VI CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
VI NATIONAL CONGRESS OF MECHANICAL ENGINEERING
18 a 21 de agosto de 2010 – Campina Grande – Paraíba - Brasil
August 18 – 21, 2010 – Campina Grande – Paraíba – Brazil

Testing a mechanical behavior of Light

Luiz Eduardo A. Sauerbronn, lsauer@ufrj.br[1]
Rodrigo José Correa, rcorrea@iq.ufrj.br[2]
Marcelo Dreux, dreux@puc-rio.br[3]
Maurício Elarrat, melarrat@poli.ufrj.br [1]

[1]Universidade Federal do Rio de Janeiro, Centro de Tecnologia, Bloco D, sala 101, Rio de Janeiro – RJ, 21941-909, Brasil
[2]Universidade Federal do Rio de Janeiro, Centro de Tecnologia, Bloco A, sala 632, Rio de Janeiro – RJ, 21941-909, Brasil
[3]Pontifícia Universidade Católica do Rio de Janeiro, Depto. de Eng. Mecânica, sala 163-L, Rio de Janeiro – RJ, 22453-900, Brasil

Abstract*. We model photons as being rigid bodies. Based only on Newtonian mechanics, we reproduce numerically the Fresnel Diffraction Experiment. In this way, a large number of rigid bodies are thrown against a single slit. The rigid bodies used are spherical and their center of mass and centroid are not coincident. Thus, each rigid body describes a cycloid (presenting amplitude, frequency and phase - as well as the DeBroglie wave). The numerical results indicate a wave pattern relatively similar to those achieved by experimental results.*

Keywords*: Rigid Bodies, Collisions, Diffraction, Fresnel, Light, Laser*

1. INTRODUCTION

We model a photon as being a non-homogeneous rigid body and thus verify if rigid bodies describing cycloids can perform a matter wave behavior (Fresnel, 1826).

In this way, we used a spherical rigid body within holes so that its centroid and its center of mass are not coincident. Thus, while the center of mass describes a straight line, the centroid describes a cycloid. The cycloid presents amplitude, frequency and phase as well as the De Broglie (1924) wave.

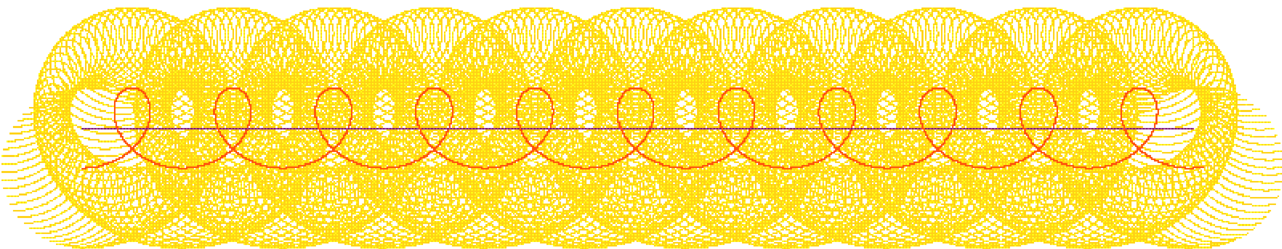

Figure 1. Our proposed photon, a spherical rigid body containing a centroid (red line) rotating towards the center of mass (black line), describing a cycloid.

In sections 2 and 3, we describe the model used to calculate the trajectory and collisions. In section 4, we describe the implementation of numerical solution. In section 5, we present experimental results. In section 6, we compare experimental and numerical results. In section 7, we present our conclusions.

2. MODELING THE TRAJECTORIES

In our approach, we modeled a bidimensional collision system, where the rigid body is a sphere A containing inside a spherical hole B, where B is not centered at the same location of A. Figure (2) shows an example of the rigid body. By applying an impulse on the sphere surface, we achieve a cycloid, as shown on Fig. (1).

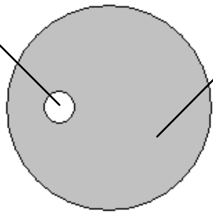

Figure 2. A hole into the sphere shifts its center of mass from its centroid.

In this way, the center of mass location is described by:

$$\begin{cases} P_x(t) = P_x(0) + v_x * t \\ P_y(t) = P_y(0) + v_y * t \end{cases}$$

and the centroid location is described by:

$$\begin{cases} Q_x(t) = P_x(t) + d * \cos(\omega t + \varphi) \\ Q_y(t) = P_y(t) + d * \sin(\omega t + \varphi) \end{cases}$$

Where:

P_x (t):	Scalar representing the coordinate x of the position of the center of mass as a function of time t
P_y (t):	Scalar representing the coordinate y of the position of the center of mass as a function of time t
P_x (0):	Scalar representing the coordinate x of the initial position of the center of mass
P_y (0):	Scalar representing the coordinate y of the initial position of the center of mass
v_x:	Scalar representing the linear velocity of the center of mass in the direction x
v_y:	Scalar representing the linear velocity of the center of mass in the direction y
Q_x (t):	Scalar representing the coordinate x of the position of the centroid as a function of time t
Q_y (t):	Scalar representing the coordinate y of the position of the centroid as a function of time t
d:	Scalar representing the distance between the centroid and the center of mass
ω:	Scalar representing the angular velocity of the center of mass
φ:	Scalar representing the initial angle of the direction given by the center of mass and the centroid
t:	Scalar representing the time spent since the launch of the photon

Figure (1) shows an example of the trajectory of the center of mass and the centroid of a rigid body describing a cycloid. For each increment, there is a collision test to locate precisely the collision point. In case of a collision, the system calculates the impulse received by the rigid body and recalculates the new trajectory.

3. MODELING THE COLLISIONS

During the bidimensional collisions, the same hypotheses were considered: the energy and the magnitude of momentum of the thrown rigid body should be constant along the whole path. This means that all collisions were treated as perfectly elastic.

The dynamics of the rigid body was calculated based on the following hypotheses:

1. The collisions are perfectly elastic.
2. The impulse transmitted by the slit to the sphere is orthogonal to the sphere's surface (thus, the direction of the impulse is given by the point of collision and the centroid).

Using a coordinate system x'y' located at the point of contact on the slit surface, where x' has the same direction of the impulse, we obtained the final constraints:

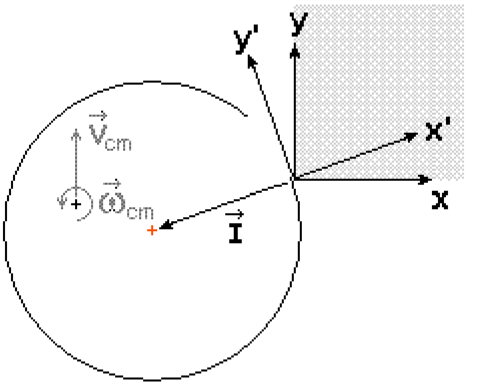

Figure 3. Calculating linear and angular velocities after collision against the internal wall of the slit. The new coordinate system x'y' helps to determine the magnitude of the impulse.

Conservation of Energy and Momentum:

$$I_{zz}\omega_1^2 + m.(v_{1x'}^2 + v_{1y'}^2) = I_{zz}\omega_0^2 + m.(v_{0x'}^2 + v_{0y'}^2) = 2K$$

$$v_{1x'} = v_{0x'} + \frac{\|\vec{I}\|}{m}$$

$$v_{1y'} = v_{0y'}$$

$$\omega_1 = \omega_0 + \frac{\|\vec{I}\times\vec{S}\|}{I_{zz}}$$

and, therefore:

$$I_{zz}\omega_1^2 + m.(v_{1x'}^2 + v_{1y'}^2) = I_{zz}\omega_0^2 + m.(v_{0x'}^2 + v_{0y'}^2)$$

$$I_{zz}\omega_1^2 + mv_{1x'}^2 = I_{zz}\omega_0^2 + mv_{0x'}^2$$

$$I_{zz}(\omega_0 + \frac{\|\vec{I}\times\vec{S}\|}{I_{zz}})^2 + m(v_{0x'} + \frac{\|\vec{I}\|}{m})^2 = I_{zz}\omega_0^2 + mv_{0x'}^2$$

$$2\omega_0\|\vec{I}\|\|\vec{S}\|\sin\theta + \frac{\|\vec{I}\|^2\|\vec{S}\|^2\sin^2\theta}{I_{zz}} + 2v_{0x'}\|\vec{I}\| + \frac{\|\vec{I}\|^2}{m} = 0$$

$$2\omega_0\|\vec{S}\|\sin\theta + \frac{\|\vec{I}\|\|\vec{S}\|^2\sin^2\theta}{I_{zz}} + 2v_{0x'} + \frac{\|\vec{I}\|}{m} = 0$$

$$\|\vec{I}\| = \frac{2\omega_0\|\vec{S}\|\sin\theta + 2v_{0x'}}{\frac{\|\vec{S}\|^2\sin^2\theta}{I_{zz}} + \frac{1}{m}}$$

Where:

$\vec{I}$: Vector representing the impulse transfered by the slit internal walls to the spherical rigid body.
$\vec{S}$: Vector starting at the point of collision and ending in the center of mass of the spherical rigid body.
$\|\vec{I}\|$: Scalar representing the magnitude of vector $\vec{I}$
$\|\vec{S}\|$: Scalar representing the magnitude of vector $\vec{S}$
m: Scalar representing the mass of the spherical rigid body.
I_{zz}: Scalar representing the moment of inertia of the spherical rigid body.
$V_{0x'}$: Scalar representing the linear velocity of the center of mass before collision in direction x'.
$V_{0y'}$: Scalar representing the linear velocity of the center of mass before collision in direction y'.
$V_{1x'}$: Scalar representing the linear velocity of the center of mass after collision in direction x'.
$V_{1y'}$: Scalar representing the linear velocity of the center of mass after collision in direction y'.
ω_0: Scalar representing the angular velocity of the center of mass before collision.
ω_1: Scalar representing the angular velocity of the center of mass after collision.
K: Total kinetic energy of the rigid body
θ: Trigonometric angle between vectors $\vec{I}$ and $\vec{S}$.

The direction of impulse is given by vector $\vec{I}$ (the direction of impulse is orthogonal to the surface of sphere). Once the magnitude of impulse was calculated, we obtained $v_{1x'}$, $v_{1y'}$ and ω_1 from the constraint's equations. Figure (4) presents the result of multiple collisions.

Thus, due to collisions that happen inside the slit, the trajectory of each sphere has to be recalculated after each collision. The impulse transmitted by the slit to the sphere provokes changes on its linear and angular velocities. The changes on its angular velocity seem to provoke the Compton (1923) effect and its interaction with the surface of an object seems to replicate the effect of color. Estimates for photon mass were obtained from Rodriguez and Spavieri (2007), Williams et al. (1971), Chernikov et al. (1992), Davis and Nieto (1975), Franken and Ampulski (1971), Accetta et al. (1985), Crandall (1983), Lakes (1998), Fishbach et al. (1994), Schaefer (1999) and Luo et al. (2003).

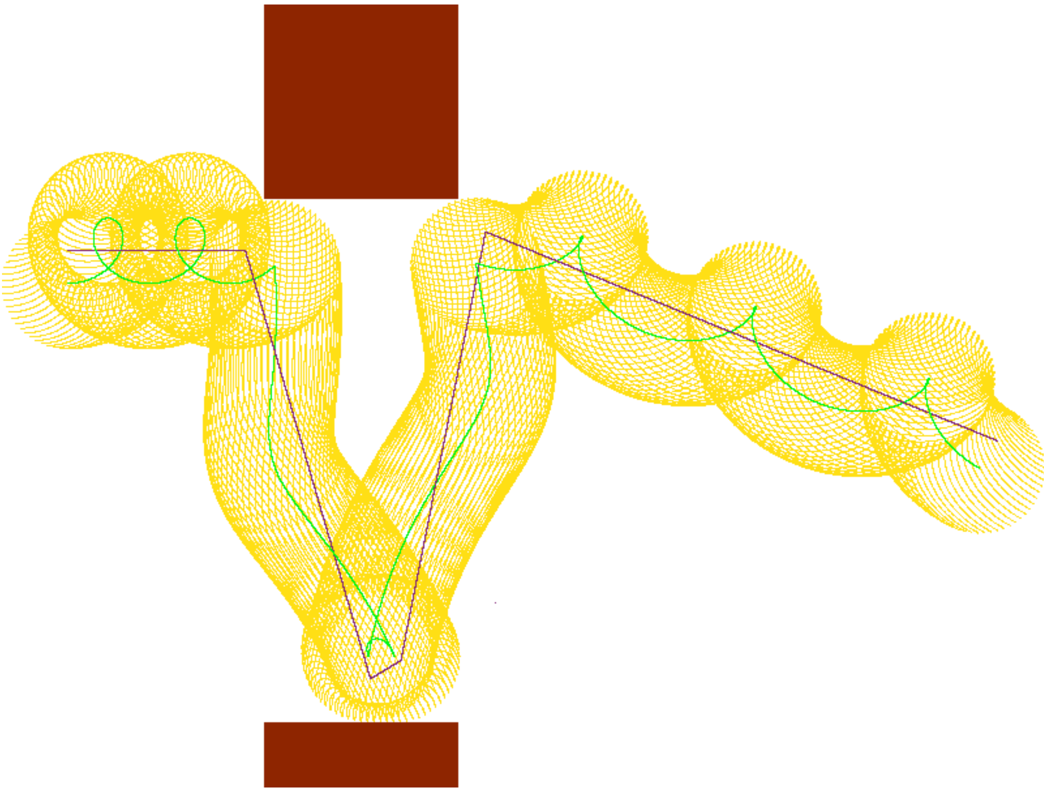

Figure 4. The sphere (yellow) collides against the walls (dark red) of the single slit.

In the next section, we present the main steps of the source code describing the collision's model.

4. C/C++ IMPLEMENTATION

The source code is divided in the following blocks:

Includes, defines, prototypes and globals;
Main loop:

a) For each rigid body launched
b) Update its location
c) Check for possible collisions (against the slit's geometry)
d) In case of collision, recalculate the trajectory
e) In case the rigid body will not cross the slit, then return to a)
f) In case the rigid body crossed the slit (and, therefore, will hit the bulkhead):
   f1) Calculates its collision point (precise position at the bulkhead).
   f2) Updates file containing the location of all final collisions
   f3) Create image file containing the whole trajectory, as seen in Fig. (4)
   f4) Return to a)
g) Return to b)

The source code can be found at:
http://www.deg.ee.ufrj.br/docentes/sauer/collisions.c

5. ACQUIRING DATA TO TEST NUMERICAL MODEL

We believe that light is ballistic and that its behavior of wave is caused by its non-homogeneous mass distribution, which generates cycloids and, consequently, a wave behavior. What we want to demonstrate is that rigid bodies that are spherical and non-homogeneous can provide the same results provided by light. In order to test this mechanical model, we selected a widely known experiment that showcases the wave properties of light, the Fresnel diffraction experiment.

In Fig. (5), we present the basic physical arrangement used in the Fresnel diffraction experiment, which includes a coherent source of light (Laser), a slit and a bulkhead. The emitted light crosses the single slit and collides against the bulkhead. After turning off the ambient lights, we took a picture of the light pattern that collides against the bulkhead.

Figure (6) shows the resulting wave pattern. To obtain the intensity of light distribution in the bulkhead, we took a picture, as shown in Fig. (6), and analyzed one line of the image file obtained that best describes the wave pattern. The graph shown in Fig. (6), shows the luminance of each pixel of the photograph taken.

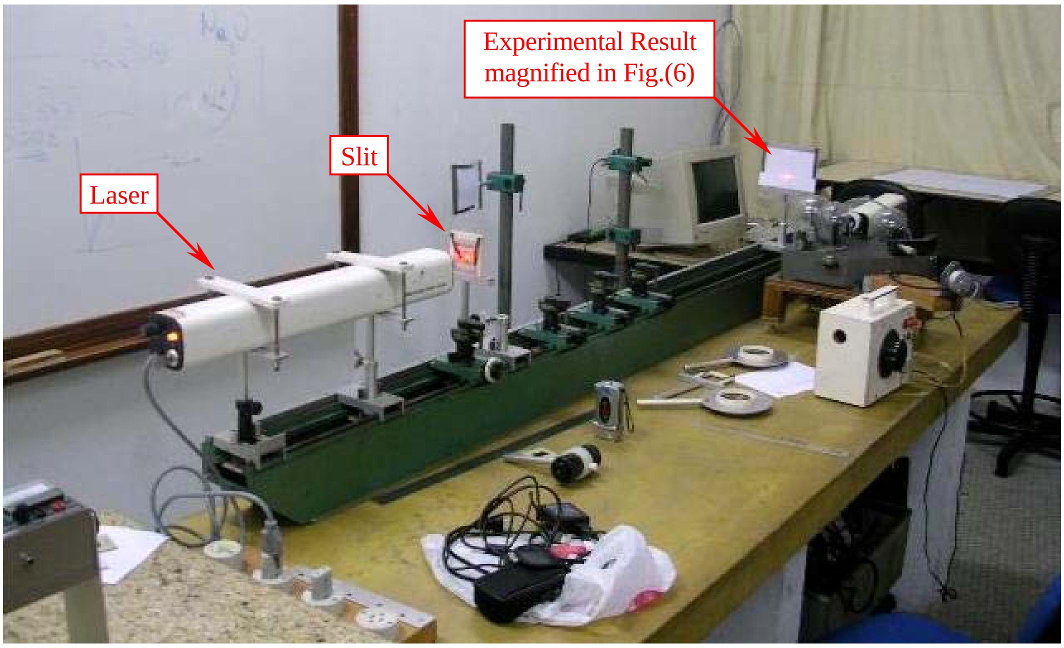

Figure 5. Setting up apparatus to compare numerical model against experimental results.

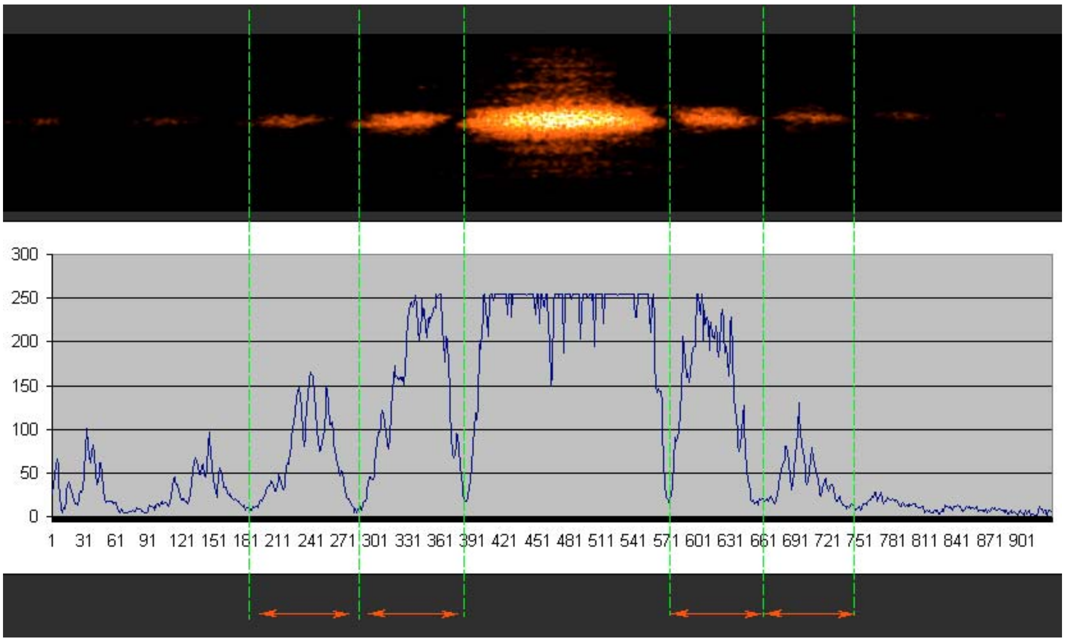

Figure 6. The diffraction of light produces a wave pattern.

6. RESULTS

After applying only rigid body dynamics, we achieved the results shown in Fig. (7). In Fig. (8), we compare the wave pattern obtained numerically against the wave pattern obtained experimentally.

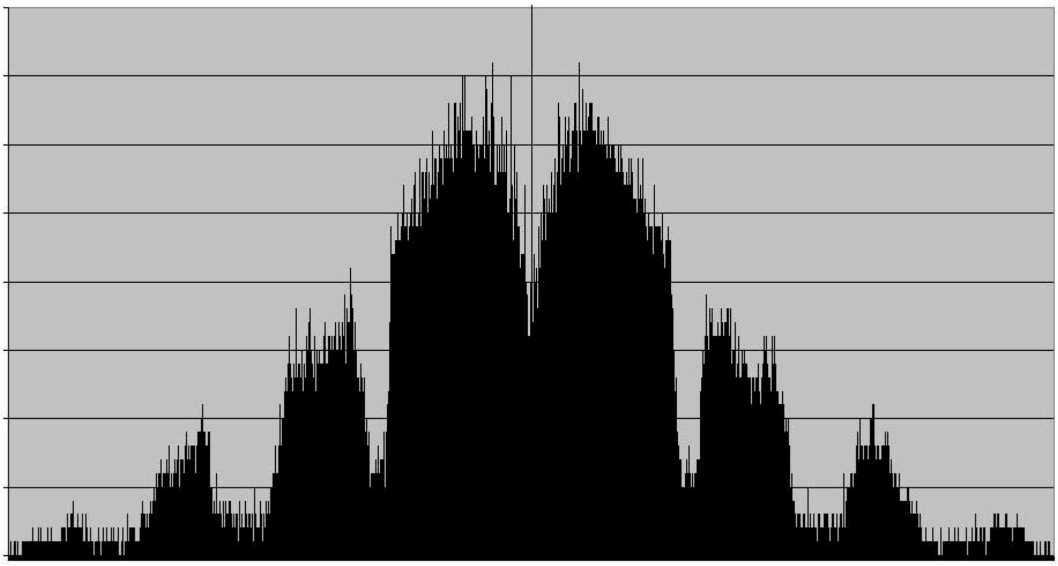

Figure 7. When we throw a large number of non-homogeneous spheres against a slit, we also obtain a wave pattern.

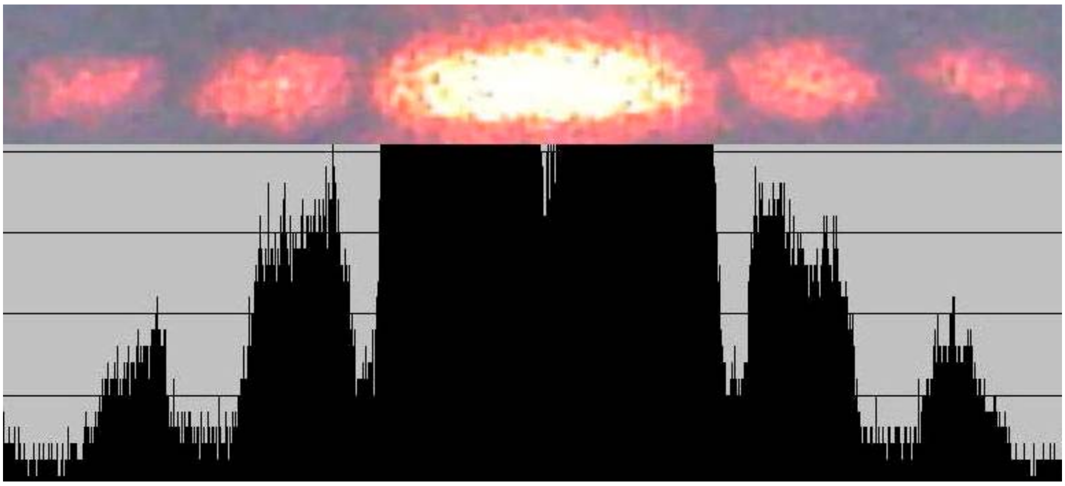

Figure 8. To visually compare both results (numerical and experimental) we present them in a single frame. Images are in different scales. While the experimental results presents a width of 20 cm, the numerical results presents a width of 20 m.

The results were considered astonishing because a high intensity luminance on the center was expected and no clear wave pattern. Instead, we found not only a wave pattern, but a pattern that decays in a similar fashion to the experimental data shown in Fig. (6).

We noticed that when we enlarge the slit's aperture, the wave enlarges at the same proportion, indicating a clear correlation with the Fresnel experiment. However, when the aperture becomes too small, typically similar to the size of the photon, the result becomes too noisy, indicating that some other parameter must be changed.

To achieve these results, we used the values shown in Tab.(1), implemented in the MS Visual C++ 6.0 using long double precision (80 bits), which provides 64 bits of mantissa and 16 bits of exponent. The code used is available for download as shown in section 4.

Table 1. Data used to produce the wave pattern shown in Fig. (7) and Fig. (8)

Data	
Photon mass [(1)]	1.2×10^{-54} kg
Photon radius [(2)]	1.0×10^{-12} m
Distance between centroid and center of mass[(2)]	1.0×10^{-13} m
Initial angular velocity of the photon [(3)]	428×10^{15} Hz
Initial linear velocity of the photon on x direction [(4)]	3×10^{8} m/s
Initial linear velocity of the photon on y direction [(2)]	0 m/s
Slit aperture [(2)]	4×10^{-10} m
Distance between slit and bulkhead [(2)]	2.0 m
Timestep [(2)]	1.0×10^{-27} s
Distance between consecutive launches [(2)]	1.0×10^{-16} m
Phasesteps between consecutive launches [(5)]	$\pi/16$

[(1)] : obtained from Luo et al. (2003).
[(2)] : estimated.
[(3)] : frequency of monochromatic red light emitted by HeNe Laser, equivalent to 700nm.
[(4)] : the speed of light.
[(5)] : we repeated the same experiment for many angles ϕ as defined in section 2.

After varying many parameters, trying to reduce the aperture from 20m to 20cm, we noticed that the radius of the photon should be smaller than 10^{-12}m, probably varying between 10^{-15}m and 10^{-19}m. As our compiler's precision just support radius until 10^{-13}m, we will need to implement a system that allows higher precision (and this is the point where we are now).

The estimated value of radius between 10^{-15}m and 10^{-19}m arose from two different methods:

In the first method, we used different values of radius varying from 10^{-10}m to 10^{-13}m and we noticed a convergence in the result, indicating that for a radius among 10^{-17}m the convergence should be perfect and the numerical result should fit pefectly the experimental result.

In the second method, we noticed that all the substances in the periodic table provide densities varying between 10^{-2}kg/m^3 (Hydrogen, 0.08988 kg/m^3) and 10^4kg/m^3 (Lead, 11.340 kg/m^3). As the photon rest mass indicated in Tab.(2) (obtained from Luo et al., 2003) varies between 10^{-47} kg (Schaefer, 1999) and 10^{-54} kg (Luo, 2003) and considering the density of a photon as being among all known substances, the volume of a photon would be given by:

Considering:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Then:

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

Applying upper and lower limits of density:

$$\frac{\min_{\text{mass}}}{\max_{\text{density}}} < \text{volume}_{\text{photon}} < \frac{\max_{\text{mass}}}{\min_{\text{density}}}$$

$$\frac{10^{-54}[\text{kg}](\text{Luo})}{10^{4}[\text{kg}/\text{m}^3](\text{Lead})} < \text{volume}_{\text{photon}} < \frac{10^{-47}[\text{kg}](\text{Schaefer})}{10^{-2}[\text{kg}/\text{m}^3](\text{Hydrogen})}$$

$$10^{-58}[\text{m}^3] < \text{volume}_{\text{photon}} < 10^{-45}[\text{m}^3]$$

Considering the photon as being nearly spherical:

$$\text{volume}_{\text{photon}} = \frac{4}{3}\pi R^3_{\text{photon}}$$

Then:

$$10^{-19}[\text{m}] < \text{Radius}_{\text{photon}} < 10^{-15}[\text{m}]$$

So, as the precision of the implemented system just supports a radius larger than 10^{-13}m, we are now working to expand our precision so that it becomes possible to precisely determine which value of photon radius matches the experimental results.

As an analogy to understand the need of such a high precision, we could say that the required precision is equivalent of tracking a tennis ball in its way to saturn with milimetric precision. The variable that stores its position must describe not only small shifts, but also the great distances that separate the ball from saturn. The storage of this information in a single variable requires large precision. This analogy considers the tennis ball as being the photon and the distance to saturn as being the distance between the light source and the bulkhead.

Table (2) provides different methods and estimates of the photon rest mass. These values were used to estimate the radius of the photon and are shown below.

Table 2. Several important photon mass experiments (1)

Author	Date	Method	Upper limit of m_γ (2)
Williams et al.	1971	Test of Coulomb's Law	2 x 10^{-50} kg
Crandall	1983	Test of Coulomb's Law	8 x 10^{-51} kg
Chernikov et al.	1992	Test of Ampere's Law	8,4 x 10^{-49} kg
Schaefer	1999	Measurement of the speed of light	4,2 x 10^{-47} kg
Fishback et al.	1994	Analysis of Earth's magnetic field	1 x 10^{-51} kg
Davis et al.	1975	Analysis of Jupiter's magnetic field	8 x 10^{-52} kg
Lakes	1998	Static torsion balance	2 x 10^{-53} kg
Luo et al	2003	Dynamic torsion balance	1.2 x 10^{-54} kg

(1) : source: Luo et al (2003)

(2) : m_γ is the mass of the photon.

7. CONCLUSION

As can be seen in Fig. (7), the launch of a large amount of non-homogeneous spheres against a single slit indicates a wave pattern relatively similar to those achieved by experimental results, as seen in Fig. (6).

Thus, it seems reasonable to apply rigid body dynamics to model photon in numerical problems related to scattering and diffraction. Further, it seems reasonable to propose that photons describe cycloids, since cycloids simultaneously present matter and wave behaviors, as well as the De Broglie wave.

For future work, we intend to use higher precision tests to propose a value to the radius of a photon that perfectly matches the experimental results. For now, we estimate the radius of a photon between 10^{-15}meters and 10^{-19}meters.

As a final remark, this work was inspired by the model of light of the atomist *Lucretius (1992, 1995).* Our contribution is in the proposal of a non-uniform distribution to the photon's internal mass and in the implementation of a numerical model to test this proposal against experimental results achieved by the Fresnel experiments.

8. ACKNOWLEDGEMENTS

The authors thank CNPq for financial support and Cristina Marlasca, André Assis, Simon Garden, Pierre Mothé Esteves, Luiz Guilherme Sauerbronn and Henrique Bertulani for helpful comments.

9. REFERENCES

Accetta, F., Ryan J. J., and Austin, R. H. , 1985, "Cryogenic photon-mass experiment", Phys. Rev. D 32, 802.

Compton, A. H., 1923, "A Quantum Theory of the Scattering of X-Rays by Light Elements", Physical Review, 21 (5), 483-502.

Chernikov, M. A., Gerber, C. J., Ott, H. R. and Gerber, H.J., 1992, "Low-temperature upper limit of the photon mass: Experimental null test of Ampère's law", Phys. Rev. Lett. 68, 3383.

Crandall, R. E., 1983, "Photon mass experiment", Amer. J. of Physics, 51, 698.

Davis, L. and Nieto, M. M. , 1975, "Limit on the Photon Mass Deduced from Pioneer-10 Observations of Jupiter's Magnetic Field", Phys. Rev. Lett. 35, 1402.

de Broglie, L., 1924, "Recherches sur la theorie des quanta" (Researches on the quantum theory), Ph.D. thesis, Paris Univ.

Fischbach, E., Kloor, H., Langel, R. A., Lui, A. T. Y. and Peredo, M., 1994, " New geomagnetic limits on the photon mass and on long-range forces coexisting with electromagnetism", Phys. Rev. Lett. 73, 514.

Franken, P. A. and Ampulski, G. W., 1971, “Photon Rest Mass”, Phys. Rev. Lett. 26, 115.
Fresnel, A., “Mémoire sur la diffraction de la lumiére”, Annales de la Chemie et de Physique,1816,1, 239-281.
Lakes, R., 1998, “Experimental Limits on the Photon Mass and Cosmic Magnetic Vector Potential”, Phys. Rev. Lett. 80, 1826.
Lucretius, 1995, “On the Nature of Things: De rerum natura”, translated by Anthony M. Esolen Baltimore: The Johns Hopkins University Press, ISBN 0-8018-5055-X.
Lucretius, 1992, “On the Nature of Things: De rerum natura”, translated by W. H. Rouse, rev. by M.F. Smith., Cambridge, Massachusetts.: Harvard University Press, reprint with revisions of the 1924 edition, ISBN 0-674-99200-8.
Luo, J., Tu, L. C., Hu, Z. K. and Luan, E. J., 2003, “New Experimental Limit on the Photon Rest Mass with a Rotating Torsion Balance”, Phys. Rev. Lett. 90, 081801.
Rodriguez, M. and Spavieri, G., 2007, “Photon mass and quantum effects of the Aharonov-Bohm type”, Phys. Rev. A, 75, 052113.
Schaefer, B. E., 1999, “Severe Limits on Variations of the Speed of Light with Frequency”, Phys. Rev. Lett. 82, 4964.
Williams, E. R., Faller, J. E. and Hill, H. A., 1971, “New Experimental Test of Coulomb’s Law: A Laboratory Upper Limit on the Photon Rest Mass”, Phys. Rev. Lett., 26, 721.

10. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.