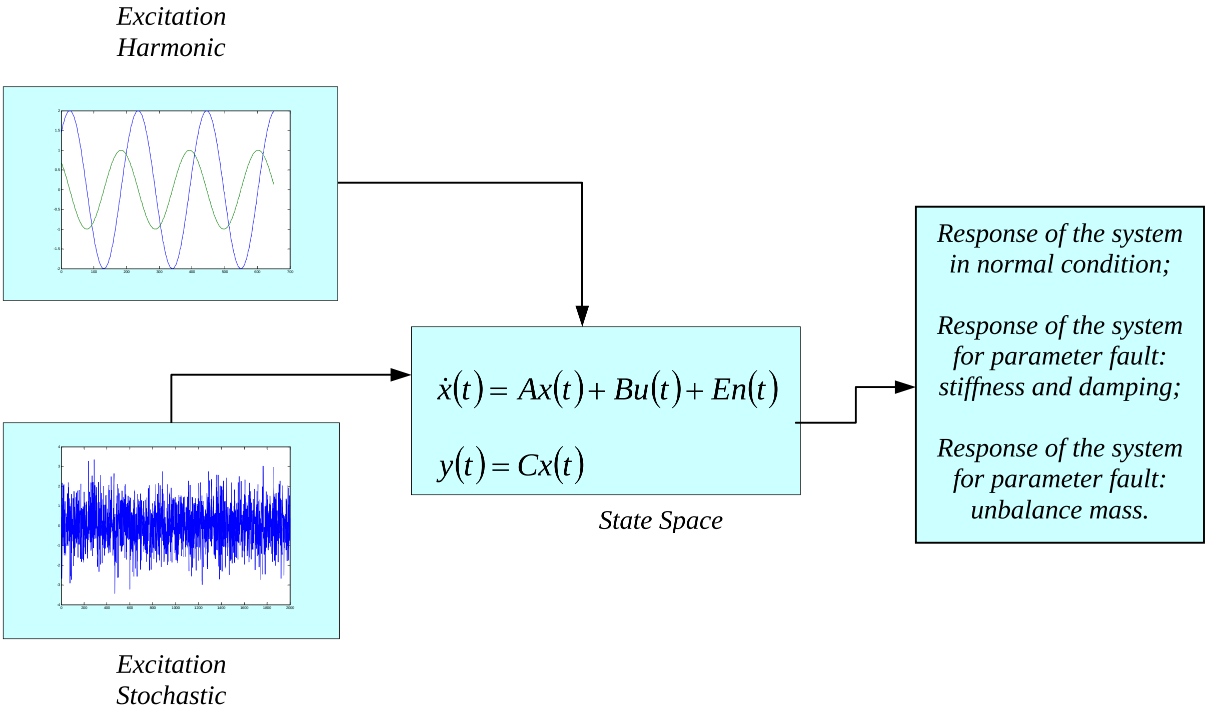

Figure 3.1– Bloch diagram using MATLAB® and SIMULINK.

In the Figure 3.1 the rotating system is excited by harmonic and stochastic forces. The state space equation is obtained for conditions:

- Normal (no fault);
- Fault parameters (stiffness and damping);
- Fault parameter mass unbalance.

4. Results

In order to simulate the conditions of faults in rotating system, ten-speed rotations conditions were obtained for parameters faults (stiffness, damping and mass unbalance) was studied (Figure 4). In the present work, the time domain vibration data was transformed to the frequency domain (FFTs) to obtain their spectra. The Figure 4.1 shows the responses obtained for fault mass unbalance.

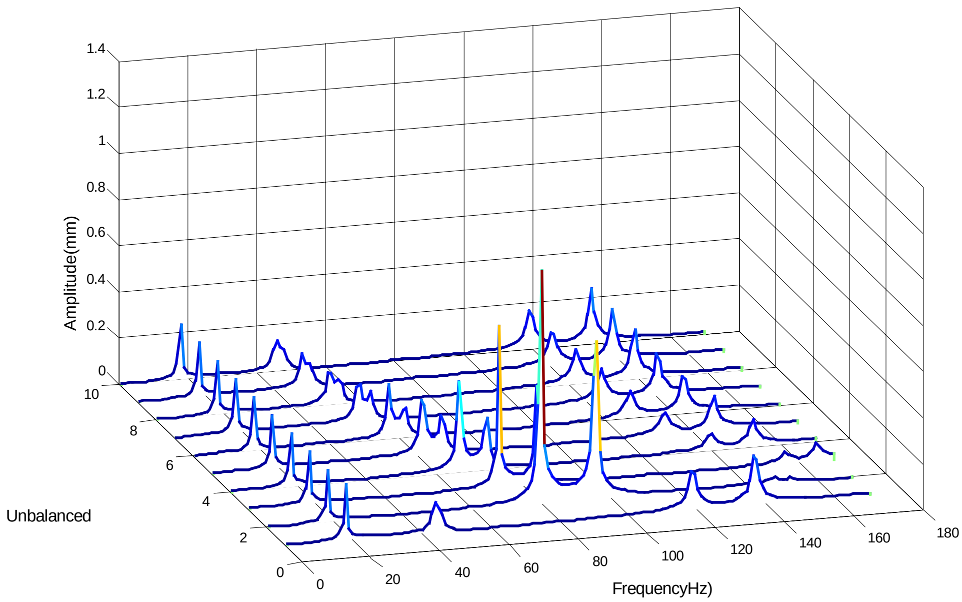

Figure 4.1 – Responses of system for the fault unbalance mass

It is interesting to note that amplitude (Figure 4) presents a constant variation (about of frequency 20 Hz). The others frequencys not occuried.

In the frequency 80 and 120Hz, the amplitude presents significant variation (conditions unbalanced 2, 4 and 6) in the order of 60 - 90%.

The Figure 4.2 shows the responses obtained for fault stiffness of bearing journal (direction y).

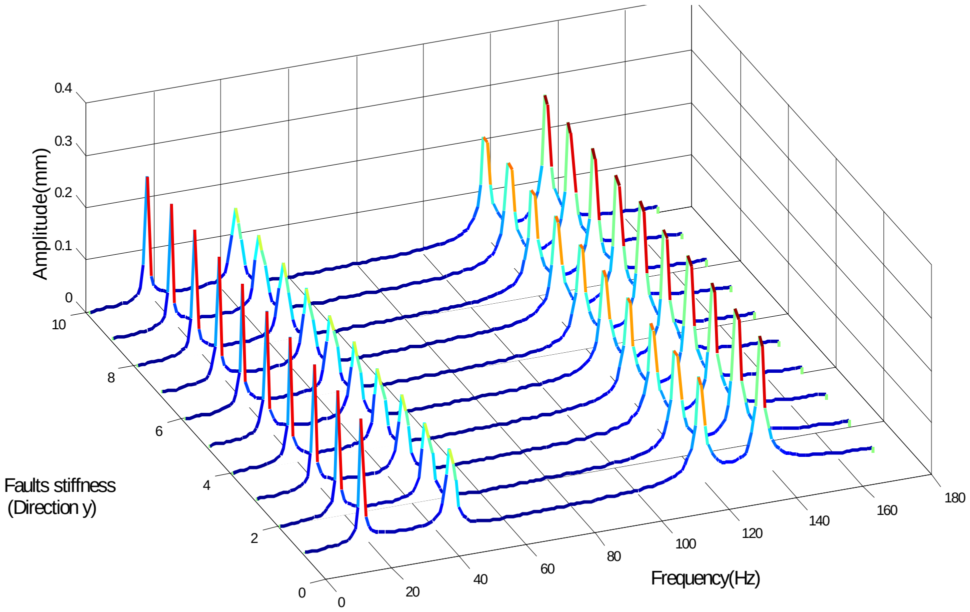

Figure 4.2- Responses of system for fault parameter stiffness of bearing journal

It is interesting to note that amplitude (Figure 4.2) presents a significant variation (about of frequency 20, 40, 120 and 140 Hz). The others frequencys not occuried.

5. Conclusions

This paper has presented model-based techniques in the context of modeling and simulation of mechanical fault rotating machinery. The aim of this paper was to contribute for the development and application of programs computational (Matlab® and Simulink) capable to proceed to the dynamic behavior of rotating machinery, with emphasis in the modellling and simulation of mechanical fault.

A model based fault detection method for a stationary mechanical system was presented. The simulated model of a rotating system was utilized to demonstrated rotor parameter monitoring. Fault in the rotor can be detected by monitoring the variation of the physical parameters due to a comparison of the responses obtained. The analysis of the results is made by rotor system modeled with six degrees of freedom.

Through these examples with good results, the viability of furthering the studies along these lines is demonstrated.

6. Acknowledgement

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