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EVALUATION OF A SIMPLE APPROACH FOR THE OPTIMIZATION OF THE STACKING SEQUENCE, THICKNESS AND ANGLES OF COMPOSITE LAMINATED STRUCTURES

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***Abstract**. The use of composite materials has been increasing in the aeronautical industry for use in primary structures, such as wing and fuselage, where it can significantly save structural weight. Composite laminate optimization can be done as a discrete variable problem constrained by material strength, buckling and natural frequency. The solution for discrete variable problem based on combinatorial optimization techniques is too costly and alternatives must be explored. In the present work a simple methodology has been applied to determine the optimal stacking sequence and the number of layers for composite plates using a technique based on the use of discrete variables applied to the PCOMP property cards of composite laminates (typical of commercial finite element codes). The structural optimization software GENESIS® is used to implement the ideas for the composite laminate optimization. A simple composite plate is optimized to validate the methodology.*

***Keywords**: composite structures, structural optimization, stacking sequence*

1. INTRODUCTION

The use of composite materials has been more attractive since last decade especially in aerospace and aeronautical industry, due to the high relation stiffness per weight, good corrosion properties and low thermal expansion coefficient. One of the great advantages with composite materials is that elastic material properties can be tailored by proper choice of ply thickness, angles and materials. However, efficient and simple optimization strategies to find such properties are not easily available in current literature. Therefore, the purpose of this paper is to explore a simple strategy for optimization of laminate composite elastic properties, of discrete nature, such as stacking sequence, ply thickness and orientation angle. The strategy is based on the use of the PCOMP bulk card common to commercial codes. In this study the structural optimization code GENESIS is used. This software has a proprietary way of dealing with discrete variables and is based on the use of approximation concepts in structural optimization Schmit (1973). Therefore, the optimization is very efficient with respect to the number of complete finite element analysis and sensitivity necessary to achieve convergence of optimization iterations.

Structure optimization starts with the natural necessity to reduce mass in structures developed the strategy to optimize structures modeled with finite elements using non linear programming methodology (MPNL). Methods like this are very attractive because they have generalities, but usually demands a good number of math calculation to objective functions, design constraints and your respective gradients which depends generically of their structural analysis.

For structures with thousand degrees of freedom structural and sensibility analysis costs could be very high, this motivates the study of approximations concepts, which are used through math programming methods in structural optimization.

Many works were done to optimize composite laminate structures. Yamazaki (1996) proposed to maximize the buckling and frequency performance of a composite plate through gradient-based optimization using design variables to approximate near the optimum discrete design. A two-step approach was proposed by Todoroki and Haftka (1998) to maximize buckling load of a composite plate. Leiva (2002) developed a new approach in stacking sequence optimization of composite laminates using GENESIS structural analysis and optimization software. Layup optimization for maximization of the buckling load using laminations parameters as design variables and including their feasible region was proposed by Diaconu and Sekine (2004).

The aim of the present article is to provide the effectiveness of a simple methodology to optimize composite laminates plates. A simple supported composite plate will be optimized for buckling load in order to compare results with analytical solutions. Although a simple structure is used as example, larger structures submitted to different loads could be also optimized using the same approach.

2. DESCRIPTION OF THE METHODOLOGY

This approach is generic and can be used in any type of software that uses the bulk data property card for composite materials similar to 'PCOMP' NASTRAN® bulk card. The laminate property bulk data card 'PCOMP' defines the layup for a laminate composite as shown in Table 1.

Table 1. PCOMP Bulk Data

PCOMP	PID	Z0	NSM	SB	FT	TREF	GE	LAM
	MID_1	T_1	θ_1	$SOUT_1$	MID_2	T_2	θ_2	$SOUT_2$
	MID_3	T_3	θ_3	$SOUT_3$	MID_4	T_4	θ_4	$SOUT_4$
	….	….	….	….	….	….	….	….
	MID_{N-1}	T_{N-1}	θ_{N-1}	$SOUT_{N-1}$	MID_N	T_N	θ_N	$SOUT_N$

The meaning of the PCOMP bulk card fields are as follows:

- PID – PCOMP Property identification number
- Z0 – Distance of lower plate from reference plane
- NSM – Non structural mass per mass unit
- SB – Bound Shear Allowable
- TREF – Reference Temperature of Laminate composite Material
- GE – Damping Coefficient
- LAM – Lamination options
- MID – Ply identification number
- T – ply thickness
- θ – ply angle
- SOUT – Stress Output option

The important fields to be defined in the optimization for each PCOMP card are the thickness and angles of each ply. These quantities will be associated to discrete design variables, as is shown in Table 2, where the thickness are now represented by design variables VTj and the angles by design variables Vθj. These design variables take their values from lists with discrete values, such as [0, t, 2t] and [0º, ±45º, 90º]. The stacking sequence will be consequence of the optimal choice of thickness and angles, since the thicknesses can be eliminated assuming the zero (approximately) value.

For practical reasons ply angles are limited to the discrete set 0º,90º, ±45º and the thickness values are integer multiples of the commercially available ply thickness..

The implementation of the strategy was carried out in two steps:

1. Defininition of the initial layup, preferably after a previous structural analysis, in order to avoid an unnecessary high number of design variables. In our case, symmetric and balanced laminates were imposed. With this configuration the number of variables number is half of the number of plies.
2. Association of PCOMP card thickness and angles fields to respective thickness and angle design variables. The Table 2 shows a typical symmetric and balanced composite laminate using optimization variables, where for the LAM field (see Table 1) of the card is changed to SYM.

Table 2. Balanced and Symmetric PCOMP Bulk Data writing using optimization variables

PCOMP	PID	Z0	NSM	SB	FT	TREF	GE	SYM
	MID_1	VT_1	$V\theta_1$	$SOUT_1$	MID_1	VT_1	$-V\theta_1$	$SOUT_1$
	MID_2	VT_2	$V\theta_2$	$SOUT_2$	MID_2	VT_2	$-V\theta_2$	$SOUT_2$
	….	….	….	….	….	….	….	….
	MID_N	VT_N	$V\theta_N$	$SOUT_N$	MID_N	$-VT_N$	$-V\theta_N$	$SOUT_N$

3. OPTIMIZATION STRATEGY

The mathematical optimization problem of mass minimization of a composite material structure under buckling load factor constraint is the following:

Minimize:

$$M\left(Vt_i, V\theta_i\right) \tag{1}$$

Subjected to

$$\lambda \geq 1 \tag{2}$$

Where the discrete variables take values from the following lists:

$$Vt_i = \left[10^{-4}, t, 2t\right] \quad \text{i=1,2,..,N} \tag{3}$$

$$V\theta_i = \left[0^0, \pm 45^0, 90^0\right] \quad \text{i=1,2,..,N} \tag{4}$$

A remark is necessary in that the value of t in the list of Eq.(3) is fixed, so that the variable VT_i must be taken from one of the three discrete values in the list. It was found that these three discrete values are sufficient for the purpose of optimization. The 10^{-4} thickness is used to represent a thickness that should be eliminated from the PCOMP card, but can not because it would cause an error in the finite element code.. In fact, this option of near zero value for the thickness variable is the key to allow staking sequence optimization.

The Figure 1 represents a typical structural optimization process using finite element model.

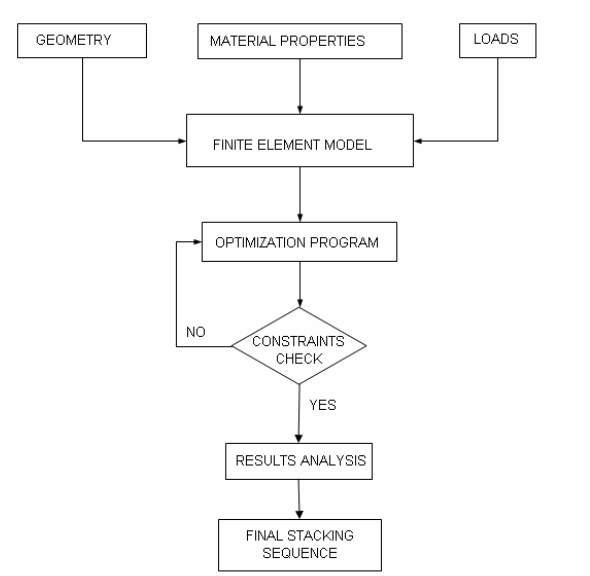

Figure 1. Structure Optimization Flowchart

4. COMPOSITE PLATE PROBLEM

The simply supported laminated plate in Figure 2, is under compression load of 5DaN/mm. The lateral dimensions are 200mm width and 400mm length. The laminate is assumed symmetric and balanced.

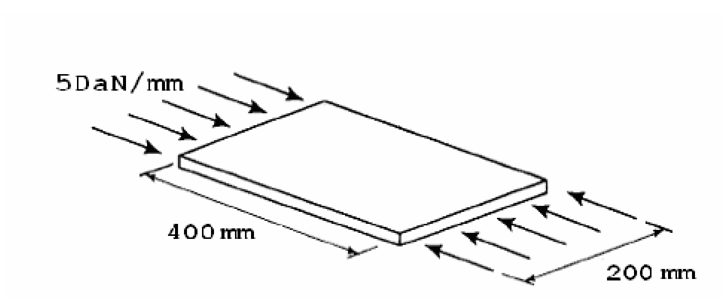

Figure 2. Simple Supported Plate Detailed

The graphite-epoxy with the properties detailed in Table 3 is used for the plate model.

Table 3. Graphite-epoxy material Properties

Material Properties		
Longitudinal Youngs's modulos	E_1	15,5 GPa
Transverse Young's modulos	E_2	0,85 GPa
Major Shear Modulus	G_{12}	450 MPa
Poisson's Ratio	υ_{12}	0.31
Density	ρ	$1{,}6.10^{-4}$ Kg/mm^3
Ply thickness	t_{ply}	0,195 mm

In order to make possible a comparison of optimal results using the strategy presented in the last section, an optimal solution based on analytical classical simpler formulation will be used. This formulation is presented in the Appendix where the critical buckling load is obtained from Eq.12, which does not consider the bend-twist stiffness coupling terms D_{16} and D_{26}, which are however present in the finite element formulation of the commercial code used here.

First the plate is optimized to minimal mass subject to the constraint of unit buckling load factor using the discrete thickness and angles of the lists in Eqs. 3-4, using the formulation in Eq.12. In fact, the process is just a sweeping using different combinations of stacking sequences, leading to the optimal laminate [45/-45/45/-45/0]s, with the buckling load factor found of approximately 1.0 (λ_{cr_an}=0.998).

The finite element optimization analysis is run in the software GENESIS®, where the strategy of Section 2 is implemented in the FE model of Fig. 1.

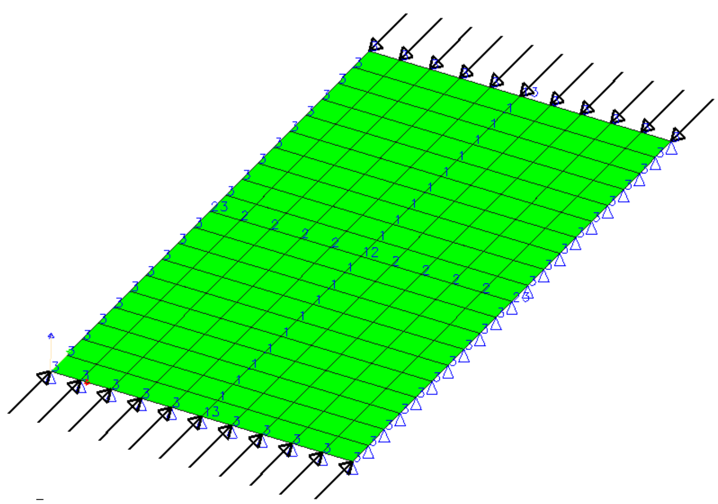

Figure 3. Simply support plate finite element description

The result of the buckling load factor of the optimal design [45/-45/45/-45/0]s from the classical solution obtained with FEM model is λ=0.9. Therefore the constraint of Eq.(2) is reformulated to $\lambda \geq 0.9$

The initial design for the finite element optimization analyses has two more plies than the analytical optimal result. Thus, fourteen design variables are used, seven to represent the possibilities for thickness and more seven to represent the possible angles related to each ply. The discrete values used for the design variables are those in Eq. 5 and Eq. 6. The initial design is a $[0_3/-45/\,45/-45/45]_s$ layup, whose initial weigh is 349g.

$$Vt_i = [0.001,\ 0.1945,\ 0.38]\text{mm} \tag{5}$$

$$V\theta_i = \left[0^0, \pm 45^0, 90^0\right] \tag{6}$$

In Fig. 5 the optimization history is presented with values of the objective function and maximum constraint violation per optimization cycle. It is interesting to see that in the initial design cycles the constraint is satisfied, becoming violated later and then finally converging to a feasible solution.

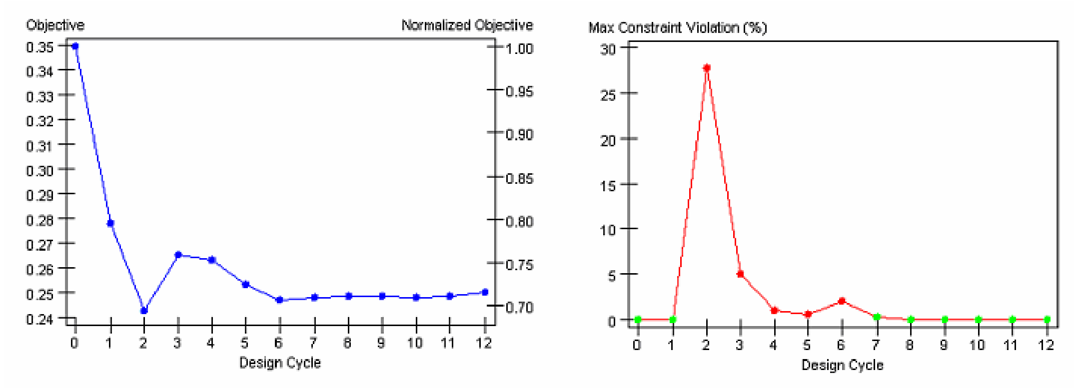

Figure 4. Optimization Results for Objective function and Constraints in each design cycle

Final results for the design variables are presented in Table 4, where the variables t1 and t2 near zero mean that these two plies must be removed and the final optimal laminate is a [45/-45/45/-45/0]s layup, which is the same obtained with the classical theory optimization.. The optimal buckling load factor is λ_{cr_fe}=0.917.

Table 4. Design Variables Results

Thickness Design Variable	Results	Angle Design Variable	Results
~~t1~~	~~0.001~~	~~a1~~	~~-45.0~~
~~t2~~	~~0.001~~	~~a2~~	~~0.0~~
t3	0.195	a3	45.0
t4	0.195	a4	-45.0
t5	0.195	a5	45.0
t6	0.195	a6	-45.0
t7	0.195	a7	0.0

The critical buckling mode shape is shown in Figure 5, from where it can be seen that it has two half waves in y direction (m=2) and one half wave in x direction (n=1), which by the way is identical to the buckling mode obtained from the approximate optimal solution.

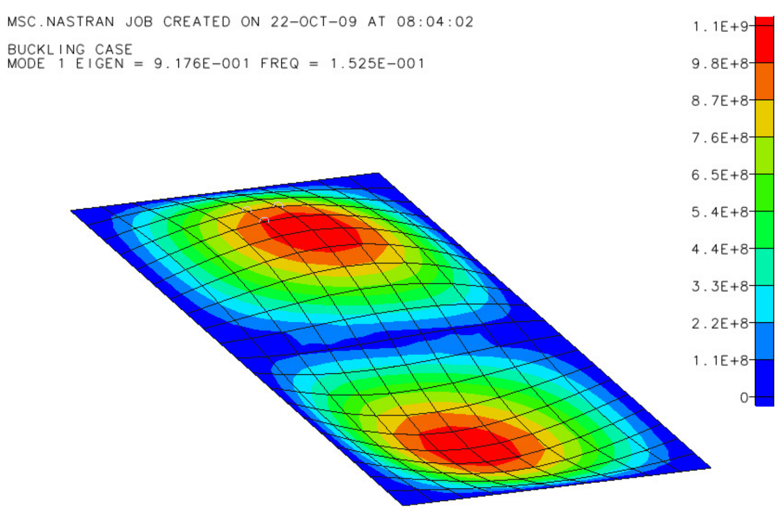

Figure 5. Finite Element Composite Laminate Panel Buckling displacement analysis

5. CONCLUSION

A method to optimize stacking sequence and number of layers for unstiffened anisotropic laminated fiber composite plates was presented, which is very simple to implement in commercial codes. The approach was validated by comparison of the finite element optimization solution obtained with commercial codes and the use of the closed form solution of buckling load factor of classical composite laminate theory. The results showed exactly the same staking sequence for the analytical analysis and finite element analysis for a simple supported composite plate.

APPENDIX (CLASSICAL SOLUTION)

Analytical analysis will describe only symmetric laminates. The symmetry characteristic simplifies considerably the general stiffness equations. Because angle and thickness symmetry in the ply there is no bending-extension coupling, all Bij expressed in Eq. 7 are equal to zero. It is important for two reasons. First, the analysis of such laminate is too much easy to analyze and second because symmetric laminates don't have the tendency to bend or twist from the inevitable thermally induced contractions.

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \gamma_6^0 \\ \kappa_1 \\ \kappa_2 \\ \kappa_6 \end{Bmatrix} \tag{7}$$

Balanced Laminates has other important simplification, the terms A_{16} e A_{26} are null and there is no shear-extension coupling. In the optimization problem, symmetric and balanced special laminates are disered, once the number of variables is half of plies number.

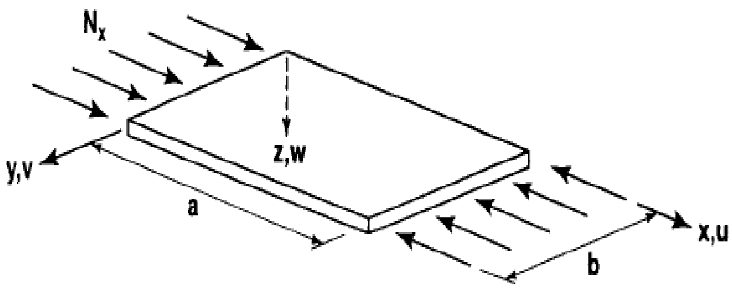

Figure 6. Simply support buckling analysis description

The buckling analysis of a simply supported laminate plate, showed in Figure 1, loaded in X direction is represented accord with Eq. 8.

$$\begin{aligned} &D_{11}\delta w,_{xxxx} + D_{16}\delta w,_{xxxy} + 2(D_{12} + 2D_{66})\delta w,_{xxyy} + \\ &4D_{26}\delta w,_{xxyy} + D_{22}\delta w,_{yyyy} + \overline{N}_x \delta w,_{xx} = 0 \end{aligned} \tag{8}$$

The Eq. 9 and Eq. 10 comes from boundary conditions: (x=0 → δw=0, δM_x=0 and y=0 → δw=0, δM_y=0)

$$-D_{11}\delta w,_{xx} - D_{12}\delta w,_{yy} - 2D_{16}\delta w,_{xy} = 0 \tag{9}$$

$$-D_{12}\delta w,_{xx} - D_{22}\delta w,_{yy} - 2D_{26}\delta w,_{xy} = 0 \tag{10}$$

There is not a closed form solution due to terms D_{xs} e D_{ys}. The variation in lateral displacement δw cannot be describe as a separated function of x plus a function of y. Otherwise, Ashton and Waddoups (1969) obtained an approximate Rayleigh-Rits solution substituting the variation in lateral displacement in the expression for second variation of potential energy and relating it with A_{mn} arriving in Eq. 11.

$$\delta w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{b}\right) \tag{11}$$

"Equation 11" satisfies the first boundary condition for all edges ($\delta w=0$) but not the natural conditions of the problem ($\delta M_n=0$).

The bend-twist coupling makes the especially orthotropic approximation for buckling load factor defined in Eq. 12 unconservative, however this work will consider the results from Eq. 12 always compared with the finite element analysis results with a correcting factor.

$$\lambda\,(m,n)=\pi^2\frac{\left[D_{11}\left(\frac{m}{a}\right)^4+2(D_{12}+2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2+D_{22}\left(\frac{n}{b}\right)^4\right]}{\left(\frac{m}{a}\right)^2N_x+\left(\frac{n}{b}\right)^2N_y} \tag{12}$$

The resulting D matrix for the optimal solution of the simply supported plate is the following, leading to λ=0,998.

$$[D]=\begin{bmatrix}2880.0 & 2325.4 & 655.2\\ 2325.4 & 2952.8 & 655.2\\ 655.2 & 655.2 & 2439.8\end{bmatrix} \tag{13}$$

6. ACKNOWLEDGEMENTS

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