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Vibration Control And Optimization Of Flexible Piezoeletric Structures

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The present advances in mechanical, automotive and aerospace engineering, justifies the necessity of designing systems that are lighter, more efficient and affordable. Technologically, this advances are possible due to investments that have been done in the last few years in the development of high-performance materials, allowing more sophisticated engineering design. In the context of this work, one important property is demanded, namely the simultaneous use of the material as a sensor and as an actuator. The piezoelectric ceramics, particularly the lead zirzonate titanate (PZT) is an example of such a material. The use of piezoelectric materials in vibration damping is a recent approach: one of the first contributions in this topic was done by Crawley and de Luis (1987), who formulated the beam pin-forced model used in the present paper.

The present work considers the use of intelligent materials for attenuating vibrations in flexible structures, as illustrated by an Euler-Bernoulli beam. The mathematical model is based in the continuum system theory as described in equation (1) and on state-space analysis that will be used for handling the differential equations that describe the dynamics of the controlled system, Equation (2), considering a finite number of vibration modes. The control forces are determined in such a way that the saturation of the piezoelectric ceramic is avoided.

$$E_b I_b \frac{\partial^4 y}{\partial x^4} + \rho_b A_b \frac{\partial^2 y}{\partial t^2} = M[\delta'(x-x_2) - \delta'(x-x_1)] \qquad (1)$$

$$\begin{aligned} \dot{z}(t) &= \mathbf{A}z(t) + \mathbf{B}v(t) \\ V_s(t) &= \mathbf{C}z(t) \end{aligned} \qquad (2)$$

The pole placement method of optimization is chosen for determining the geometry of the piezoelectric elements in such a way that damping is maximized uniformly in the retained modes. This means that the poles of the system (or the eigenvalues of dynamic matrix **A)** are placed as far into the left half of the complex plane as possible. Mathematically, this goal can be expressed as:

$$\min \max \mathrm{Re}\left[\lambda_i(\mathbf{A})\right] \qquad (3)$$

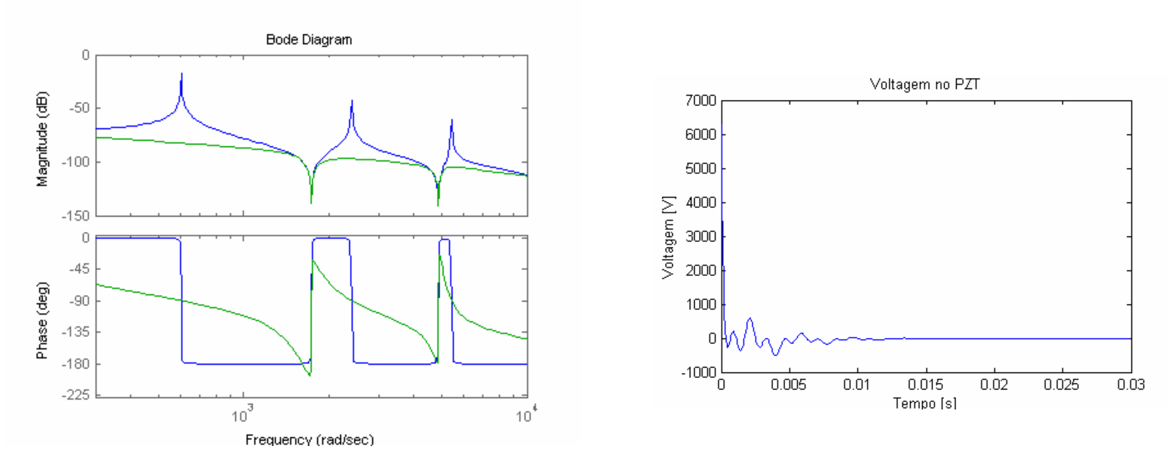
where λ_i(**A**) is the *i*-th eingenvalue of **A**. The unconstrained minimization is performed by using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) method using the Matlab optimization Toolbox. Other available optimization routines could be used as well.

Optimum control techniques are used to define the control law, such as the H_∞ optimum and sub-optimum control as formulated by *Linear Matrix Inequalities* (LMI) as in equation (4). The convex optimization problem is solved to obtain the feedback gain vector, **K**. For comparison purposes a *linear quadratic regulator* (LQR) controller is also designed as in equation (5).

$$\begin{bmatrix} -\mathbf{AP}-\mathbf{PA}^T-\mathbf{B}_2\mathbf{Z}-\mathbf{Z}^T\mathbf{B}_2{}^T & \mathbf{B}_1 & \mathbf{PC}^T \\ \mathbf{B}_1{}^T & \mathbf{I} & \mathbf{0} \\ \mathbf{CP} & \mathbf{0} & \delta\mathbf{I} \end{bmatrix} \geq 0 \quad \Longrightarrow \quad \mathbf{K}=\mathbf{Z}\mathbf{P}^{-1} \tag{4}$$

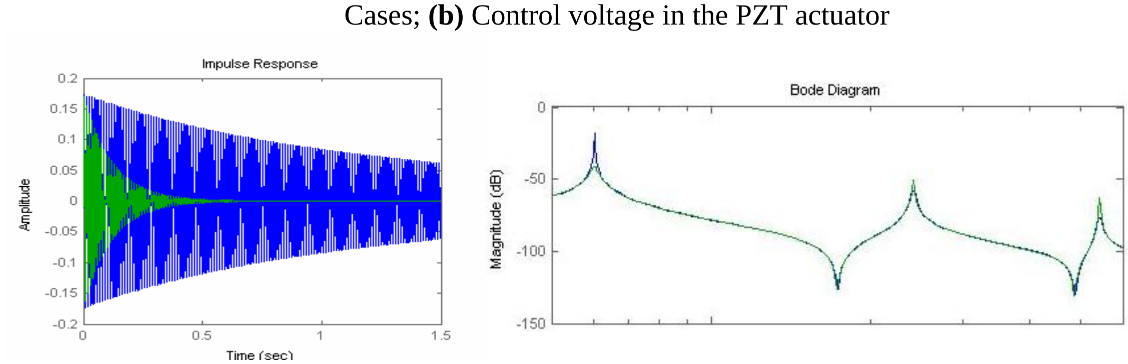
$$J=\int_0^{\infty}(z^T\mathbf{Q}z+v^T\mathbf{R}v)dt \quad \Longrightarrow \quad \mathbf{K}=\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{5}$$

Figures 1 (a) and (b) show, respectively, the Bode diagram for the H_∞ optimum control and the applied voltage in PZT actuator. It can be observed that an extreme high control effort should be used in this case. This problem is overcome by using an H_∞ sub-optimum control law. Figure 2 shows the responses for the LQR control, which are similar to the results obtained for the H_∞ sub-optimum control (for the sake of conciseness they are not presented here).



(a) (b)

Figure 1 – (a) Bode diagram for the uncontrolled (blue) and controlled (green) Cases; **(b)** Control voltage in the PZT actuator



(a) (b)

Figure 2 – (a) Impulsive response, LQR control **(b)** Bode diagram, LQR control

We conclude that optimum control laws represent an efficient and modern approach for the design of state-space systems. The presented results are encouraging in the sense of obtaining high performance engineering structures.

REFERENCES

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