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ADAPTIVE FUZZY SLIDING MODE CONTROL WITH APPLICATION TO A CHAOTIC PENDULUM

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Abstract. *Chaos control may be understood as the use of tiny perturbations for the stabilization of unstable periodic orbits embedded in a chaotic attractor. The idea that chaotic behavior may be controlled by small perturbations of physical parameters allows this kind of behavior to be desirable in different applications. In this work, the application of a variable structure controller to second order nonlinear systems is discussed. The approach is based on the sliding mode control strategy and enhanced by an adaptive fuzzy algorithm to cope with modeling inaccuracies and external disturbances. The general procedure is applied to a nonlinear pendulum and numerical results are presented in order to demonstrate the control system performance. A comparison between the stabilization of general orbits and unstable periodic orbits embedded in chaotic attractor is carried out showing that the chaos control can confer flexibility to the system by changing the response with low power consumption. Since noise contamination is unavoidable in experimental data acquisition, it is important to evaluate its effect on chaos control procedures. This work also investigates the effect of noise on the proposed control scheme, verifying the influence on the system stabilization and on the required control action.*

Keywords: *Adaptive algorithms, Chaos control, Fuzzy logic, Nonlinear pendulum, Noisy signal, Sliding modes*

1. INTRODUCTION

Chaotic response is related to a dense set of unstable periodic orbits (UPOs) and the system often visits the neighborhood of each one of them. Moreover, chaos has sensitive dependence to initial conditions, which implies that the system evolution may be altered by small perturbations. Chaos control is based on the richness of chaotic behavior and may be understood as the use of tiny perturbations for the stabilization of an UPO embedded in a chaotic attractor. It makes this kind of behavior to be desirable in a variety of applications, since one of these UPO can provide better performance than others in a particular situation.

The first chaos control method has been proposed by Ott *et al.* (1990), nowadays known as the OGY (Ott-Grebogi-Yorke) method. This is a discrete technique that considers small perturbations applied in the neighborhood of the desired orbit when the trajectory crosses a specific surface, such as some Poincaré section. The delayed feedback control (Pyragas, 1992), on the other hand, was the first continuous method proposed for controlling chaos, which states that chaotic systems can be stabilized by a feedback perturbation proportional to the difference between the present and the delayed state of the system.

Literature presents some contributions related to the analysis of chaos control in pendulum systems (Pereira-Pinto *et al.*, 2004, 2005; Wang and Jing, 2004; Yagasaki and Yamashita, 1999) using different approaches. De Paula and Savi (2009b) propose a multiparameter semi-continuous method based on OGY approach to perform the chaos control of a nonlinear pendulum. Afterwards, De Paula and Savi (2009a) use a continuous delayed-feedback scheme and Bessa *et al.* (2009) propose an adaptive fuzzy sliding mode based approach to control chaos in the same nonlinear pendulum.

Sliding mode control is a very attractive control scheme because of its robustness against both structured and unstructured uncertainties as well as external disturbances. Nevertheless, the discontinuities in the control law must be smoothed out to avoid the undesirable chattering effects. The adoption of properly designed boundary layers have proven effective in completely eliminating chattering, however, leading to an inferior tracking performance.

In this work, a control scheme based on the sliding mode strategy and enhanced by an adaptive fuzzy algorithm is employed to chaos control. The adaptive fuzzy inference system approximates the unknown system dynamics within boundary layer of a smooth sliding mode controller, improving the tracking performance. As an application of the general procedure, the chaos control of a nonlinear pendulum that has a rich response, presenting chaos and transient chaos (De Paula *et al.*, 2006), is treated. Numerical simulations are carried out illustrating the stabilization of some UPOs of the chaotic attractor showing an effective response. Unstructured uncertainties related to unmodeled dynamics and structured uncertainties associated with parametric variations are both considered in the robustness analysis. A comparison between the stabilization of a general orbit and unstable periodic orbits embedded in chaotic attractor is performed showing the less energy consumption related to UPOs. Moreover, the analysis from noisy time series is also conducted showing the effectiveness of the controller to stabilize unstable orbits.

2. ADAPTIVE FUZZY SLIDING MODE CONTROL

As demonstrated by Bessa and Barrêto (2010), adaptive fuzzy algorithms can be properly embedded in sliding mode controllers to compensate for modeling inaccuracies, in order to improve the trajectory tracking of uncertain nonlinear systems. It has also been shown that adaptive fuzzy sliding mode controllers are suitable for a variety of applications ranging from remotely operated underwater vehicles (Bessa *et al.*, 2008) to space satellites (Guan *et al.*, 2005).

On this basis, let us consider a second order dynamical system represented by the following equation of motion:

$$\ddot{\phi} = f(\phi, \dot{\phi}, t) + hu + p(\phi, \dot{\phi}) \tag{1}$$

where ϕ and $\dot{\phi}$ represent the state variables, u is the control input, h is the control gain, $f : \mathbb{R}^3 \to \mathbb{R}$ is a nonlinear function that represents system dynamics and p represents modeling inaccuracies.

Now, let $S(t)$ be a sliding surface defined in the state space by the equation $s(e, \dot{e}) = 0$, with the function $s : \mathbb{R}^2 \to \mathbb{R}$ satisfying

$$s(e, \dot{e}) = \dot{e} + \lambda e \tag{2}$$

where $e = \phi - \phi_d$ is the tracking error, $\dot{e}$ is the first time derivative of e, ϕ_d is the desired trajectory and λ is a strictly positive constant.

The control of the system dynamics (1) is done by assuming a sliding mode based approach, defining a control law composed by an equivalent control $\hat{u} = \hat{h}^{-1}(-\hat{f} - \hat{p} + \ddot{\phi}_d - \lambda\dot{e})$ and a discontinuous term $-K\,\mathrm{sgn}(s)$:

$$u = \hat{h}^{-1}(-\hat{f} - \hat{p} + \ddot{\phi}_d - \lambda\dot{e}) - K\,\mathrm{sgn}(s) \tag{3}$$

where $\hat{h}$, $\hat{f}$, and $\hat{p}$ are estimates of h, f and p, respectively, K is a positive control gain and $\mathrm{sgn}(\cdot)$ is defined as

$$\mathrm{sgn}(s) = \begin{cases} -1 & \text{if} \quad s < 0 \\ 0 & \text{if} \quad s = 0 \\ +1 & \text{if} \quad s > 0 \end{cases} \tag{4}$$

Regarding the development of the control law, the following assumptions should be made:

Assumption 1 *The states ϕ and $\dot{\phi}$ are available.*

Assumption 2 *The desired trajectories ϕ_d and $\dot{\phi}_d$ are once differentiable in time. Furthermore ϕ_d, $\dot{\phi}_d$ and $\ddot{\phi}_d$ are available and with known bounds.*

Assumption 3 *The function f is unknown but bounded, i.e., $|\hat{f} - f| \leq \mathcal{F}$.*

Assumption 4 *The input gain h is unknown but positive and bounded, i.e.,* $0 < h_{\min} \leq h \leq h_{\max}$.

Assumption 5 *The term p is unknown but bounded, i.e., $|p| \leq \mathcal{P}$.*

Based on Assumption 4 and considering that the estimate $\hat{h}$ could be chosen according to the geometric mean $\hat{h} = \sqrt{h_{\max} h_{\min}}$, the bounds of h may be expressed as $\mathcal{H}^{-1} \leq \hat{h}/h \leq \mathcal{H}$, where $\mathcal{H} = \sqrt{h_{\max}/h_{\min}}$.

Under this condition, the gain K should be chosen according to

$$K \geq \mathcal{H}\hat{h}^{-1}(\eta + |\hat{p}| + \mathcal{P} + \mathcal{F}) + (\mathcal{H} - 1)|\hat{u}| \tag{5}$$

here η is a strictly positive constant related to the reaching time.

At this point, it should be highlighted that the control law (3), together with (5), is sufficient to impose the sliding condition

$$\frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s| \tag{6}$$

and, consequently, the finite time convergence to the sliding surface S. For a proof of the convergence properties of the aforementioned controller (3) considering the gain (5), the reader is referred to (Bessa *et al.*, 2009).

In order to obtain a good approximation to p, the estimate $\hat{p}$ is computed directly by an adaptive fuzzy algorithm. The adopted fuzzy inference system is the zero order TSK (Takagi–Sugeno–Kang), whose rules can be stated in a linguistic manner as follows:

$$\textit{If } \phi \textit{ is } \Phi_r \textit{ and } \dot{\phi} \textit{ is } \dot{\Phi}_r \textit{ then } \hat{p}_r = \hat{P}_r \; , \quad r = 1, 2, \ldots, N$$

where Φ_r and $\dot{\Phi}_r$ are fuzzy sets, whose membership functions could be properly chosen, and $\hat{P}_r$ is the output value of each one of the N fuzzy rules.

Considering that each rule defines a numerical value as output $\hat{P}_r$, the final output $\hat{p}$ can be computed by a weighted average:

$$\hat{p}(\phi, \dot{\phi}) = \hat{\mathbf{P}}^{\mathrm{T}}\mathbf{\Psi}(\phi, \dot{\phi}) \tag{7}$$

where $\hat{\mathbf{P}} = [\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_N]$ is the vector containing the attributed values $\hat{P}_r$ to each rule r, $\mathbf{\Psi}(\phi, \dot{\phi}) = [\psi_1, \psi_2, \ldots, \psi_N]$ is a vector with components $\psi_r(\phi, \dot{\phi}) = w_r / \sum_{r=1}^{N} w_r$ and w_r is the firing strength of each rule, which can be computed from the membership values with any fuzzy intersection operator (t-norm).

The estimation of $\hat{p}$ is done by considering that the vector of adjustable parameters can be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{P}}} = \varphi s \mathbf{\Psi}(\phi, \dot{\phi}) \tag{8}$$

where φ is a strictly positive constant related to the adaptation rate.

Now, in order to overcome the undesirable chattering effects, a thin boundary layer, S_ϵ, in the neighborhood of the switching surface can be adopted (Slotine, 1984):

$$S_\epsilon = \left\{ (e, \dot{e}) \in \mathbb{R}^2 \;\middle|\; |s(e, \dot{e})| \leq \epsilon \right\}$$

where ϵ is a strictly positive constant that represents the boundary layer thickness.

The boundary layer is achieved by replacing the sign function by a continuous interpolation inside S_ϵ. There are several options to smooth out the ideal relay but the most common choice is the saturation function:

$$\mathrm{sat}(s/\epsilon) = \begin{cases} \mathrm{sgn}(s) & \text{if} \quad |s/\epsilon| \geq 1 \\ s/\epsilon & \text{if} \quad |s/\epsilon| < 1 \end{cases}$$

In this way, to avoid chattering, a smooth version of Eq. (3) is defined:

$$u = \hat{h}^{-1}(-\hat{f} - \hat{p} + \ddot{\phi}_d - \lambda\dot{e}) - K\,\mathrm{sat}(s/\epsilon) \tag{9}$$

Nevertheless, it should be emphasized that the substitution of the discontinuous term by a smooth approximation inside the boundary layer turns the perfect tracking into a tracking with guaranteed precision problem, which actually means that a steady-state error will always remain. According to Bessa (2009) and considering a second order system with a smooth sliding mode controller, the tracking error vector will exponentially converge to a closed region $\Lambda = \{(e, \dot{e}) \in \mathbb{R}^2 \mid |s(e, \dot{e})| \leq \epsilon$ and $|e| \leq \lambda^{-1}\epsilon$ and $|\dot{e}| \leq 2\epsilon\,\}$.

3. NONLINEAR PENDULUM

As an application of the control procedure, a nonlinear pendulum is investigated. This pendulum is based on an experimental set up, previously analyzed by Franca and Savi (2001) and Pereira-Pinto *et al.* (2004). De Paula *et al.* (2006)

presented a mathematical model to describe the dynamical behavior of the pendulum and the corresponding experimentally obtained parameters.

The schematic picture of the considered nonlinear pendulum is shown in Fig. 1. Basically, the pendulum consists of an aluminum disc (1) with a lumped mass (2) that is connected to a rotary motion sensor (4). This assembly is driven by a string-spring device (6) that is attached to an electric motor (7) and also provides torsional stiffness to the system. A magnetic device (3) provides an adjustable dissipation of energy. An actuator (5) provides the necessary perturbations to stabilize this system by properly changing the string length.

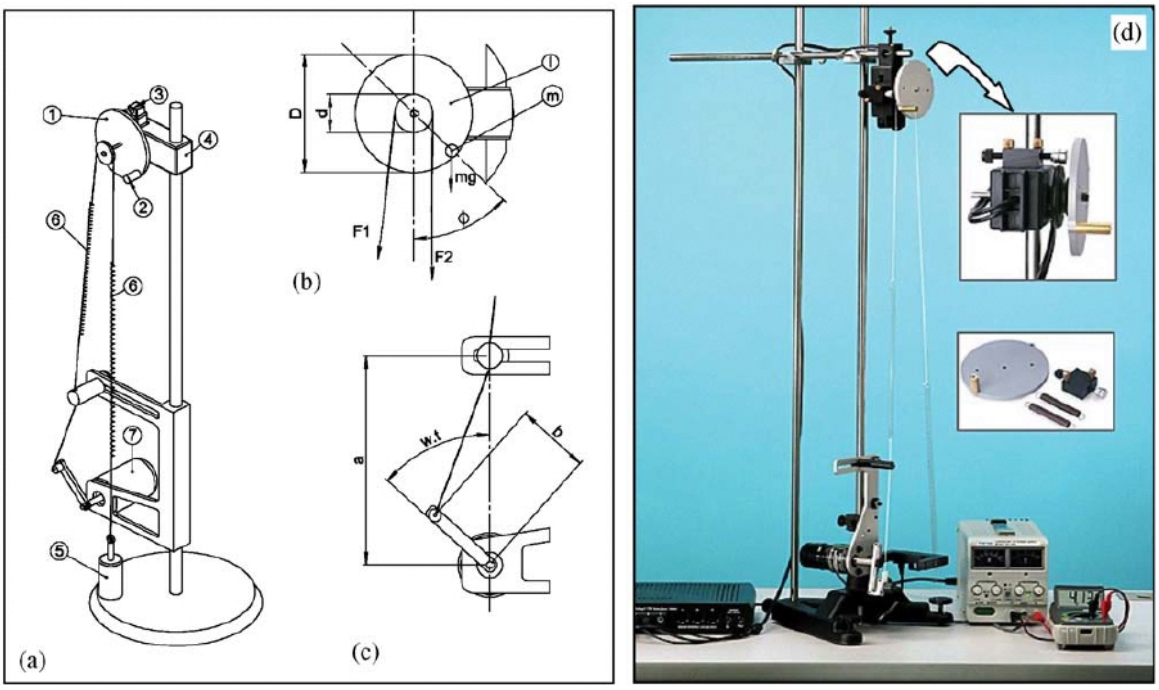


Figure 1. (a) Nonlinear pendulum – (1) metallic disc; (2) lumped mass; (3) magnetic damping device; (4) rotary motion sensor (PASCO CI-6538); (5) anchor mass; (6) string-spring device; (7) electric motor (PASCO ME-8750). (b) Parameters and forces on metallic disc. (c) Parameters from driving device. (d) Experimental apparatus.

In order to obtain the equations of motion of the experimental nonlinear pendulum it is assumed that system dissipation may be expressed by a combination of a linear viscous dissipation together with dry friction. Therefore, denoting the angular position as ϕ, the following equation is obtained (De Paula *et al.*, 2006):

$$\ddot{\phi} + \frac{\zeta}{I}\dot{\phi} + \frac{\mu\,\mathrm{sgn}(\dot{\phi})}{I} + \frac{kd^2}{2I}\phi + \frac{mgD\sin(\phi)}{2I} = \frac{kd}{2I}\left(\sqrt{a^2+b^2-2ab\cos(\omega t)} - (a-b) - \Delta l\right) \quad (10)$$

where ω is the forcing frequency related to the motor rotation, a defines the position of the guide of the string with respect to the motor, b is the length of the excitation crank of the motor, D is the diameter of the metallic disc and d is the diameter of the driving pulley, m is the lumped mass, ζ represents the linear viscous damping coefficient, while μ is the dry friction coefficient; g is the gravity acceleration, I is the inertia of the disk-lumped mass, k is the string stiffness and Δl is the length variation in the spring provided by the linear actuator (5).

De Paula *et al.* (2006) show that this mathematical model presents results that are in close agreement with experimental data. The pendulum equation can be expressed in terms of Eq. (1) by assuming that $h = kd/2I$, $u = -\Delta l$, f can be obtained from Eq. (1) and Eq. (10), and the term p represents modeling inaccuracies.

4. CONTROLLING THE NONLINEAR PENDULUM

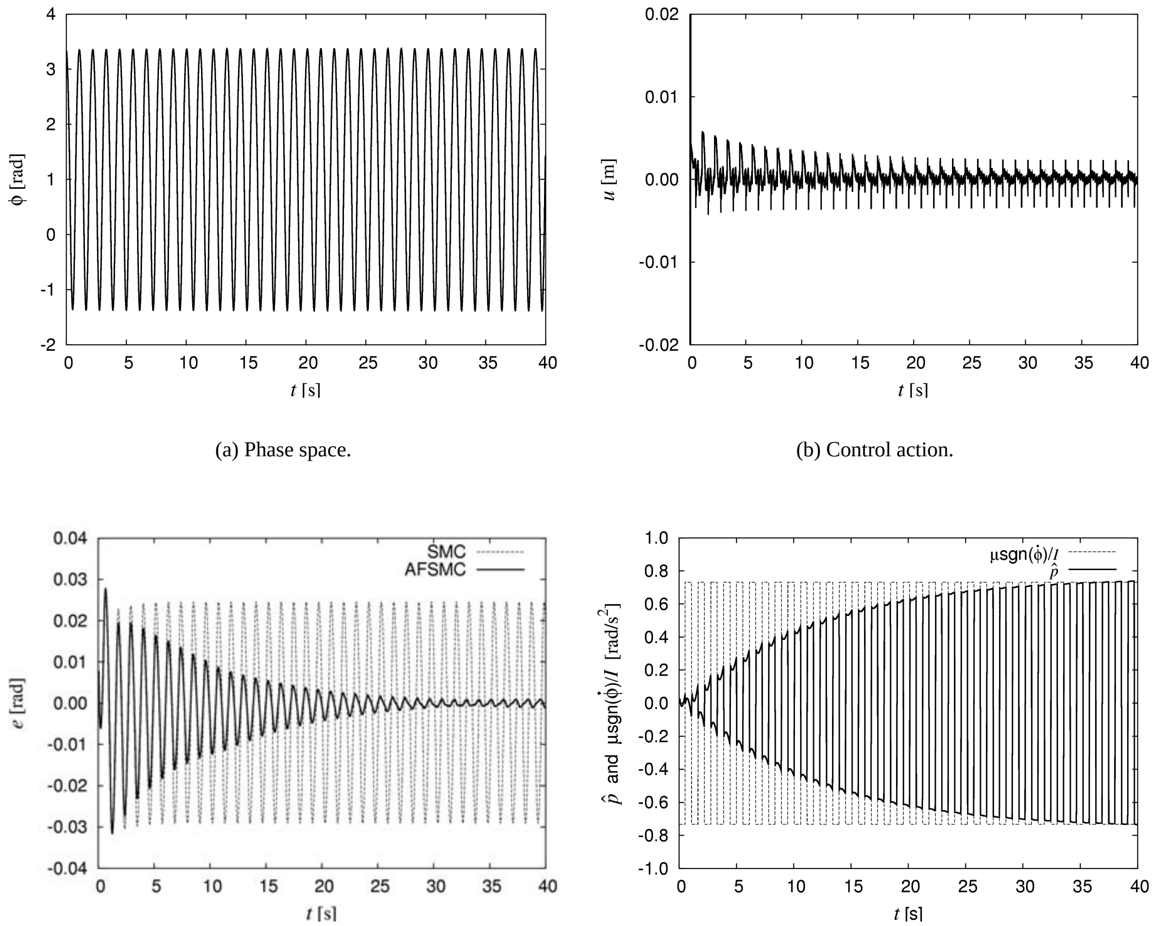
The controller capability is now investigated by considering numerical simulations. The fourth order Runge-Kutta method is employed and sampling rates of 107 Hz for control system and 214 Hz for dynamical model are assumed. The model parameters are chosen according to (De Paula *et al.*, 2006): $I = 1.738 \times 10^{-4}$ kg m^2; $m = 1.47 \times 10^{-2}$ kg; $k = 2.47$ N/m; $\zeta = 2.368 \times 10^{-5}$ kg m^2/s; $\mu = 1.272 \times 10^{-4}$ N m; $a = 1.6 \times 10^{-1}$ m; $b = 6.0 \times 10^{-2}$ m; $d = 4.8 \times 10^{-2}$ m; $D = 9.5 \times 10^{-2}$ m and $\omega = 5.61$ rad/s.

For tracking purposes, different UPOs are identified using the close return method (Pereira-Pinto *et al.*, 2004) and two of these are chosen as desired trajectories in the numerical studies that follows.

To ratify the robustness of the proposed control scheme against modeling imprecisions, the term $\mu\,\mathrm{sgn}(\dot{\phi})/I$ in Eq. (10) will be treated as unmodeled dynamics and not considered in controller design. In this way, regarding controller parameters, the following values were chosen: $\mathcal{F} = 1.2$; $\mathcal{P} = 1.1$; $\mathcal{H} = 1.0$; $\epsilon = 1.0$; $\lambda = 0.8$; $\eta = 0.05$ and $\varphi = 3.0$.

Concerning the fuzzy system, triangular and trapezoidal membership functions are adopted for both Φ_r and $\dot{\Phi}_r$, with the central values defined respectively as $C_0 = \{0.0\}$ and $C_1 = \{-10.0\,;\,-1.0\,;\,-0.1\,;\,0.0\,;\,0.1\,;\,1.0\,;\,10.0\} \times 10^{-1}$. The chosen fuzzy intersection operator was the product t-norm. It is also important to emphasize that the vector of adjustable parameters is initialized with zero values, $\hat{\mathbf{P}} = \mathbf{0}$, and updated at each iteration step according to Eq. (8).

In order to evaluate the control system performance, a period-1 UPO was identified using the close return method De Paula *et al.* (2006) and chosen to be stabilized. The obtained results are presented in Fig. 2.



(a) Phase space.

(b) Control action.

(c) Tracking error.

(d) Convergence of $\hat{p}$ to $\mu\,\mathrm{sgn}(\dot{\phi})/I$.

Figure 2. Tracking of period-1 UPO.

As observed in Fig. 2, even in the presence of modeling inaccuracies, the adaptive fuzzy sliding mode controller (AFSMC) is capable to provide the trajectory tracking with a small associated error. It should be emphasized that the control action u represents the length variation in the string and only tiny variations are required to provide such different dynamic behaviors, which actually allows a great flexibility for the controlled nonlinear system.

It can be also verified that the proposed control law provides a smaller tracking error when compared with the conventional sliding mode controller (SMC), Fig. 2(c). By considering simulation purposes, the AFSMC can be easily converted to the classical SMC by setting the adaptation rate to zero, $\varphi = 0$. The improved performance of AFSMC over SMC is due to its ability to recognize and compensate the modeling imprecisions, Fig. 2(d). The time evolution of the input-output surface of the fuzzy inference system is shown in Fig. 3 after four different iteration steps.

Now, in order to demonstrate that the adopted control scheme can deal with both structured (parametric) and unstructured uncertainties (unmodeled dynamics), an uncertainty of $\pm 20\%$ over the value of the viscous damping coefficient, ζ, is considered and the dry friction is treated as unmodeled dynamics and not taken into account within the design of the control law. On this basis, the estimates $\hat{\zeta} = 1.9 \times 10^{-5}$ kg m^2/s and $\hat{\mu} = 0$ are assumed. The other estimates in both $\hat{f}$

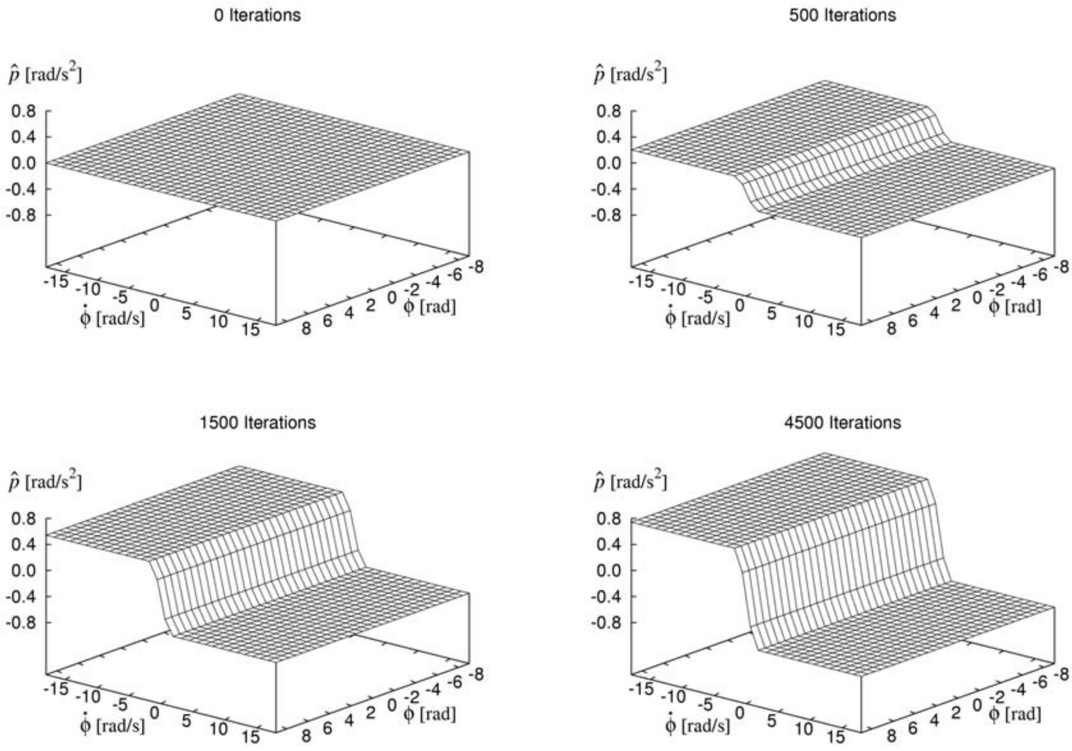


Figure 3. Convergence of the input-output surface of the fuzzy inference system after four different iteration steps.

and $\hat{h}$ are chosen based on the assumption that model coefficients are perfectly known.

The idea of the UPO control is interesting since these orbits are embedded in the chaotic attractor and, therefore are natural orbits related to the system dynamics. Hence, it is an important task to evaluate a comparison of the control action required to stabilize some UPOs and a general orbit (artificial or non-natural). Basically, three different situations are treated. In the first case, Fig. 4(a) and Fig. 4(d), a general artificial orbit $[\phi_d, \dot{\phi}_d] = [1.0 + 2.35\sin(2\pi t), 4.70\pi\cos(2\pi t)]$ is considered. A second case, on the other hand, stabilizes a period-1 UPO, Fig. 4(b) and Fig. 4(e). Although both orbits are similar, it should be highlighted that the controller requires less effort to stabilize the UPO. Even with more complicated orbits, as is the case of the period-4 UPO shown in Fig. 4(c), the amplitude of the control action, Fig. 4(f), is significantly smaller when compared with the control effort required to stabilize the general orbit. The control of unstable periodic orbits is the essential aspect to be explored in chaos control that can confer flexibility to the system with low energy consumption.

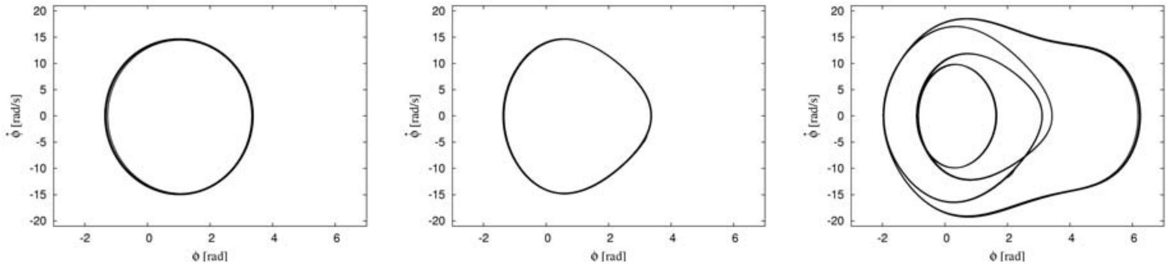
Since noise contamination is unavoidable in experimental data acquisition, it is important to evaluate its effect on chaos control procedures. In order to simulate experimental noisy data sets, a white Gaussian noise is introduced in the signal:

$$\bar{x}(t) = x(t) + \varepsilon \tag{11}$$

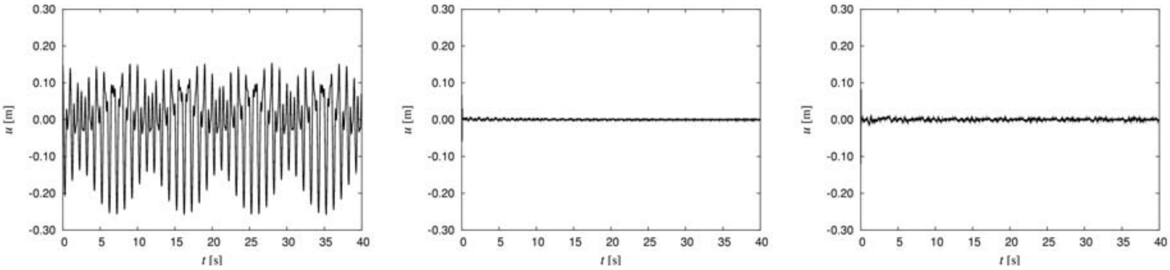
where $\bar{x}$ represents the measured state variable, x the clean signal and ε the white Gaussian noise. White Gaussian noise is generated using the polar form of Box-Muller transformation (Box and Muller, 1958). The noise level is parameterized by the standard deviation of the clean signal (S_{signal}). Therefore, the standard deviation of the noise, S_{noise}, is a fraction γ of S_{signal}:

$$\gamma = \frac{S_{\text{noise}}}{S_{\text{signal}}} \times 100 \quad (\%) \tag{12}$$

Figures 5–13 shows the stabilization of a period-1 UPO, a period-2 UPO and a period-4 UPO with three different values of γ: 1%, 3% and 5%. The phase space, the control action and the Poincaré section embedded in the related noisy strange attractor are presented.

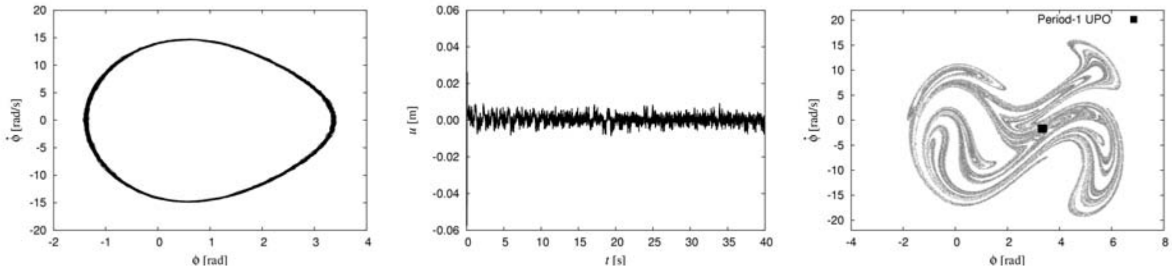


(a) General orbit. (b) Period-1 UPO. (c) Period-4 UPO.



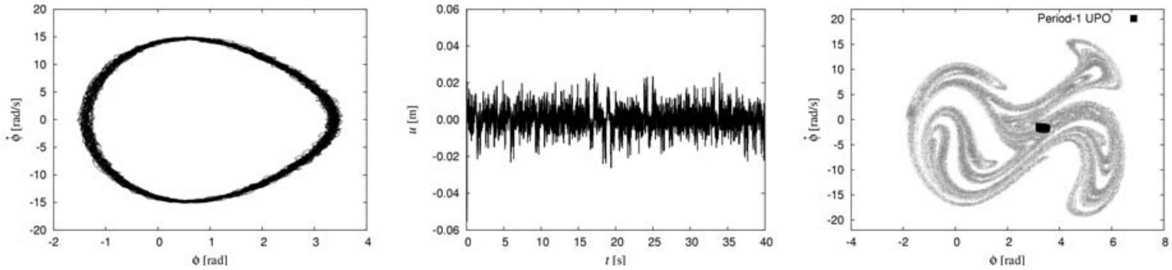
(d) u for general orbit. (e) u for period-1 UPO. (f) u for period-4 UPO.

Figure 4. Control action required to stabilize a general orbit and 2 different UPOs.



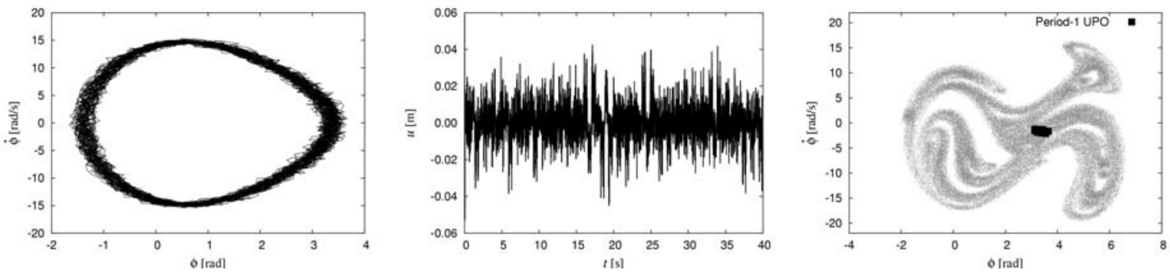
(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 5. Tracking of a period-1 UPO with $\gamma = 1\%$.



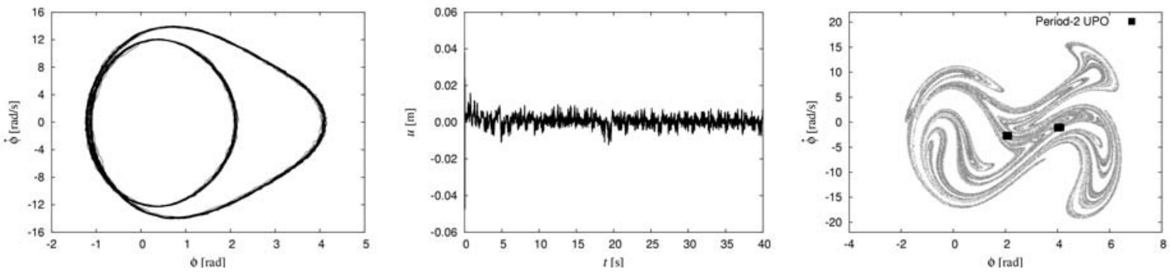
(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 6. Tracking of a period-1 UPO with $\gamma = 3\%$.



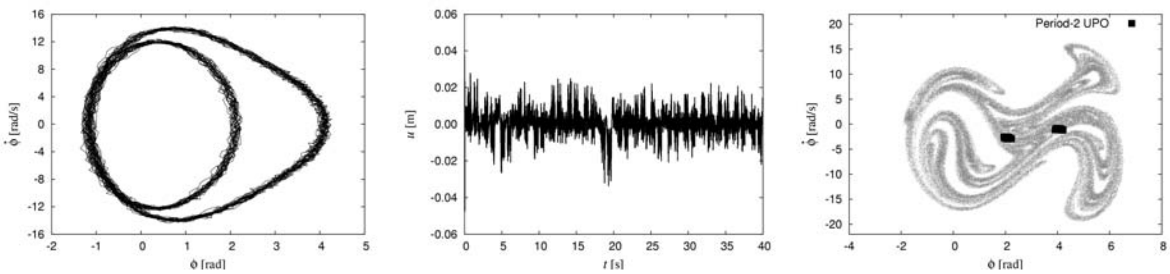
(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 7. Tracking of a period-1 UPO with $\gamma = 5\%$.



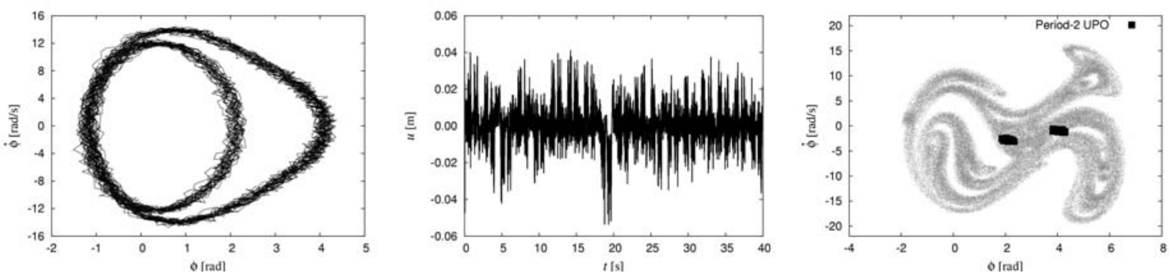
(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 8. Tracking of a period-2 UPO with $\gamma = 1\%$.



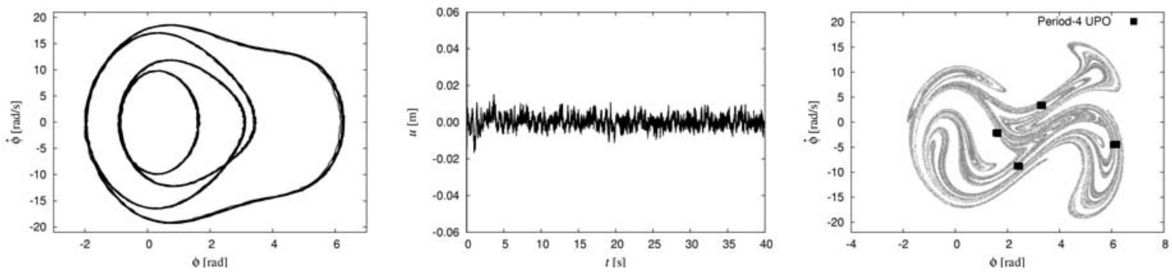
(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 9. Tracking of a period-2 UPO with $\gamma = 3\%$.



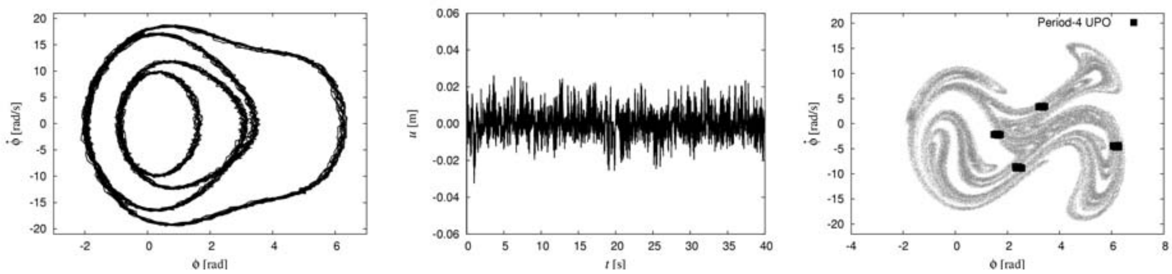
(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 10. Tracking of a period-2 UPO with $\gamma = 5\%$.



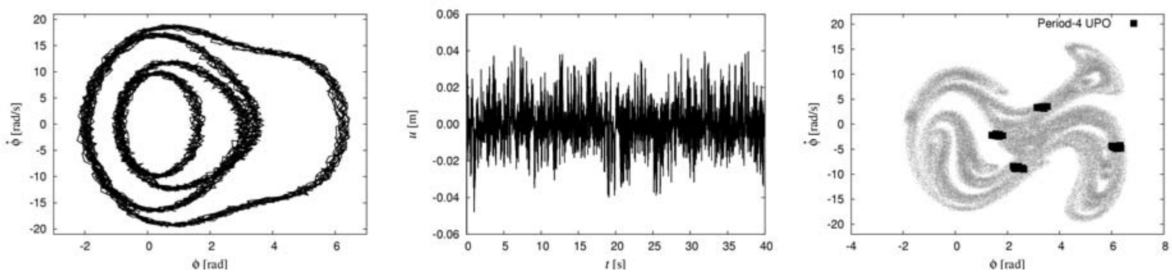
(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 11. Tracking of a period-4 UPO with $\gamma = 1\%$.



(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 12. Tracking of a period-4 UPO with $\gamma = 3\%$.



(a) Phase space. (b) Control action. (c) Poincaré section.

Figure 13. Tracking of a period-4 UPO with $\gamma = 5\%$.

As observed in Figs. 5–13, the proposed control scheme allows the UPOs stabilization even when noisy signals are considered. Nevertheless, it can be verified that the increase of the noise amplitude causes a proportional increase of the control effort and a decrease in the tracking performance.

5. CONCLUSIONS

The present contribution presents the application of an adaptive fuzzy sliding mode controller for chaos control. Numerical simulations of a nonlinear pendulum with chaotic response is of concern. The control system performance is investigated showing the tracking of a generic orbit as well as for UPO stabilization. It is shown that the controller needs less effort to stabilize an UPO when compared with a general non-natural orbit. This is an essential point related to chaos control that can confer flexibility to the system dynamics changing response with low power consumption. The robustness of the proposed control scheme against modeling inaccuracies are investigated evaluating both unstructured and parametric uncertainties. Noisy signals are also investigated showing the controller capability to deal with this kind of uncertainty. In general, the proposed procedure is able to perform chaos control even in situations where high uncertainties are involved.

6. ACKNOWLEDGEMENTS

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