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OPTIMIZATION OF PARAMETERS IN A SYSTEM OF VIBRATIONS ISOLATORS FOR A POWER GENERATOR

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Abstract: *When designing the supporting structure of a power generator group, under predefined data relative to loadings and frequency of operation of the group or parts to be protected, some special requeriment is the definition of a specific system of vibration isolators for the supporting structure. The objective of this work is the computational development of an optimal system of vibration isolators for the group constituted by the motor, the generator and the supporting structure. The methodology of the work involves the modeling of the group as a rigid body and placed over isolators, which are selected with an objective function oriented to minimize the maximum natural frequency of the group in order to be far away from any excitement frequency. Some results of this development include the optimal definition of the stiffness and damping parameters of the vibration isolators for safe operation of the group.*

Keywords: *vibration isolator, motor generator, nonlinear optimization*

1. INTRODUCTION

Three basic arguments can be argued about control of vibrations in mechanical equipments.

Depending on the excitement level of the equipment, what corresponds to the first argument, the equipment can fail because of discrepancies in terms of the design conception, in the construction and operation ways, as well as due to the usage of no certificated items.

A correct understanding of the transmission path, what corresponds to the second argument, makes possible to reduce the vibration energy propagation, through the utilization of secondary elements.

The relative vibrations among parts of the equipment can be caused by an external source, the own equipment or due to the action of forces generated during the equipment operation. This last argument can be represented by forced vibrations, which can excite some natural frequency of the equipment.

Inman (2007), Rivin (2006) and Moore (1985) present different elements for reducing or isolating the vibrations generated in the equipment, and develops a methodological sequence in order to find the most appropriate element for isolating or absorbing some vibration. After evaluation of the equipment response and identification of the design vibrational parameters, the next step is to adjust these parameters for satisfying the design response.

This work seeks optimal values of parameters for the isolators system of a motor generator group under certain conditions of the equipment to be protected.

2. CENTER OF MASS OF THE POWER GROUP

The power group is constituted by the motor, the generator and the supporting structure, the last also serves as fixation medium for the isolators.

A relevant model of the group under vibration involves a rigid body with distributed mass and restricted through isolators. In this case, the equilibrium of forces and moments must be satisfied during the movement of the body.

The group is considered as a rigid body with center of mass *CM*, as shown in Fig. 1.

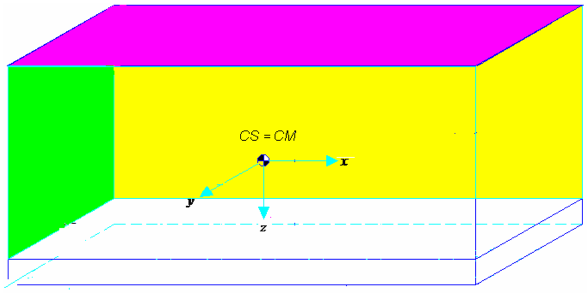

Figure 1. Origin of the coordinate system *SC* and the center of mass *CM*

Under applied forces, the body can experiment translational (longitudinal x, transversal y and vertical z) or rotational (roll ψ_x, pitch ψ_y and yaw θ) displacements. In most cases, the vertical/roll displacements are decoupled, while the longitudinal/pitch and the transversal/roll displacements are coupled.

The center of mass is illustrated in Fig. 2.

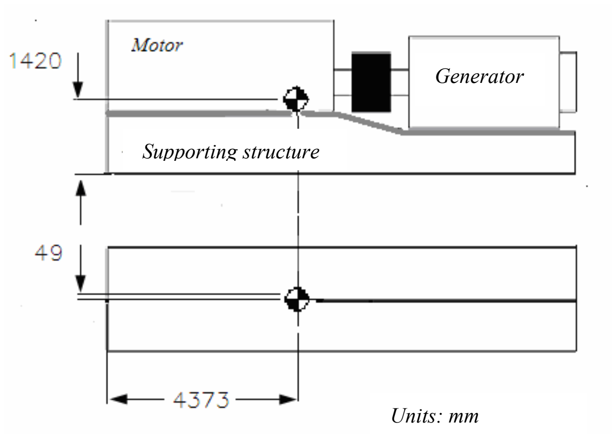

Figure 2. Center of mass location.

3. DYNAMIC EQUATIONS OF FREE VIBRATION

The power group is supported with a discrete number of isolators. The movement equations of free vibration can be developed with and without damping. The group is assumed as a rigid body in according to Fig. 1 and can vibrate with up to six degrees of freedom. The analysis considers the following conditions (Moore, 1985):

- The reference axes for the body will be chosen such that products of inertia $I_{xy}=I_{yz}=I_{zx}=0$. The origin of the reference axes passes through the *CM* and the moments of inertia I_{xx}, I_{yy}, I_{zz} will be the principal moments of inertia.
- The supporting springs are aligned such their load axes are parallel to the reference axes.
- For the supporting springs, $k_{xy}=k_{yz}=k_{zx}=0$, where for example, k_{xy} is the lateral stiffness of the spring with longitudinal axes in the x direction.
- The stiffness of the foundation is infinity.

The motion equations for damped free vibration, considering the Fig. 3, are expressed as:

$$m\,\ddot{x} + \sum c_{xi}\dot{x} + \sum(c_{xi}l_{zi})\dot{\psi}_y - \sum(c_{xi}l_{yi})\dot{\theta} + \sum k_{xi}x + \sum(k_{xi}l_{zi})\psi_y - \sum(k_{xi}l_{yi})\theta = 0 \quad (1)$$

$$m\,\ddot{y} + \sum c_{yi}\dot{y} + \sum(c_{yi}l_{zi})\dot{\psi}_x - \sum(c_{yi}l_{xi})\dot{\theta} + \sum k_{yi}y + \sum(k_{yi}l_{zi})\psi_x - \sum(k_{yi}l_{xi})\theta = 0 \quad (2)$$

$$m\,\ddot{z} + \sum c_{zi}\dot{z} + \sum(c_{zi}l_{yi})\dot{\psi}_x - \sum(c_{zi}l_{xi})\dot{\psi}_y + \sum k_{zi}z + \sum(k_{zi}l_{yi})\psi_x - \sum(k_{zi}l_{xi})\psi_y = 0 \quad (3)$$

$$\begin{aligned} &I_{xx}\ddot{\psi}_x + \sum(c_{yi}l_{zi})\dot{y} + \sum(c_{zi}l_{yi})\dot{z} + (\sum(c_{yi}l_{zi}^2) + \sum(c_{zi}l_{zi}^2))\dot{\psi}_x \\ &- \sum(c_{zi}l_{xi}l_{yi})\dot{\psi}_y - \sum(c_{yi}l_{xi}l_{zi})\dot{\theta} + \sum(k_{yi}l_{zi})y + \sum(k_{zi}l_{yi})z + (\sum(k_{yi}l_{zi}^2) + \sum(k_{zi}l_{zi}^2))\psi_x \\ &- \sum(k_{zi}l_{xi}l_{yi})\psi_y - \sum(k_{yi}l_{xi}l_{zi})\theta = 0 \end{aligned} \quad (4)$$

$$
\begin{aligned}
& I_{yy}\ddot{\psi}_y + \sum(c_{xi}l_{zi})\dot{x} - \sum(c_{zi}l_{xi})\dot{z} - \sum(c_{zi}l_{xi}l_{yi})\dot{\psi}_x \\
& + (\sum(c_{xi}l_{zi}^2) + \sum(c_{zi}l_{xi}^2))\dot{\psi}_y - \sum(c_{xi}l_{yi}l_{zi})\dot{\theta} + \sum(k_{xi}l_{zi})x - \sum(k_{zi}l_{xi})z - \sum(k_{zi}l_{xi}l_{yi})\psi_x \\
& + (\sum(k_{xi}l_{zi}^2) + \sum(c_{zi}l_{xi}^2))\dot{\psi}_y - \sum\left(k_{xi}l_{yi}l_{zi}\right)\theta = 0
\end{aligned}
\quad (5)
$$

$$
\begin{aligned}
& I_{zz}\ddot{\theta} - \sum(c_{xi}l_{yi})\dot{x} + \sum(c_{yi}l_{xi})\dot{y} - \sum(c_{yi}l_{xi}l_{zi})\dot{\psi}_x \\
& - \sum(c_{xi}l_{yi}l_{zi})\dot{\psi}_y + (\sum(c_{xi}l_{yi}^2) + \sum(c_{yi}l_{xi}^2))\dot{\theta} - \sum(k_{xi}l_{yi})x + \sum(k_{yi}l_{xi})y - \sum(k_{yi}l_{xi}l_{zi})\psi_x \\
& - \sum(k_{xi}l_{yi}l_{z})\psi_y + (\sum(k_{xi}l_{yi}^2) + \sum(k_{yi}l_{xi}^2))\theta = 0
\end{aligned}
\quad (6)
$$

where,

m : total mass of body (group), equal to 70000 kg

l_{xi}, l_{yi}, l_{zi} : distance from the *CM* of the body to the isolator *i*, in axes *x, y, z*, respectively

x, y, z : translation in axes *x, y, z*, respectively

$\dot{x}, \dot{y}, \dot{z}$: velocity in axes *x, y, z*, respectively

$\ddot{x}, \ddot{y}, \ddot{z}$: acceleration in axes *x, y, z*, respectively

k_{xi}, k_{yi}, k_{zi} : stiffness of the isolator *i* in axes *x, y, z*, respectively

c_{xi}, c_{yi}, c_{zi} : viscous damping coefficient of the isolator *i* in axes *x, y, z*, respectively

ψ_x, ψ_y, θ : angular rotation around axes *x, y, z*, respectively

$\dot{\psi}_x, \dot{\psi}_y, \dot{\theta}$: angular velocity around axes *x, y, z*, respectively

$\ddot{\psi}_x, \ddot{\psi}_y, \ddot{\theta}$: angular acceleration around axes *x, y, z*, respectively

I_{xx}, I_{yy}, I_{zz} : principal moments of inertia relative to axes *x, y, z*; equal to 142000, 540000, 493000 N.m^2, respectively

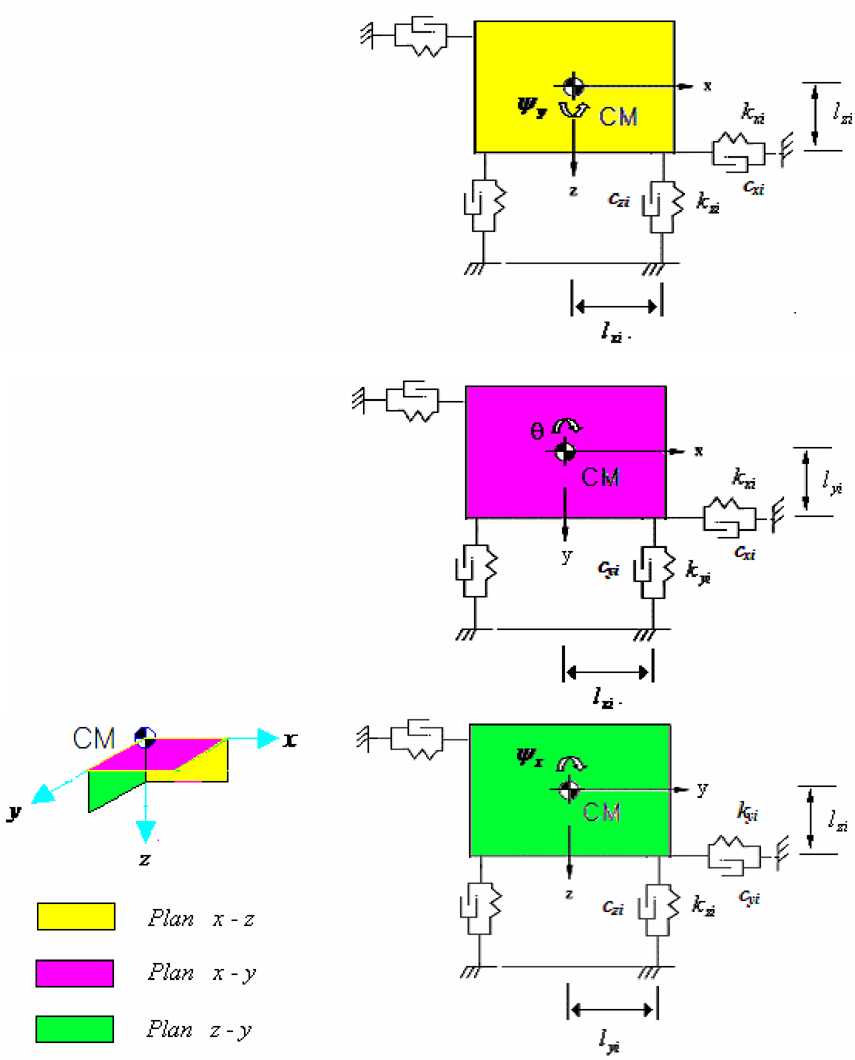

Figure 3. Tri-dimensional supporting system for the rigid body

These equations can be re-written in matrix form as,

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{c}\dot{\mathbf{x}} + \mathbf{k}\mathbf{x} = \mathbf{0} \tag{7}$$

where
m : mass matrix
k : stiffness matrix
c : damping matrix
x, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$: displacements, velocity and acceleration vector, respectively

$$\mathbf{m} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix} \tag{8}$$

$$\mathbf{k} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \tag{9}$$

$$\mathbf{c} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \tag{10}$$

The elements of the stiffness matrix **k** are originated from the displacement coefficients in Eqs. (1) to (6) and expressed in Eqs. (11) to (16) as,

$$k_{11} = \sum_{i=1}^{16} k_{xi}, \quad k_{12} = k_{13} = k_{14} = 0, \quad k_{15} = \sum_{i=1}^{16} k_{xi} l_{zi}, \quad k_{16} = -\sum_{i=1}^{16} k_{xi} l_{yi} \tag{11}$$

$$k_{21} = k_{23} = k_{25} = 0, \quad k_{22} = \sum_{i=1}^{16} k_{yi,} \quad k_{24} = \sum_{i=1}^{16} k_{yi} l_{zi}, \quad k_{26} = -\sum_{i=1}^{16} k_{yi} l_{xi} \tag{12}$$

$$k_{31} = k_{32} = k_{36} = 0, \quad k_{33} = \sum_{i=1}^{16} k_{zi,} \quad k_{34} = \sum_{i=1}^{16} k_{zi} l_{yi}, \quad k_{35} = \sum_{i=1}^{16} k_{zi} l_{xi} \tag{13}$$

$$k_{41} = 0, \quad k_{42} = \sum_{i=1}^{16} k_{yi} l_{zi}, \quad k_{43} = \sum_{i=1}^{16} k_{zi} l_{yi}, \quad k_{44} = \sum_{i=1}^{16} k_{yi} l_{zi}^2 + \sum_{i=1}^{16} k_{zi} l_{zi}^2,$$
$$k_{45} = -\sum_{i=1}^{16} k_{zi} l_{xi} l_{yi}, \quad k_{46} = -\sum_{i=1}^{16} k_{yi} l_{xi} l_{zi} \tag{14}$$

$$k_{51} = \sum_{i=1}^{16} k_{xi} l_{zi}, \quad k_{52} = 0, \quad k_{53} = -\sum_{i=1}^{16} k_{zi} l_{xi}, \quad k_{54} = -\sum_{i=1}^{16} k_{zi} l_{xi} l_{yi},$$
$$k_{55} = \sum_{i=1}^{16} k_{xi} l_{zi}^2 + \sum_{i=1}^{16} k_{zi} l_{xi}^2, \quad k_{56} = -\sum_{i=1}^{16} k_{xi} l_{yi} l_{zi} \tag{15}$$

$$k_{61} = -\sum_{i=1}^{16} k_{xi} l_{yi}, \quad k_{62} = \sum_{i=1}^{16} k_{yi} l_{xi}, \quad k_{63} = 0, \quad k_{64} = -\sum_{i=1}^{16} k_{yi} l_{xi} l_{zi},$$
$$k_{65} = -\sum_{i=1}^{16} k_{xi} l_{yi} l_{zi}, \quad k_{66} = \sum_{i=1}^{16} k_{xi} l_{yi}^2 + \sum_{i=1}^{16} k_{yi} l_{xi}^2 \tag{16}$$

The elements of the damping matrix **c** are originated from the displacement coefficients in Eqs. (1) to (6) and expressed in Eqs. (17) to (22) as,

$$c_{11} = \sum_{i=1}^{16} c_{xi}, \quad c_{12} = c_{13} = c_{14} = 0, \quad c_{15} = \sum_{i=1}^{16} c_{xi} l_{zi}, \quad c_{16} = -\sum_{i=1}^{16} c_{xi} l_{yi} \tag{17}$$

$$c_{21} = c_{23} = c_{25} = 0, \quad c_{22} = \sum_{i=1}^{16} c_{yi,} \quad c_{24} = \sum_{i=1}^{16} c_{yi} l_{zi}, \quad c_{26} = -\sum_{i=1}^{16} c_{yi} l_{xi} \tag{18}$$

$$c_{31} = c_{32} = c_{36} = 0, \quad c_{33} = \sum_{i=1}^{16} c_{zi,} \quad c_{34} = \sum_{i=1}^{16} {}_{zi} l_{yi}, \quad c_{35} = \sum_{i=1}^{16} c_{zi} l_{xi} \tag{19}$$

$$\begin{gathered} c_{41} = 0, \quad c_{42} = \sum_{i=1}^{16} c_{yi} l_{zi}, \quad c_{43} = \sum_{i=1}^{16} c_{zi} l_{yi}, \quad c_{44} = \sum_{i=1}^{16} c_{yi} l_{zi}^2 + \sum_{i=1}^{16} c_{zi} l_{zi}^2, \\ c_{45} = -\sum_{i=1}^{16} c_{zi} l_{xi} l_{yi}, \quad c_{46} = -\sum_{i=1}^{16} c_{yi} l_{xi} l_{zi} \end{gathered} \tag{20}$$

$$\begin{gathered} c_{51} = \sum_{i=1}^{16} c_{xi} l_{zi}, \quad c_{52} = 0, \quad c_{53} = -\sum_{i=1}^{16} c_{zi} l_{xi}, \quad c_{54} = -\sum_{i=1}^{16} c_{zi} l_{xi} l_{yi}, \\ c_{55} = \sum_{i=1}^{16} c_{xi} l_{zi}^2 + \sum_{i=1}^{16} c_{zi} l_{xi}^2, \quad c_{56} = -\sum_{i=1}^{16} c_{xi} l_{yi} l_{zi} \end{gathered} \tag{21}$$

$$\begin{gathered} c_{61} = -\sum_{i=1}^{16} c_{xi} l_{yi}, \quad c_{62} = \sum_{i=1}^{16} c_{yi} l_{xi}, \quad c_{63} = 0, \quad c_{64} = -\sum_{i=1}^{16} c_{yi} l_{xi} l_{zi}, \\ c_{65} = -\sum_{i=1}^{16} c_{xi} l_{yi} l_{zi}, \quad c_{66} = \sum_{i=1}^{16} c_{xi} l_{yi}^2 + \sum_{i=1}^{16} c_{yi} l_{xi}^2 \end{gathered} \tag{22}$$

4. NATURAL FREQUENCIES OPTIMIZATION

This work must search the best selection of isolators' parameters, reducing or minimizing the sixth natural frequency of the system and permitting that the group operation be far from the excitement frequency. The function NLPSolve of the program Maple 10 is used in this work. The stiffness constants and damping coefficients of isolators are chosen as the variables for the non linear optimization problem with constraints.

The minimization of the sixth natural frequency has to satisfy two constraints. The first is relative to limits over the stiffness constants, and the second one is relative to limits over the damping coefficients, according to technological and manufacturing specifications. The optimization problem is established as,

$$\begin{aligned} &\text{Minimize } f(\mathbf{k},\mathbf{c}) = -\omega_6(\mathbf{k},\mathbf{c}) \\ &\mathbf{k} \in \Re^r, \mathbf{c} \in \Re^r \\ &\text{subject to: } \mathbf{k}^{\min} \le \mathbf{k} \le \mathbf{k}^{\text{máx}} \\ &\qquad\qquad\quad \mathbf{c}^{\min} \le \mathbf{c} \le \mathbf{c}^{\text{máx}} \end{aligned} \tag{23}$$

where r=16 is the number of isolators. An optimization without parameters' constraints produces null points. On the other hand, the constraints $\mathbf{k} \le \mathbf{k}^{\max}$ and $\mathbf{c} \le \mathbf{c}^{\max}$ can be disregarded using great values of $\mathbf{k}^{\max}$ and $\mathbf{c}^{\max}$ guaranteeing the inequality. During the optimization process, there is no control over cross of natural frequencies. Table 1 gives the coordinates of each isolator (i=1…16) relative to *CM* as shown in Fig. 4.

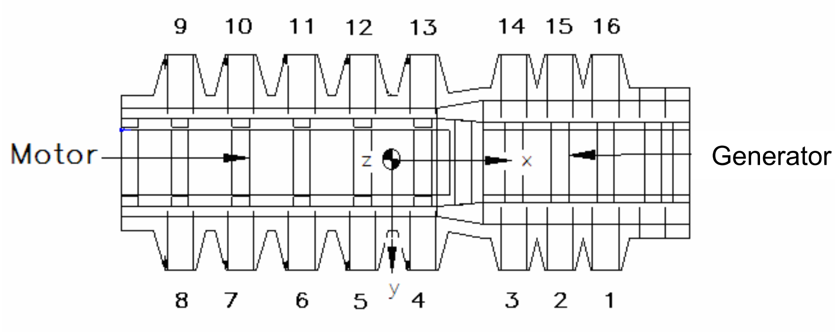

Figure 4. Position of the isolator relative to CM

Table 1. Coordinates position of the isolators

Isolator (i =1... 16)	Coordinates from CM (m)		
	l_{xi}	l_{yi}	l_{zi}
1	2,999	1,464	1,440
2	2,299	1,464	1,440
3	1,599	1,464	1,440
4	0,226	1,464	1,440
5	-0,693	1,464	1,440
6	-1,613	1,464	1,440
7	-2,533	1,464	1,440
8	-3,453	1,464	1,440
9	-3,453	-1,364	1,440
10	-2,533	-1,364	1,440
11	-1,613	-1,364	1,440
12	-0,693	-1,364	1,440
13	0,226	-1,364	1,440
14	1,599	-1,364	1,440
15	2,299	-1,364	1,440
16	2,999	-1,364	1,440

The optimal stiffness constant and damping coefficient of the r=16 isolators must be obtained after running the optimization of Eq. (23), adopting 12 isolators type ISO/A ($k^{\text{mín}} = 3.52\text{x}10^6$ N/m and $k^{\text{máx}} = 4.30\text{x}10^6$ N/m), and 4 isolators type ISO/B ($k^{\text{mín}} = 3.95\text{x}10^6$ N/m and $k^{\text{máx}} = 4.83\text{x}10^6$ N/m). The damping coefficients of all elements must satisfy $c^{\text{mín}} = 2.7\text{x}10^4$ kg $\cdot$ rad/s and $c^{\text{máx}} = 4.15\text{x}10^4$ kg $\cdot$ rad/s . The transversal and longitudinal stiffness constant and damping coefficient are equal to 20% of the corresponding vertical one value, as exposed by Rivin (2006) for precision equipment. Table 2 gives the founded optimal parameters and Table 3 shows the natural frequencies.

Table 2. Optimized parameters of isolator

Isolator (i =1...16)	Type	Constant stiffness (N/m)			Damping coefficient (kg rad/s)		
		k_{zi}	$k_{xi} = 0.2k_{zi}$	$k_{yi} = 0.2k_{zi}$	c_{zi}	$c_{xi} = 0.2\ c_{zi}$	$c_{yi} = 0.2\ c_{zi}$
1, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 16	ISO/A	$3.91\text{x}10^6$	$7.82\text{x}10^5$	$7.82\text{x}10^5$	$3.43\text{x}10^4$	$6.85\text{x}10^3$	$6.85\ \text{x}10^3$
2, 6, 11, 15	ISO/B	$4.39\text{x}10^6$	$8.78\text{x}10^5$	$8.78\text{x}10^5$	$3.68\text{x}10^4$	$7.35\text{x}10^3$	$7.35\ \text{x}10^3$

Table 3. Natural frequencies after optimization

Mode	1	2	3	4	5	6
Damped natural frequency (Hz)	1,929	1,998	2,081	4,075	4,711	5,804

5. CONCLUSIONS

This work applies a numerical optimization to obtain the optimal parameters of isolators for reducing the maximum natural frequency of the group, so that the system operates far from the frequency of operation. The stiffness constant and damping coefficient of the isolators are the variables of the nonlinear optimization problem with constraints.

For the geometrical conditions of the problem, the optimization algorithm supplies the following parameters: stiffness constant equal to $3.91\text{x}10^6$ N/m for ISO/A and $4.39\text{x}10^6$ N/m for ISO/B isolators, while the damping coefficient is equal to 3.43 kg·rad/s for all isolators.

The natural frequencies for the group becomes between 1.92 and 5.80 Hz.

6. REFERENCES

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