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A CONSTITUTIVE MODEL FOR TRANSFORMATION INDUCED PLASTICITY IN SHAPE MEMORY ALLOYS

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Abstract. *Transformation Induced Plasticity (TRIP) is widely defined in the literature as the plastic flow arising from solid state phase transformation processes involving volume and/or shape changes without overlapping the yield surface. This phenomenon occurs in the Shape Memory Alloys (SMAs) having significant influence over their macroscopic thermomechanical behavior. Recent studies propose some constitutive theories to describe this phenomenon and its particular features inherent to SMAs. The present contribution presents a macroscopic one-dimensional constitutive model with internal constraints, which accounts for TRIP, plasticity, plastic-phase transformation coupling and tension-compression asymmetry. Comparisons between numerical and experimental results attest the model's capability to capture TRIP phenomenon. Moreover, numerical simulations are conducted in order to evaluate saturation mechanisms during cyclic loadings.*

Keywords*: Shape memory alloy, constitutive model, transformation induced plasticity, numerical simulation.*

1. INTRODUCTION

Shape Memory Alloys (SMAs) present complex thermomechanical behaviors related to different physical processes. Besides the most common phenomena presented by this class of material, such as: pseudoelasticity, shape memory effect, which may be one-way (SME) or two-way (TWSME), and phase transformation due to temperature variation, there are more complicated phenomena that have significant influence over its overall thermomechanical behavior – for instance: plastic behavior, tension-compression asymmetry, plastic-phase transformation coupling, transformation induced plasticity, among others. All these phenomena take place at a microscopic level but most of them affect SMAs' macroscopic response; therefore, should not be neglected while modeling their phenomenological behavior (Lagoudas, 2008; Paiva & Savi, 2006).

Experimental studies (Greenwood & Johnson, 1965; Magee, 1966; Abrassart, 1972; Desalos, 1981; Denis *et al.*, 1982; Olson & Cohen, 1986) reveal the growth of a nonlinear irreversible (plastic) strain amount while solid state phase transformations take place. This deformation mechanism is known in the literature as Transformation Induced Plasticity (TRIP) and results from internal stresses arising not only from the volume change associated with the transformation but also from the accompanying shape change (Marketz & Fischer, 1994) without reaching the yield surface of the weaker phase involved.

A clear distinction can be made between classical plasticity and TRIP. While classical plasticity arises from applied stress or temperature variation, TRIP is caused by phase proportions variation – even for low constant stresses level (Leblond *et al.*, 1989 ; Gautier *et al.*, 1989 ; Gautier, 1998; Fischer *et al.*, 2000, 1996; Tanaka & Sato, 1985).

TRIP phenomenon is widely attributed to occur due to two distinct physical mechanisms, which have been proposed by Greenwood & Johnson (1965) and Magee (1966). The Greenwood-Johnson effect is admitted to be due an accommodation process of the microplasticity associated with a volume change. The Magee effect, on the other hand, is due to an orientation effect that arises from a shear internal stress state, which favors the thermodynamically preferred orientation direction for martensite formation in the presence of an external stress field, involving shape change.

According to Greenwood & Johnson (1965), transformation plasticity is due to the compactness difference between parent (austenite) and product (martensite) phases' lattice structure. During martensitic transformation, this difference results in a volume change, producing a microscopic internal stress state, responsible for microscopic plasticity in the weaker phase (that with the lower yield stress). Without external applied stress, the average of these internal stresses generally vanishes and, from macroscopic point of view, only global variation of the volume will be observed. In order to understand the Magee effect, consider the martensitic formation process on cooling. During its nucleation, martensite develops plates that grow and generate large amounts of shear (deviatoric) stress in the parent austenitic matrix (Fischer et al. 1998). When no external load is applied, the plate's orientation is generally random, which makes the (macroscopic) resultant of the microscopic internal stresses average out null. An (even low and constant) external applied load, triggers an alignment of these martensitic plates with the loading stress direction. This external load is responsible for an internal stress state increase that no more will macroscopically average out null (Taleb *et al.*, 2001).

Many efforts have been done so far towards adequate TRIP modeling (Leblond *et al.*, 1989; Leblond 1989; Stringfellow *et al.*, 1992; Marketz & Fischer, 1994; Zwigl & Dunand 1997; Cherkaoui *et al.*, 1998; Ganghoffer & Simonsson, 1998; Fischer *et al.*, 1998, Taleb *et al.*, 2001; Taleb & Sidoroff, 2003; Homberg; 2004). The great majority of these models focus on micro scale features of the TRIP. Moreover, some of them discard the Magee effect under some reasonable considerations for particular studies. The present contribution presents a one-dimensional constitutive model with internal constraints, which accounts for TRIP, (linear hardening) plasticity, plastic-phase transformation coupling and tension-compression asymmetry. The target of this work is to describe the macroscopic manifestation of the TRIP phenomenon and also to explore the TRIP importance under cyclic loadings. Comparisons between numerical and experimental results are used to demonstrate the model's ability to capture TRIP phenomenon. Moreover, numerical simulations are conducted in order to evaluate saturation mechanisms during cyclic loadings.

2. CONSTITUTIVE MODEL

This work revisits the model with internal constraints previously presented in different references (Savi *et al.*, 2002; Baêta-Neves *et al.*, 2004; Paiva *et al.*, 2005; Savi & Paiva, 2005; Monteiro Jr. *et al.*, 2009), which is based on Fremond's model (Fremond, 1996). This one-dimensional macroscopic model considers (linear hardening) plasticity, plastic-phase transformation coupling, tension-compression asymmetry and different material properties. The herein presented model includes a new feature to allow TRIP phenomenon description.

A Helmholtz free energy density (ψ) considers each phase separately adopting four state variables: elastic strain (ε_e) and temperature (T) for themoelasticity description and two internal variables (γ and μ) that help the plastic phenomenon description, being associated with the isotropic and kinematic hardening, respectively.

$$\boldsymbol{M+}: \quad \rho\psi_1(\varepsilon_e,T,\gamma,\mu)=\frac{1}{2}E_M\,\varepsilon_e{}^2-\alpha^T\varepsilon_e-\Lambda_M^T-\Omega_M(T-T_0)\varepsilon_e+\frac{1}{2}K_M\,\gamma^2+\frac{1}{2H_M}\mu^2 \tag{1}$$

$$\boldsymbol{M}-: \quad \rho\psi_2(\varepsilon_e,T,\gamma,\mu)=\frac{1}{2}E_M\,\varepsilon_e{}^2+\alpha^C\varepsilon_e-\Lambda_M^C-\Omega_M(T-T_0)\varepsilon_e+\frac{1}{2}K_M\,\gamma^2+\frac{1}{2H_M}\mu^2 \tag{2}$$

$$\boldsymbol{A}: \quad \rho\psi_3(\varepsilon_e,T,\gamma,\mu)=\frac{1}{2}E_A\,\varepsilon_e{}^2-\Lambda_A-\Omega_A(T-T_0)\varepsilon_e+\frac{1}{2}K_A\,\gamma^2+\frac{1}{2H_A}\mu^2 \tag{3}$$

$$\boldsymbol{M}: \quad \rho\psi_4(\varepsilon_e,T,\gamma,\mu)=\frac{1}{2}E_M\,\varepsilon_e{}^2+\Lambda_M-\Omega_M(T-T_0)\varepsilon_e+\frac{1}{2}K_M\,\gamma^2+\frac{1}{2H_M}\mu^2 \tag{4}$$

In the previous equations, subscript M is related to martensitic phase while A is associated with austenite. Moreover, superscript T is related to tensile parameters while C is associated with compressive parameters. Observing these indexes, notice that α's are material parameters related to the vertical size of the stress-strain hysteresis loop, while Λ's are associated with phase transformations stress levels and are temperature dependent (as will be later discussed); E's represent the elastic moduli, Ω's are related to the thermal expansion coefficients, K's are the plastic modulus while H's are the kinematic hardening moduli; T_0 is a reference temperature and ρ is the density.

For the thermomechanical description of SMAs behavior, the proposed model formulation establishes the free energy of the whole mixture ($\tilde{\psi}$), which is written weighting each energy potential with the corresponding volume fraction. Under this assumption, four volume fractions are considered: β_1 is associated with tensile detwinned martensite ($M+$), β_2 is related to compressive detwinned martensite ($M-$), β_3 represents austenite (A) and β_4 corresponds to twinned martensite (M). Obviously, it is possible to consider only three volume fractions, since $\beta_1 + \beta_2 + \beta_3 + \beta_4 = 1$. Thus, writing β_4 as a function of β_1, β_2 and β_3, one finds

$$\begin{aligned}
&\rho\psi\left(\varepsilon_e,T,\gamma,\mu,\beta_i,\xi_i\right)=\rho\tilde{\psi}\left(\varepsilon_e,T,\gamma,\mu,\beta_i\right)+J_\pi\left(\beta_i\right)+J_\tau\left(\xi_i\right)= \\
&=\beta_1\left[-\alpha^T\varepsilon_e-\left(\Lambda_M+\Lambda_M^T\right)\right]+\beta_2\left[\alpha^C\varepsilon_e-\left(\Lambda_M+\Lambda_M^C\right)\right]+ \\
&+\beta_3\left[\frac{1}{2}\left(E_A-E_M\right)\varepsilon_e{}^2-\left(\Lambda_M+\Lambda_A\right)-\left(\Omega_A-\Omega_M\right)\left(T-T_0\right)\varepsilon_e+\frac{1}{2}\left(K_A-K_M\right)\gamma^2+\left(\frac{1}{2H_A}-\frac{1}{2H_M}\right)\mu^2\right] \\
&+\frac{1}{2}E_M\varepsilon_e{}^2+\Lambda_M-\Omega_M\left(T-T_0\right)\varepsilon_e+\frac{1}{2}K_M\gamma^2+\frac{1}{2H_M}\mu^2+J_\pi+J_\tau
\end{aligned} \tag{5}$$

where $J_\pi=J_\pi\left(\beta_i\right)$ is the indicator function of the convex set π, which establishes the constraints associated with the phases' coexistence defined as follows. From now on, the subscript index i refers to i = 1,2,3 (M+, M– and A, respectively).

$$\pi=\left\{\beta_i\in\Re\,\middle|\,0\le\beta_i\le1;\ \beta_1+\beta_2+\beta_3\le1\right\} \tag{6}$$

while $J_\tau=J_\tau\left(\xi_i\right)$ is the indicator function associated with the constraints of the saturation effect during cyclic loadings, represented by the variable ξ_i, that will be later defined. At this point, it is assumed an additive decomposition such that the elastic strain may be written as:

$$\varepsilon_e=\varepsilon-\varepsilon_p-\varepsilon_{tp}+\alpha_h^C\beta_2-\alpha_h^T\beta_1 \tag{7}$$

Note that ε_p represents the plastic strain, ε_{tp} is associated to the TRIP deformation and ε is the total strain. Moreover, α_h parameters defines the horizontal width of the stress-strain hysteresis loop. As a result, the total free energy in its final form is expressed by:

$$\begin{aligned}
&\rho\psi\left(\varepsilon,\varepsilon_p,\varepsilon_{tp},T,\gamma,\mu,\beta_n,\xi_n\right)=\beta_1\left[-\alpha^T\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}+\alpha_h{}^C\beta_2-\alpha_h{}^T\beta_1\right)-\left(\Lambda_M+\Lambda_M^T\right)\right]+ \\
&+\beta_2\left[\alpha^C\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}+\alpha_h{}^C\beta_2-\alpha_h{}^T\beta_1\right)-\left(\Lambda_M+\Lambda_M^C\right)\right]+ \\
&+\beta_3\left[\frac{1}{2}\left(E_A-E_M\right)\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}+\alpha_h{}^C\beta_2-\alpha_h{}^T\beta_1\right)^2-\left(\Lambda_A+\Lambda_M\right)-\right. \\
&\left.-\left(\Omega_A-\Omega_M\right)\left(T-T_0\right)\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}+\alpha_h{}^C\beta_2-\alpha_h{}^T\beta_1\right)+\frac{1}{2}\left(K_A-K_M\right)\gamma^2+\left(\frac{1}{2H_A}-\frac{1}{2H_M}\right)\mu^2\right]+ \\
&+\frac{1}{2}E_M\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}+\alpha_h{}^C\beta_2-\alpha_h{}^T\beta_1\right)^2+\Lambda_M-\Omega_M\left(T-T_0\right)\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}+\alpha_h{}^C\beta_2-\alpha_h{}^T\beta_1\right)+ \\
&+\frac{1}{2}K_M\gamma^2+\frac{1}{2H_M}\mu^2+J_\pi+J_\tau
\end{aligned} \tag{8}$$

State equations can be obtained from the Helmholtz free energy as follows:

$$\sigma=\rho\frac{\partial\psi}{\partial\varepsilon}=E\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}+\alpha_h{}^C\beta_2-\alpha_h{}^T\beta_1\right)+\alpha^C\beta_2-\alpha^T\beta_1-\Omega\left(T-T_0\right) \tag{9}$$

$$\begin{aligned}
B_1\in-\rho\frac{\partial\psi}{\partial\beta_1}&=\alpha^T\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}\right)+\Lambda_1+\beta_2\left(\alpha_h{}^C\alpha^T+\alpha_h{}^T\alpha^C+E\alpha_h{}^T\alpha_h{}^C\right)- \\
&-\beta_1\left(2\alpha_h{}^T\alpha^T+E\alpha_h{}^{T\,2}\right)+\alpha_h{}^T\left[E\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}\right)-\Omega\left(T-T_0\right)\right]-\partial_{\beta_1}J_\pi
\end{aligned} \tag{10}$$

$$\begin{aligned}
B_2\in-\rho\frac{\partial\psi}{\partial\beta_2}&=-\alpha^C\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}\right)+\Lambda_2+\beta_1\left(\alpha_h{}^T\alpha^C+\alpha_h{}^C\alpha^T+E\alpha_h{}^C\alpha_h{}^T\right)- \\
&-\beta_2\left(2\alpha_h{}^C\alpha^C+E\alpha_h{}^{C\,2}\right)-\alpha_h{}^C\left[E\left(\varepsilon-\varepsilon_p-\varepsilon_{tp}\right)-\Omega\left(T-T_0\right)\right]-\partial_{\beta_2}J_\pi
\end{aligned} \tag{11}$$

$$B_3 \in -\rho\frac{\partial\psi}{\partial\beta_3} = -\frac{1}{2}(E_A - E_M)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C\beta_2 - \alpha_h^T\beta_1\right)^2 + \Lambda_3 +$$
$$+ (\Omega_A - \Omega_M)(T - T_0)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C\beta_2 - \alpha_h^T\beta_1\right) - \frac{1}{2}(K_A - K_M)\gamma^2 - \quad (12)$$
$$-\left(\frac{1}{2H_A} - \frac{1}{2H_M}\right)\mu^2 - \partial_{\beta_3} J_\pi$$

$$X = -\rho\frac{\partial\psi}{\partial\varepsilon_p} = E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C\beta_2 - \alpha_h^T\beta_1\right) + \alpha^C\beta_2 - \alpha^T\beta_1 - \Omega(T - T_0) = \sigma \quad (13)$$

$$Y = -\rho\frac{\partial\psi}{\partial\gamma} = -K\gamma \quad (14)$$

$$Z = -\rho\frac{\partial\psi}{\partial\mu} = -\frac{1}{H}\mu \quad (15)$$

$$R = -\rho\frac{\partial\psi}{\partial\varepsilon_{tp}} = E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C\beta_2 - \alpha_h^T\beta_1\right) + \alpha^C\beta_2 - \alpha^T\beta_1 - \Omega(T - T_0) = \sigma \quad (16)$$

$$S_i \in -\rho\frac{\partial\psi}{\partial\xi_i} = -\partial_{\xi_i} J_\tau \quad (17)$$

where the thermodynamic forces B_i are associated with β_i, X, Y and Z are related to the plasticity phenomenon; R and S_i correspond to the TRIP effect. σ represents the uniaxial stress and $\partial_{\beta_i} J_\pi$ are the sub-differentials with respect to β_i (Rockafellar, 1970). Furthermore, the parameters E, Ω, K and $1/H$ are defined from their correspondent values for austenitic and martensitic phases, as follows:

$$(\,) = (\,)_M - \beta_3[(\,)_M - (\,)_A] \quad (18)$$

while Λ_1, Λ_2 and Λ_3 are linearly dependent on temperature and are defined as follows:

$$\Lambda_1 = \Lambda_M + \Lambda_M^T = \begin{cases} -L_0^T + \dfrac{L^T}{T_M}(T - T_M) & \text{if} \quad T > T_M \\ -L_0^T & \text{if} \quad T \le T_M \end{cases} \quad (19)$$

$$\Lambda_2 = \Lambda_M + \Lambda_M^C = \begin{cases} -L_0^C + \dfrac{L^C}{T_M}(T - T_M) & \text{if} \quad T > T_M \\ -L_0^C & \text{if} \quad T \le T_M \end{cases} \quad (20)$$

$$\Lambda_3 = \Lambda_M + \Lambda_A = \begin{cases} -L_0^A + \dfrac{L^A}{T_M}(T - T_M) & \text{if} \quad T > T_M \\ -L_0^A & \text{if} \quad T \le T_M \end{cases} \quad (21)$$

where T_M is the temperature below with martensite is stable.

In order to describe the dissipation processes, it is necessary to introduce a pseudo-potential of dissipation Φ. In general, it is possible to split Φ into an intrinsic part (ϕ) and a thermal part (ϕ_T). Here, the interest is focused on the mechanical part of the potential and, for convenience, is expressed in terms of its dual (ϕ^*).

$$\phi^*(B_i, X, Y, Z, R, S_i) = \frac{1}{2\eta_1}(B_1 + \eta_{ci}Y + \eta_{ck}Z)^2 + \frac{1}{2\eta_2}(B_2 + \eta_{ci}Y + \eta_{ck}Z)^2 +$$
$$+ \frac{1}{2\eta_3}(B_3 - \eta_{ci}Y - \eta_{ck}Z)^2 + R^2\{(M_{31}\beta_1 + M_{13}\beta_3)\dot\beta_1 + (M_{32}\beta_2 + M_{23}\beta_3)\dot\beta_2 + \quad (22)$$
$$+ [M_{43}\beta_3 + M_{34}(1 - \beta_1 - \beta_2 - \beta_3)]\dot\beta_3\} + |\dot\beta_1|S_1 + |\dot\beta_2|S_2 + |\dot\beta_3|S_3 + J_\chi(B_n) + J_f(X, Y, Z)$$

The parameters η_i are associated with the internal dissipation of the material, while η_{ci} and η_{ck} are respectively, isotropic and kinematic parameters related to plastic-phase transformation coupling. M_{13}, M_{31}, M_{23}, M_{32}, M_{34} and M_{43}, are parameters associated to the TRIP effect.

In order to take into account differences on the kinetics of phase transformation for loading and unloading processes, it is possible to consider different values to the parameter η_i during loading and unloading. Moreover, J_χ is the indicator function related to the convex set χ, which provides constraints associated with phase transformations evolution, such as internal subloops due to incomplete phase transformations description and $M+ \Rightarrow M$ and $M- \Rightarrow M$ phase transformations avoidance (Savi & Paiva, 2005). The other indicator function (J_f) is related to the yield surface defined as: $f = \left|X + HZ\right| - (\sigma_Y - Y)$.

After these definitions, it is possible to write the complementary equations to describe the internal variables evolution.

$$\dot{\beta}_i \in \frac{\partial \phi}{\partial B_i} = \frac{B_i}{\eta_i} - \frac{\eta_{ci}}{\eta_i} Y - \frac{\eta_{ck}}{\eta_i} Z + \partial_{\dot{\beta}_i} J_\chi \tag{23}$$

$$\dot{\varepsilon}_p \in \frac{\partial \phi}{\partial X} = \lambda \operatorname{sign}\left(X + HZ\right) \tag{24}$$

$$\dot{\gamma} \in \frac{\partial \phi}{\partial Y} = \lambda + \eta_{ci}\left(\frac{B_1}{\eta_1} + \frac{B_2}{\eta_2} - \frac{B_3}{\eta_3}\right) \tag{25}$$

$$\dot{\mu} \in \frac{\partial \phi}{\partial Z} = \lambda H \operatorname{sign}\left(X + HZ\right) + \eta_{ck}\left(\frac{B_1}{\eta_1} + \frac{B_2}{\eta_2} - \frac{B_3}{\eta_3}\right) \tag{26}$$

$$\dot{\varepsilon}_{tp} = \frac{\partial \phi}{\partial R} = 2R\left[\left(M_{31}\beta_1 + M_{13}\beta_3\right)\dot{\beta}_1 + \left(M_{32}\beta_2 + M_{23}\beta_3\right)\dot{\beta}_2 + \left[M_{43}\beta_3 + M_{34}\left(1 - \beta_1 - \beta_2 - \beta_3\right)\right]\dot{\beta}_3\right] \tag{27}$$

$$\dot{\xi}_i = \frac{\partial \phi}{\partial S_i} = \left|\dot{\beta}_i\right| \tag{28}$$

where λ is the plastic multiplier and $\partial_{\dot{\beta}_i} J_\chi$ are the sub-differentials with respect to variables $\dot{\beta}_i$. The irreversible nature of plastic flow is represented by means of the *Kuhn-Tucker conditions* (Simo & Hughes, 1998). The yield limit σ_Y has different values for austenitic and martensitic phases, and for very high temperatures, this value tends to decrease (Paiva *et al.*, 2005).

In order to make the model able to describe the effect of TRIP it is necessary to represent the saturation effect. This behavior is taken into account by considering functions governed by exponential laws related to ξ_i, the internal variables associated with TRIP phenomenon. Under this assumption, it is possible to control the vertical size of hysteresis in stress-strain curves from parameters α^C and α^T defined as follows:

$$\alpha^T = \frac{1}{2}\overline{\alpha}^T\left[1 + \exp(-m_\alpha \xi_1)\right] \tag{29}$$

$$\alpha^C = \frac{1}{2}\overline{\alpha}^C\left[1 + \exp(-m_\alpha \xi_2)\right] \tag{30}$$

The horizontal size is controlled by α_h^C and α_h^T, defined as follows:

$$\alpha_h^T = \frac{1}{2}\overline{\alpha}_h^T\left[1 + \exp(-m_{\alpha_h} \xi_1)\right] \tag{31}$$

$$\alpha_h^C = \frac{1}{2}\overline{\alpha}_h^C\left[1 + \exp(-m_{\alpha_h} \xi_2)\right] \tag{32}$$

Moreover, it is important to define parameters of functions Λ_1, Λ_2 and Λ_3 in the same way. Therefore,

$$L_0^T = \frac{1}{2}\overline{L}_0^T[1+\exp(-m_L\xi_1)] \qquad L^T = \frac{1}{2}\overline{L}^T[1+\exp(-m_L\xi_1)] \tag{33}$$

$$L_0^C = \frac{1}{2}\overline{L}_0^C[1+\exp(-m_L\xi_2)] \qquad L^C = \frac{1}{2}\overline{L}^C[1+\exp(-m_L\xi_2)] \tag{34}$$

$$L_0^A = \frac{1}{2}\overline{L}_0^A[1+\exp(-m_L\xi_3)] \qquad L^A = \frac{1}{2}\overline{L}^A[1+\exp(-m_L\xi_3)] \tag{35}$$

The magnitude of the coupling phenomenon are given by:

$$\eta_{ci} = \frac{1}{2}\overline{\eta}_{ci}\left[1+\exp\left(-m_p\left(\xi_1+\xi_2\right)\right)\right] \qquad \eta_{ck} = \frac{1}{2}\overline{\eta}_{ck}\left[1+\exp\left(-m_p\left(\xi_1+\xi_2\right)\right)\right] \tag{36}$$

Moreover, the magnitude of TRIP deformation is described by:

$$M_{13} = \frac{1}{2}\overline{M}_{13}\left[1+\exp\left(-m_1\,\xi_1\right)\right] \qquad M_{31} = \frac{1}{2}\overline{M}_{31}\left[1+\exp\left(-m_1\,\xi_1\right)\right] \tag{37}$$

$$M_{23} = \frac{1}{2}\overline{M}_{23}\left[1+\exp\left(-m_2\,\xi_2\right)\right] \qquad M_{32} = \frac{1}{2}\overline{M}_{32}\left[1+\exp\left(-m_2\,\xi_2\right)\right] \tag{38}$$

$$M_{34} = \frac{1}{2}\overline{M}_{34}\left[1+\exp\left(-m_3\,\xi_3\right)\right] \qquad M_{43} = \frac{1}{2}\overline{M}_{43}\left[1+\exp\left(-m_3\,\xi_3\right)\right] \tag{39}$$

In order to control the amount of TRIP deformation at different temperatures, ε_{tp} should be temperature dependent as well. Thus, the parameters $\overline{M}_{13}$, $\overline{M}_{31}$, $\overline{M}_{23}$ and $\overline{M}_{32}$ are assumed to depend linearly on temperature. For instance, for $\overline{M}_{13}$, the following expression is adopted.

$$\overline{M}_{13} = \begin{cases} 0 & \text{if} \quad T < T_{TRIP} \\ \overline{M}_{13}^{\text{Ref}}\,\dfrac{\left(T-T_{TRIP}\right)}{T_F - T_{TRIP}} & \text{if} \quad T \geq T_{TRIP} \end{cases} \tag{40}$$

where $\overline{M}_{13}^{\text{Ref}}$ is a reference value of $\overline{M}_{13}$ at $T = T_F$ and T_{TRIP} is the temperature below which no transformation plasticity should exist. Analogous expressions are used for $\overline{M}_{31}$, $\overline{M}_{23}$ and $\overline{M}_{32}$. For the sake of simplicity, this article considers $M_{34} = M_{43} = 0$. These equations form a complete set of constitutive equations. Since the pseudo-potential of dissipation is convex, positive and vanishes at the origin, the Clausius-Duhen inequality is automatically satisfied if the entropy is defined as $s = -\partial\psi/\partial T$. Box 1 summarizes the set of constitutive equations for the proposed model.

Box 1. Constitutive equations.

$$\sigma = E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) + \alpha^C \beta_2 - \alpha^T \beta_1 - \Omega(T - T_0)$$

$$\dot{\beta}_1 = \frac{1}{\eta_1}\left\{\alpha^T\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) + \Lambda_1 + \beta_2\left(\alpha_h^C \alpha^T + \alpha_h^T \alpha^C + E\alpha_h^T \alpha_h^C\right) - \beta_1\left(2\alpha_h^T \alpha^T + E\,\alpha_h^{T\,2}\right) + \right.$$

$$\left. + \alpha_h^T\left[E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) - \Omega(T - T_0)\right] - \eta_{ci} K\gamma - \eta_{ck}\frac{\mu}{H} - \partial_{\beta_1} J_\pi\right\} + \partial_{\dot{\beta}_1} J_\chi$$

$$\dot{\beta}_2 = \frac{1}{\eta_2}\left\{-\alpha^C\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) + \Lambda_2 + \beta_1\left(\alpha_h^T \alpha^C + \alpha_h^C \alpha^T + E\alpha_h^C \alpha_h^T\right) - \beta_2\left(2\alpha_h^C \alpha^C + E\,\alpha_h^{C\,2}\right) - \right.$$

$$\left. - \alpha_h^C\left[E\left(\varepsilon - \varepsilon_p - \varepsilon_{tp}\right) - \Omega(T - T_0)\right] - \eta_{ci} K\gamma - \eta_{ck}\frac{\mu}{H} - \partial_{\dot{\beta}_2} J_\pi\right\} + \partial_{\dot{\beta}_2} J_\chi$$

$$\dot{\beta}_3 = \frac{1}{\eta_3}\left\{-\frac{1}{2}(E_A - E_M)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right)^2 + \Lambda_3 + (\Omega_A - \Omega_M)(T - T_0)\left(\varepsilon - \varepsilon_p - \varepsilon_{tp} + \alpha_h^C \beta_2 - \alpha_h^T \beta_1\right) - \right.$$

$$\left. - \frac{1}{2}(K_A - K_M)\gamma^2 - \left(\frac{1}{2H_A} - \frac{1}{2H_M}\right)\mu^2 + \eta_{ci} K\gamma + \eta_{ck}\frac{\mu}{H} - \partial_{\dot{\beta}_3} J_\pi\right\} + \partial_{\dot{\beta}_3} J_\chi$$

$$\dot{\varepsilon}_p = \lambda\ \text{sign}\,(\sigma - \mu) \qquad \dot{\gamma} = \left|\dot{\varepsilon}_p\right| + \eta_{ci}\left(\dot{\beta}_1 + \dot{\beta}_2 - \dot{\beta}_3\right) \qquad \dot{\mu} = H\,\dot{\varepsilon}_p + \eta_{ck}\left(\dot{\beta}_1 + \dot{\beta}_2 - \dot{\beta}_3\right)$$

$$\dot{\varepsilon}_{tp} = 2\sigma\left[(M_{31}\beta_1 + M_{13}\beta_3)\dot{\beta}_1 + (M_{32}\beta_2 + M_{23}\beta_3)\dot{\beta}_2 + \left[M_{43}\beta_3 + M_{34}(1 - \beta_1 - \beta_2 - \beta_3)\right]\dot{\beta}_3\right]$$

$$\dot{\xi}_1 = \left|\dot{\beta}_1\right| \qquad \dot{\xi}_2 = \left|\dot{\beta}_2\right| \qquad \dot{\xi}_3 = \left|\dot{\beta}_3\right|$$

3. NUMERICAL RESULTS

This item presents a fit between results obtained from the numerical model and experimental data obtained in pseudoelastic tensile tests for three temperatures (Tobushi *et al.*, 1991). This adjustment demonstrates the ability of the model to describe an alloy with shape memory at different temperatures. The model considers the properties observed at T=373K. At this temperature, it is observed a maximum residual deformation due to the TRIP effect. These properties are listed in Table 1. Material are assumed to be symmetric and therefore, tension and compression parameters are the same. Moreover, as the applied stress levels for the presented results are insufficient to promote plastic strain the plastic parameters are omitted.

Table 1. Model parameters obtained through comparison between numerical and experimental results provided by Tobushi *et al.* (1991) for a Ni-Ti SMA alloy.

E_A (GPa)	E_M (GPa)	Ω_A (MPa/K)	Ω_M (MPa/K)	$\overline{\alpha}_h^T$	$\overline{\alpha}^T$
54	42	0.74	0.17	0.0425	364
$\overline{L}_0^T$ (MPa)	$\overline{L}^T$ (MPa)	L_0^A(MPa)	L_1^A (MPa)	T_M (K)	T_0 (K)
0.15	41.5	0.63	170	291.4	307
T_A (K)	T_F (K)	T_{TRIP} (K)	m_1	η_1^L (MPa.tu)	η_1^U (MPa.tu)
307.5	423	330	0.1	1	1.7
n_2^L(MPa.tu)	n_2^U(MPa.tu)	η_3^L (MPa.tu)	η_3^U (MPa.tu)	$\overline{M}_{13}$ (GPa^{-1})	$\overline{M}_{31}$ (GPa^{-1})
1	1.7	1	1.7	9	9

Figure 1 presents both experimental and numerical results for pseudoelastic tests for three different temperatures: T=373K, T=353K and T=333K. It should be observed the good agreement between experimental data and numerical results that takes into account the transformation induced plasticity. The results confirms the model capacity to describe the effect of TRIP. It is important to highlight that there is a decrease of TRIP deformation with decreasing temperature. For the temperature T=333K the reverse transformation occurs completely.

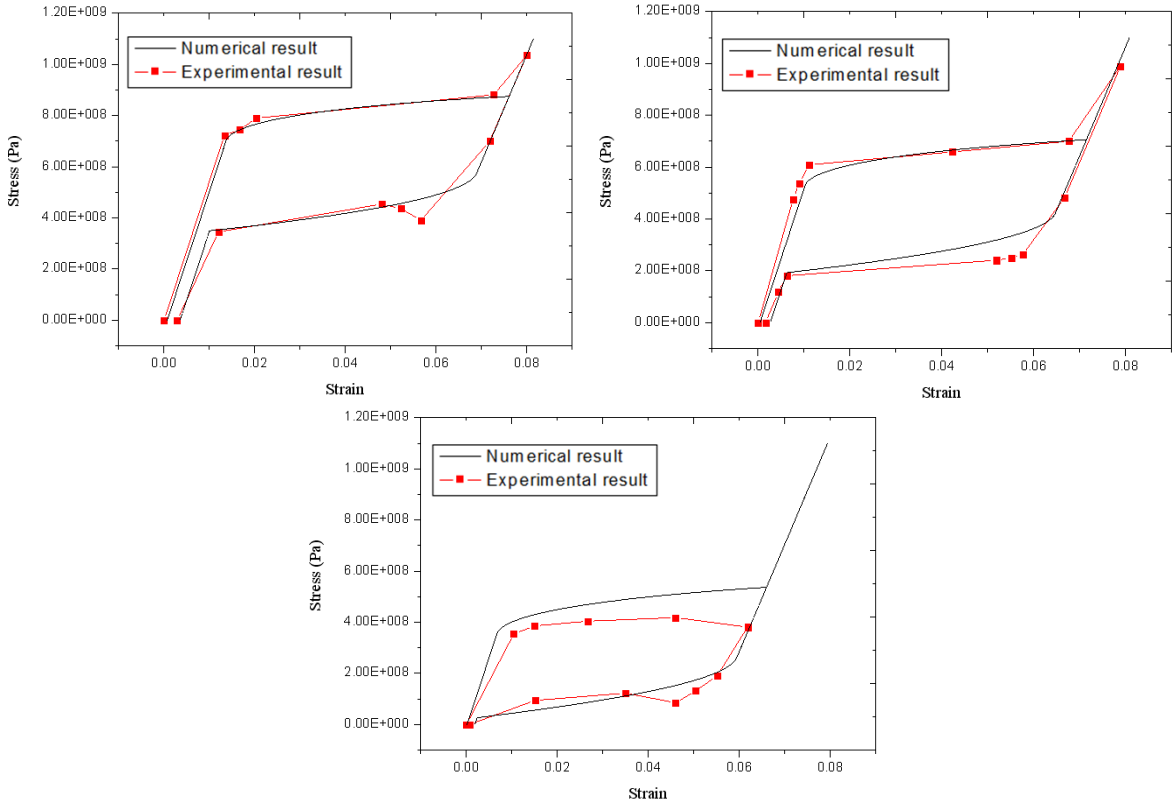

Figure 1. Numerical-experimental comparison for three different temperatures (Tobushi *et al.*, 1991): *T*=373 *K*, *T*= 353 *K* and *T=333 K*.

Cyclic loading is now of concern. Experimental results obtained for a Ni-Ti alloy subjected to cyclic loading at a temperature *T*=363K, obtained by Lagoudas *et al.* (2003), are used as a reference. Table 2 provides the values of the parameters identified from the experimental results.

Table 2. Parameters obtained through comparison between numerical and experimental results provided by Lagoudas *et al.* (2003).

E_A (GPa)	E_M (GPa)	Ω_A (MPa/K)	Ω_M (MPa/K)	$\overline{\alpha}_h^T$	$\overline{\alpha}^T$ (MPa)
72	25	0.74	0.17	0.045	200
$\overline{L}_0^T$ (MPa)	$\overline{L}^T$ (MPa)	$\overline{L}_0^A$ (MPa)	$\overline{L}^A$ (MPa)	T_M (K)	T_0 (K)
0.1	41.5	0.63	170	291.4	323
T_A (K)	T_F (K)	T_{TRIP} (K)	m_1	m_α	$m_{\alpha_h}(10^{-3})$
307.5	423	330	0.01	0.5	0.02
m_L	η_1^L (MPa.tu)	η_1^U (MPa.tu)	n_2^L(MPa.tu)	n_2^L(MPa.tu)	n_2^U(MPa.tu)
0.12	0.1	0.05	0.1	0.1	0.05
η_3^L (MPa.tu)	η_3^U (MPa.tu)	$\overline{M}_{13}$ (GPa^{-1})	$\overline{M}_{31}$ (GPa^{-1})		
0.1	0.05	17	17		

Figure 2 shows the comparison between experimental data and numerical results obtained using parameters listed in Table 2. Once again, it should be highlighted the close agreement between both results showing the ability of the model to represent the phenomenon of saturation in SMAs. After a few cycles, there is the stabilization of SMA behavior and the TRIP effect is no longer observed. This result has particular importance since it enables understanding of the

saturation in SMAs, allowing the description of training material in order to make possible their use in various applications.

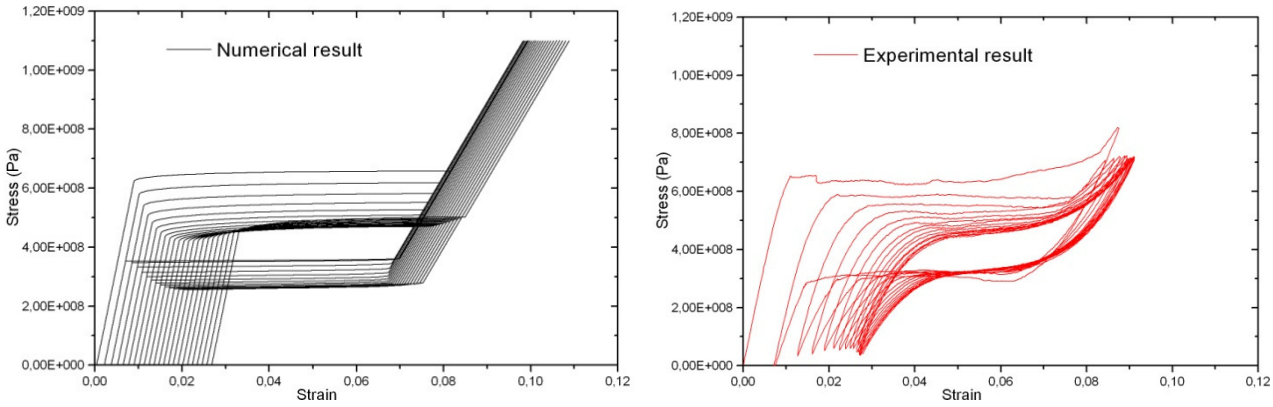

Figure 2. Numerical-experimental comparison for cyclic loading (Lagoudas *et al.*, 2003).

4. CONCLUSIONS

A macroscopic constitutive theory is developed to describe the TRIP phenomenon in SMAs. Numerical results provided by the model are in good agreement with experimental data available in literature. Besides, numerical simulations demonstrate the model's ability to capture some other features associated with TRIP such as saturation processes under cyclic tests. Experimental studies reveal that these two mechanisms are known to be present in SMAs and have great relevance during training processes. Therefore, the proposed model capture the general thermomechanical behavior related to TRIP.

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