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DESIGN OF OPTIMUM VISCOELASTIC VIBRATION ABSORBERS BASED ON THE FRACTIONAL CALCULUS MODEL

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Abstract. *The so often called vibration absorbers, which more appropriately should be called vibration neutralizers, are mechanical devices to be attached to another mechanical system, or structure, called the primary system, with the purpose of controlling, or reducing vibration and sound radiation from machines, structural surfaces and panels.*
*The cheapest and easiest way to construct a vibration neutralizer is by incorporating a viscoelastic material, both as the resilient and energy dissipating part. A major problem in the analysis and design of such neutralizers is that, when applied to a structure, they render equations with coefficients dependent on frequency. Such a difficult problem was first efficiently solved by Espíndola and Silva in 1992, with the introduction of a new concept of generalized quantities for the neutralizer. Such a concept has been used for the design of optimum viscoelastic neutralizer systems, first with the help of techniques of non-linear optimization and, subsequently, of a hybrid technique combining both genetic algorithm and non-linear optimization. A difficult question still remained after the computation of the optimum parameters***:** *the selection of a proper viscoelastic material to match such parameters. This question has been solved recently by the authors of this paper with the help of the four parameter fractional model for the viscoelastic part of the neutralizers. This paper sets out to describe such a solution. In the output, one has the identified four parameters of the optimum material, as well as the anti-resonance frequency of the neutralizers, at the peak loss factor. Only a simple geometric factor is left to be defined by the designer. A numerical example is produced and discussed.*

Keywords*: Viscoelastic Material, Fractional Calculus, Vibration Absorber, Vibration Neutralizer.*

1. Introduction

Vibration neutralizers, also much often called vibration absorbers, are mechanical devices to be attached to another mechanical system, or structure, called the primary system, with the purpose of reducing, or controlling vibrations and sound radiation. In this paper, except in the title, such devices will be referred to as vibration neutralizers, instead of vibration absorbers, because vibration absorber is a rather imprecise name (Crede, 1965).

Since neutralizers were first used to reduce rolling motions of ships (Den Hartog, 1956), many publications on the subject have steadily come to light, demonstrating their efficiency in mitigating vibrations and sound radiation in many surfaces, structures and machines.

With modern technology of viscoelastic materials, which makes it possible to tailor a particular product to meet design specifications, vibration neutralizers are easy to make and apply to almost any complex structure.

In recent times, a great deal of effort has been made to extend and generalize the theory of vibration neutralizers applied to more complex structures than the single degree of freedom undamped one, tackled by Ormondroyd and Den Hartog (1928).

Single degree of freedom neutralizers applied to particular positions of uniform beams, with particular boundary conditions, has been studied (Jacquot, 1978; Candir and Ozguven, 1986). Also mass distributed neutralizers have been analyzed (Manikahally and Crocker, 1991; Esmailzadeh and Jalili, 1998). Simply supported uniform thin plates have also been considered as a primary system (Broch, 1946; Snowdon, 1975; Korenev and Reznikov, 1993).

In the work of Espíndola and Silva (1992), a general theory for the optimum design of neutralizer systems, when applied to a most generic structure of any shape, with any amount and distribution of damping, was derived. This approach has been applied to viscoelastic neutralizers of various types (Espíndola and Silva, 1992; Freitas and Espíndola, 1993). The theory is based on the concept of equivalent generalized quantities for the neutralizers, introduced by the first author. With this concept, it is possible to write down the equations for the movement of the composite system (primary plus neutralizers) in terms of the generalized coordinates (degrees of freedom), previously chosen to describe the primary system alone, in spite of the fact that the composite system has additional degrees of freedom.

This fact was crucial in the development of the theory. It permits a coordinate transformation using the modal matrix of the primary system, which is invariant during the optimization process.

In the modal space of the composite structure, it is possible to retain only few modal equations, encompassing the band of frequencies of interest. If coupling is not considered between these equations, then the neutralizer system can be designed to be optimum for a particular mode, in parallel with Den Hartog's simple optimization method. If a set of coupled modal equations is retained, covering a particular frequency band, then a nonlinear optimization technique can be used to design the neutralizer system to be optimum (in a certain sense) over that frequency band.

In recent years, the concept of fractional derivative has been applied to the construction of parametric models for viscoelastic materials (Bagley and Torvik, 1979; Torvik and Bagley, 1987; Liebst and Torvik, 1996; Rossikhin and Shitikova, 1998; Espíndola *et alii*., 2004).

This paper adds an important step to the above review**:** instead of looking, at the final design stage, for published data of a commercial material that suits the design parameters just computed, the parametric fractional model allows the specification of the optimum material, represented by the four or five fractional parameters.

So, in the end of the optimization process, the design parameters for the neutralizers are known, together with the fractional parameters for the viscoelastic material, which model the material itself.

2. Equivalent generalized quantities for the simple (one degree of freedom) neutralizer

For completeness, a brief review of the concept of generalized quantities is presented here.

The simple neutralizer, that is, one with a single degree of freedom, has one lump of mass (m_a) connected to a rigid massless basis through a resilient device, assumed as having a viscoelastic nature (figure. 1), with complex stiffness ($K_c(\Omega)$) equal to (Espíndola, 2003)**:**

$$K_c(\theta,\Omega) = \vartheta G_c(\theta,\Omega) = \vartheta G(\theta,\Omega)\left[1 + i\eta(\theta,\Omega)\right] \qquad (1)$$

In the above expression, $G_c(\theta, \Omega)$ is the complex shear modulus of the viscoelastic material, $G(\theta, \Omega)$ is the dynamic shear modulus, $\eta(\theta, \Omega)$ is the loss factor of such a material, θ is the temperature, Ω is the circular frequency and ϑ is a geometric factor of the viscoelastic device.

The rigid, massless basis is conceived here to connect the neutralizer to the primary structure, at a point described by one generalized coordinate (in the particular example to be seen, a transverse displacement).

According to the fractional derivative model with four parameters, the complex shear modulus is defined as

$$G_c(\Omega) = \frac{G_0 + G_\infty (ib_1\Omega)^\alpha}{1 + (ib_1\Omega)^\alpha}, \qquad (2)$$

where α is the fractional derivative order, b_1 is the relaxation time, and G_0 and G_∞ are the inferior and superior asymptotes of the dynamic shear modulus, respectively.

For simplicity of notation, the letter θ, standing for temperature, will be omitted from now on.

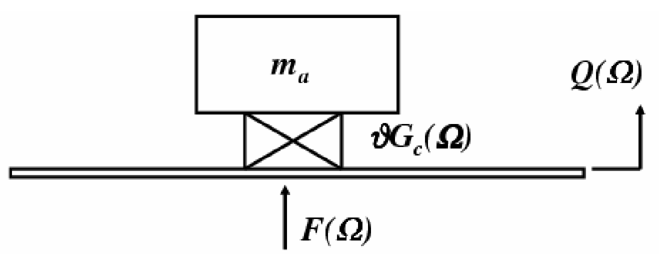

Figure 1. Scheme of a simple neutralizer

In figure 1, $Q(\Omega)$ and $F(\Omega)$ stand for the Fourier transform of the basis displacement $q(t)$ and the applied force $f(t)$, respectively.

It is a simple matter to verify that the impedance $Z_a(\Omega)$, and the dynamic mass $M_a(\Omega)$, at the attachment (massless) plate, are given by**:**

$$Z_a(\Omega) = \frac{-im_a \Omega \vartheta G_c(\Omega)}{m_a \Omega^2 - \vartheta G_c(\Omega)} \tag{3}$$

$$M_a(\Omega) = -m_a \frac{\vartheta G_c(\Omega)}{m_a \Omega^2 - \vartheta G_c(\Omega)} \tag{4}$$

The anti-resonant frequency of the simple neutralizer is defined as the one such that, in the absence of damping, makes the denominator of expressions (3) or (4) equal to zero:

$$\Omega_a{}^2 = \frac{\vartheta G(\Omega_a)}{m_a} \tag{5}$$

In expression (5), Ω_a stands for the anti-resonant frequency of the neutralizer. Note that, in absence of damping, $G_c(\Omega)=G(\Omega)$.

Since one can write $\vartheta\, G_c(\Omega)=\vartheta\, G(\Omega_a) r_a(\Omega)$, expressions (3) and (4) can be rewritten as:

$$Z_a(\Omega) = -im_a \Omega_a \frac{\varepsilon_a r_a(\Omega)\left[1+i\eta(\Omega)\right]}{\varepsilon_a{}^2 - r_a(\Omega)\left[1+i\eta(\Omega)\right]} \tag{6}$$

$$M_a(\Omega) = -m_a \frac{r_a(\Omega)\left[1+i\eta(\Omega)\right]}{\varepsilon_a{}^2 - r_a(\Omega)\left[1+i\eta(\Omega)\right]}, \tag{7}$$

where $r_a(\Omega)=G(\Omega)/G(\Omega_a)$ and $\varepsilon = \Omega/\Omega_a$.

The equivalent generalized viscous damping is defined as the real part of the impedance (expression (6)) and, for this simple neutralizer, it is:

$$c_{eq}(\Omega) = m_a \Omega_a \frac{r_a(\Omega)\eta(\Omega)\varepsilon_a{}^3}{\left[\varepsilon_a{}^2 - r_a(\Omega)\right]^2 + \left[r_a(\Omega)\eta(\Omega)\right]^2} \tag{8}$$

In an analogous way, the equivalent generalized mass is the real part of expression (7):

$$m_{eq}(\Omega) = -m_a \frac{r_a(\Omega)\left\{\varepsilon_a{}^2 - r_a(\Omega)\left[1+\eta^2(\Omega)\right]\right\}}{\left[\varepsilon_a{}^2 - r_a(\Omega)\right]^2 + \left[r_a(\Omega)\eta(\Omega)\right]^2} \tag{9}$$

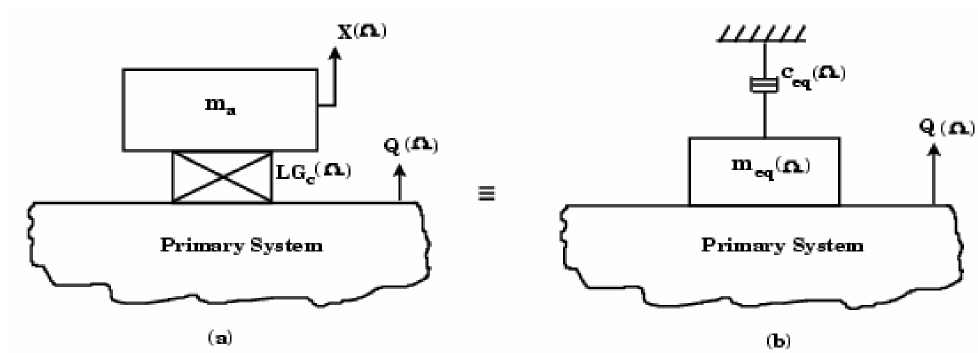

Figure 2. Equivalent systems

In figure 2a, a simple neutralizer (see figure 1) is fixed to the primary structure through the rigid massless basis.

Now, it is a simple task to verify that both schemes shown in figure 2a and figure2b are dynamically equivalent (Espíndola and Silva, 1992). The primary system "feels" the neutralizer as a mass $m_{eq}(\Omega)$ attached to it along a generalized coordinate $q(t)$ and a viscous dashpot (even if the damping is solid) of constant $c_{eq}(\Omega)$ linked to earth.

The dynamics of the resultant system (primary + neutralizers) can then be formulated in terms of the original physical generalized coordinates alone, although it has now added degrees of freedom. This is the main advantage of the concept of equivalent generalized quantities.

Notice that expressions (3) and (4) (or (8) and (9)) contain all the parameters of the fractional viscoelastic model.

The above described generalized equivalent quantities correspond to a generic dynamic neutralizer. Particular cases, such as viscous or viscoelastic damping, can be analyzed by inserting the pertinent expression into the general equations.

If many such neutralizers are added, the equation of motion can be written as:

$$\left[-\Omega^2\tilde{M} + i\Omega\tilde{C} + K\right]Q(\Omega) = F(\Omega) \tag{10}$$

In the above equation, K is the ordinary stiffness matrix of the primary system; $\tilde{M}$ and $\tilde{C}$ are the mass and viscous damping matrices of the primary system, modified by the neutralizers. $F(\Omega)$ and $Q(\Omega)$ now represent the vector of generalized forces and displacements, respectively, relative to the primary system.

Note also that the effects of the added neutralizers lie in modifying the mass and damping matrices. The vector of generalized coordinates of the primary system remains unchanged.

In order to illustrate the above ideas, assume that p simple neutralizers are attached along p physical generalized coordinates $q_{k1}, q_{k2}, \ldots, q_{kp}$. Their equivalent generalized masses and damping are m_{eq1}, $m_{eq2}, \ldots m_{eqp}$ and c_{eq1}, c_{eq2}, $\ldots, c_{eqp}$.

The modified mass and damping matrices will be:

$$\tilde{M} = M + \begin{bmatrix} 0\ddots & & & & \\ & m_{e1} & & 0 & \\ & & \ddots & & \\ & 0 & & m_{ep} & \\ & & & & \ddots 0 \end{bmatrix} = M + M_A, \quad \tilde{C} = C + \begin{bmatrix} 0\ddots & & & & \\ & c_{e1} & & 0 & \\ & & \ddots & & \\ & 0 & & c_{ep} & \\ & & & & \ddots 0 \end{bmatrix} = C + C_A \tag{11}$$

Now, in equation (10), assume the transformation:

$$Q(\Omega) = \Phi\hat{P}(\Omega), \tag{12}$$

where Φ is the modal matrix of the primary system, obtained numerically or experimentally, and is of order $n\,x\,\hat{n}$, where n is its number of degrees of freedom and $\hat{n}$ is the number of eigenvectors actually computed, or measured.

Normally, $\hat{n} \ll n$.

If expression (12) is taken into equation (10), one gets, assuming proportional damping in the primary system:

$$\left\{-\Omega^2\left[I_{\hat{n}} + \hat{M}_A(\Omega)\right] + i\Omega\left[\Gamma_{\hat{n}} + \hat{C}_A(\Omega)\right] + \Lambda_{\hat{n}}\right\}\hat{P}(\Omega) = \hat{N}(\Omega). \tag{13}$$

where

$$\Gamma_{\hat{n}} = diag(2\xi_j\Omega_j); \quad \Lambda_{\hat{n}} = diag(\Omega_j^2); \quad \Phi^T K\Phi = \Lambda_{\hat{n}}; \quad I_{\hat{n}} = I; \tag{14}$$

$$\hat{M}_A(\Omega) = \Phi^T M_A\Phi; \quad \hat{C}_A(\Omega) = \Phi^T C_A\Phi; \quad \hat{N}(\Omega) = \Phi^T F(\Omega). \tag{15}$$

In the above expressions, it was assumed that the eigenvectors of the primary structure are orthonormal in relation to the mass matrix M. The $\Omega_i, i = 1, \hat{n}$, are natural frequencies of the primary structure and ξ_j are the modal damping.

Equation (13) represents a small system of $\hat{n} \ll n$ coupled equations and can be solved directly with use of expressions (8) and (9). Returning back to expression (12), the solution in physical coordinates is accomplished.

From expressions. (13), (14) and (15), it is easy to show that:

$$Q(\Omega) = \Phi\hat{\mathrm{A}}(\Omega)\Phi^T F(\Omega), \tag{16}$$

where

$$\hat{\mathrm{A}}(\Omega) = [D_0(\Omega) - \Omega^2 \hat{M}_A(\Omega) + i\Omega\hat{C}_A(\Omega)]^{-1}. \tag{17}$$

and

$$D_0(\Omega) = diag\left(k_j - m_j\Omega^2 + i\Omega c_j\right) \tag{18}$$

or

$$D_0(\Omega) = diag\left(\Omega_j^{\,2} - \Omega^2 + i2\xi_j\Omega_j\Omega\right), \tag{19}$$

in case of the eigenvectors of the primary system are orthonormals.

From expression (16), the receptance matrix of the primary system, after the neutralizers have been attached, can be seen to be:

$$\mathrm{A}(\Omega) = \Phi\hat{\mathrm{A}}(\Omega)\Phi^T. \tag{20}$$

A particular member of this matrix is

$$\alpha_{ks}(\Omega) = \sum_{j=1}^{\hat{n}} \sum_{l=1}^{\hat{n}} a_{jl}(\Omega)\phi_{sl}\phi_{kj}, \tag{21}$$

where $a_{jl}(\Omega)$ and ϕ_{sl} are entries of $\hat{\mathrm{A}}(\Omega)$ and Φ, respectively.

This can be compared with the receptances before the attachment of the neutralizers

$$\mathrm{A}_0(\Omega) = \Phi D_0^{-1}(\Omega)\Phi^T. \tag{22}$$

The pertinent response ratios can then be computed**:**

$$R_{ks}(\Omega) = \frac{\alpha_{ks}(\Omega)}{\alpha_{ks_0}(\Omega)}. \tag{23}$$

The modulus of the response ratios can be taken as a measure of the efficiency of the neutralizer system.

For primary systems with one degree of freedom, the recommended mass-ratio between neutralizer and primary structure by Den Hartog (1956) is $\mu = m_a / m_s$ = 0.1 to 0.25.

The use of the modal mass-ratio concept has been proposed by Espíndola and Silva (1992) for a primary system of multiple degrees of freedom as**:**

$$\mu_j = \frac{m_a \sum_{i=1}^{p} \Phi_{k_i j}^2}{m_j}. \tag{24}$$

(Note that, if the eigenvectors are orthonormalized, $m_j = 1$).

In expression 24 it was assumed, for simplicity, that all the neutralizers have the same mass m_a. It will also be assumed, from now on, that all neutralizers are made with the same viscoelastic material. These assumptions are not compulsory, but they make a good engineering sense.

3. Optimization for a frequency range

To better profit from the inherent damping of the viscoelastic material, it is convenient that the anti-resonant frequency of the neutralizers lays as close as possible to the frequency where the loss factor is maximum.

It is easy to show that this frequency is:

$$\Omega_a = \frac{1}{b_1}\left(G_0 / G_\infty\right)^{\frac{1}{2\alpha}} . \tag{25}$$

This expression should be considered as a constraint imposed on the anti-resonant frequency of the neutralizers.

Expression (13) can be written as

$$\hat{P}(\Omega) = \hat{A}(\Omega)\hat{N}(\Omega) \tag{26}$$

where $\hat{A}\left(\Omega\right) \in \mathbb{C}^{\hat{n}\times\hat{n}}$, already defined in expression (17), is a receptance matrix linking the modal response $\hat{P}(\Omega)$ to the modal excitation $\hat{N}(\Omega)$.

Note that $\hat{A}(\Omega) \in \mathbb{C}^{\hat{n}x\hat{n}}$ is assumed to be of very small order (because $\hat{n} \ll n$).

Assume now that a set of fractional parameters {α, b, G_0, G_∞} is available so that Ω_a can be computed (see expression (25)). In this circumstance, $r_a(\Omega) = G(\Omega)/G(\Omega_a)$ can also be computed for any frequency Ω. Also the equivalent quantities $c_{eq}(\Omega)$ and $m_{eq}(\Omega)$ (expression (8) and (9)) are known and the same can be said for the receptance matrix $\hat{A}(\Omega)$.

Said that, an objective function can be defined as

$$f(x) = \max_{\Omega_1 \le \Omega \le \Omega_2} \left\|\hat{A}(\Omega)\right\|_F , \tag{27}$$

subjected to the following constraints: 0< α <1, $b_0 < b_1 < b_2$, $G_{01} < G_o < G_{02}$ and $G_{\infty 1} < G_\infty < G_{\infty 2}$.

In expression (27), $\| . \|_F$ stands for the Frobenius norm and $x^T = \left[\alpha \quad b_1 \quad G_0 \quad G_\infty\right]$

After the minimization procedure of f(x), the four parameters α, b_1, G_0 and G_∞ are known together with the anti-resonance Ω_a.

Since m_a was given as an input parameter, the stiffness of the viscoelastic element can be computed at frequency Ω_a.

This leads easily to the design of the neutralizers.

4. Numerical example

The above theory was applied to the transverse vibration of a freely supported rectangular steel plate as the primary system. The dimensions of the plate were 240x360x6 mm and it had a total mass of 4.0 kg.

This plate was divided into fifty-four elements and model parameters of the first $\hat{n}$ modes were identified. It was imagined that four absorbers were fixed at the plate, one at each corner through their rigid massless basis (see figure 1)

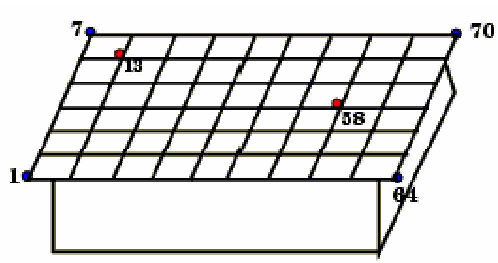

Figure 3. Primary system

Figure 3 shows a sketch of the plate with its experimental model analysis mesh. Table 1 shows some identified natural frequencies and modal loss factor. The plate was actually supported on a soft cushion of foam, to simulate the free flexural condition.

Table 1- Some natural frequencies and modal loss factors for plate of figure 3

Ω (Hz)	262.6	275	594.4	619.6	825.1	1086.2	1240.4	1495.8
η	0.0204	0.0225	0.0138	0.0158	0.0127	0.0073	0.0065	0.0073

In this example, five successive modes, starting from the third one (see dots at the peaks), were taken to have their peaks reduced. Many other choices have been computed, some including the first and second modes, but are not included here to save space. In all cases, the selected modes have been substantially reduced (10dB or more) in their peaks. The choice of the modes (frequencies) to be peak reduced depend on the particular problem at hand, that is, on the actual modes contained in the experimental band of frequencies responsible, for instance, for great acoustic noise radiation .

The restrictions imposed on some design variables were as follows:

$$1s^{-1} \leq \Omega_a \leq 10^4\ s^{-1};\ 0 < \alpha < 1;\ 10^{-6}\ s < b_1 < 10^{-3} s;\ 10^3\ Pa < G_0 < 10^6\ Pa;\ Pa\ 10^4 < G_\infty < 10^{8Pa}.$$

The limits in the above variables restrictions are imposed by the fractional model theory on α . For other parameters, the limits are dictated by years of experience of the authors in viscoelastic dynamic property measurements and vibration control and by the bulk of published data. For all the modes, it has been taken $\mu_j = 0.1$. The results, after identification, were: $\alpha = 0.69$; $b_1 = 1.23\text{x}10^{-4}$s; $G_0 = 3.52\text{x}10^5$ Pa; G_∞=$8.85\text{x}10^5$ Pa and $\Omega_a = 4.22\text{x}10^3\ s^{-1} \approx 671.5$ Hz (for all the four neutralizers). The total mass of the four neutralizers amounts to nearly four percent of the plate mass.

Figure 4 compares one of the measured FRF of the primary plate (with no neutralizer attached) with the same FRF, after the attachment of the neutralizers. One can appreciate the remarkable reduction in the FRF peaks, amounting to 10 dB or more, as anticipated above. Figure 5 represents the dynamic shear modulus of the material and the loss factor.

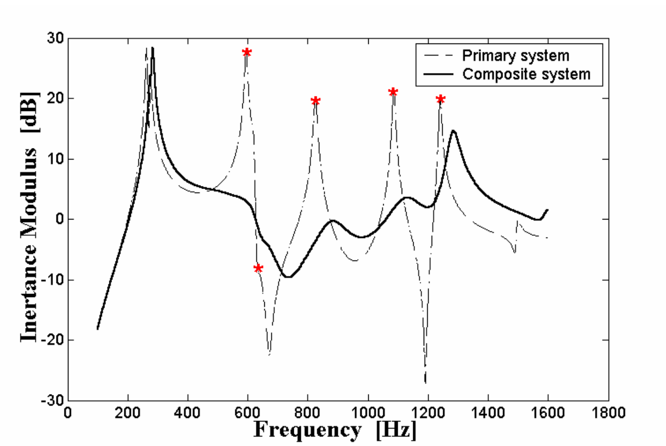

Figure 4. Response of the composite system (primary plus neutralizers).

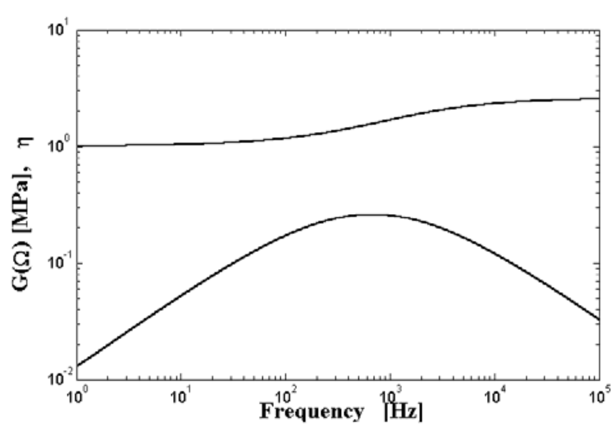

Figure 5. Dynamic shear modulus and loss factor of the optimum viscoelastic material.

5. Conclusions

The concept of equivalent quantities can lead to a representation of the modal space of the composite system in terms of the modal parameters of the primary system. Retaining only a limited number of modal equations, dictated by a practical problem, an optimization scheme was devised, which leads to optimum parameter selection of a system of viscoelastic neutralizers.

This procedure is general and independent of the geometrical complexity of the linear primary structure, which is represented by its modal model. For best performance, the viscoelastic material should work in the frequency transition zone.

The use of the fractional model, besides leading to a optimum design of a neutralizer system, allows the identification of the optimum fractional parameters, that is, of the most suitable viscoelastic material. Having these parameters, that is, the dynamic characteristics of the viscoelastic material, modern technology is able to tailor it accordingly.

The performance of an optimum neutralizer system can be remarkable, at the expense of only a small increase in overall weight, which makes this technique particularly valuable for light structures and panels.

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7. Responsibility notice