

## **SIMULATION AND ANALYSIS OF A COLUMN OF STEMS PCP PUMP USING ANALOGY WITH MULTIPLE PENDULUMS**

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**Abstract.** *This paper presents a sets of simulations and analysis to development a monitoring system of a column of stems for Pump Cavities Progressive (PCP) in oil wells onshore using analogy with the movements of multiple pendulums, that in this case consider a one hundred pendulums (stems). The Interface was developed in Matlab for simulate the movements of the column of stems inside the pipe production developed from the engine rotation. The results of time domain and frequency domain are presents from the direct kinematics. Two different situations was considered for the column of stems: constant presence of the gravitational acting constant and gravitational acting variable. The analysis of obtained results only shows the possibility of monitoring a pump system by its vibration transmitted through the tubing. The main objective in the present research is to know the dynamics of the column of stems and to determine the frequencies that are developed in the bending vibrations against the tubing. The displacement of column stems causes beating in the inner production pipe wall and difficult the monitoring. This beating could damage the structure leading to production stops and an increase on well interventions. The analysis and investigation of the dynamic behavior of the column stem can help in the development of monitoring vibration systems for the reduction on both the maintenance costs and petroleum well intervention.*

**Keywords:** *PCP pump, simulator, stems, analogy.*

### **1. INTRODUCTION**

In the acoustic and vibration analysis of operation oil wells, mainly in a column of stems PCP pump for onshore oil wells, should consider the different sources of noises, sometimes, indicate the responsible factors for breaks of those stems. The dynamics of the stems PCP pump when oscillate can be compared with multiple pendulums, that when oscillate causes beating in the inner production pipe wall. In this paper we present, initially, results of dynamic analysis of one hundred pendulums due the constant gravitational acting and variable gravitational acting over the column of stems. We will present the results with detailed discussion for changes in the pendulums (stems) frequency with initial angle and we will suggest that those disturbances (frequency variation) interferes in the oscillation of stems PCP.

#### **1.1. Progressive Cavities Pump (PCP)**

Recently, Rodrigues (2009) developed a simulation and dynamic analysis in a column of stems PCP pump using analogy with multiple pendulums, where was observed that the column of stems caused shocks (beaten) in the inner wall of the production piping, then the simulation showed the column stems movements in the production column. Those beats can cause damages in the structure provoking the stop of oil well production, interventions in the well to change of stems and noises that cause interference in the data acquisition of vibration for monitoring. The Progressive Cavities Pump has a column of stems that is submitted a rotation generated by an electric motor at the surface. The column stems works in the production tubing.

Each stems has 7.62 meters of length connected at the end by screw-glove. The rotation movements combined with the great length of column, which depends on the depth of the well, causing bending movements of column of stems inside the tubing (Thomas, 2001). For the monitoring of vibrations in this system is necessary to know the natural frequencies. Beyond the natural frequencies of vibration torsional, bending, longitudinal and flow of oil, this work comes to identify noises that disturbs the pump system. In the Rodrigues (2009), the use of analogy with pendulums

becomes easy to make experimental tests in place of system mass-spring. Figure 1, present the configuration of oil well and equipments.

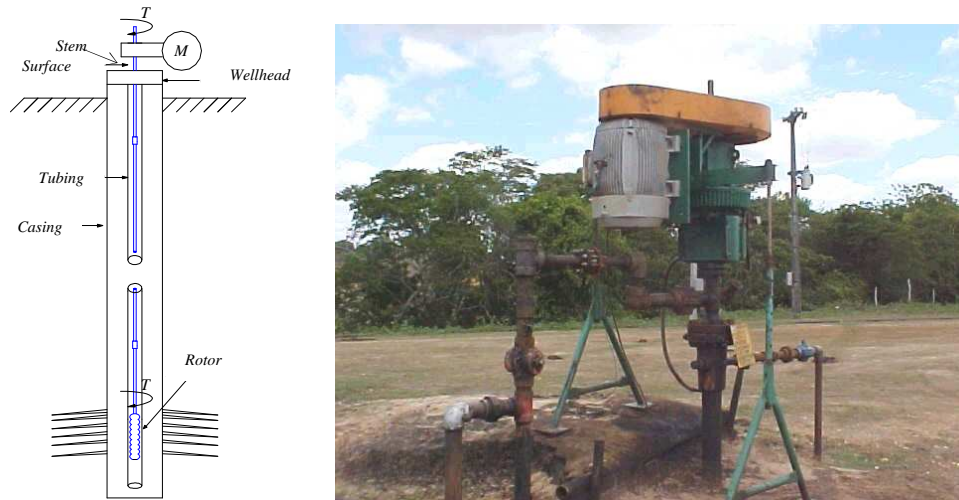


Figure 1. Configuration of oil well equipped with typical PCP installation (electric motor and headstock).

## 2. OSCILLATION OF THE SIMPLE PENDULUM

Suppose a pendulum of length  $z$ , fixed for a wire with a mass  $m$ . Let us yet that wire makes a vertical angle  $\theta$  and that  $g$  is the acceleration of gravity (Craig, 1986). The differential equation representing the motion of simple pendulum is given by:

$$\ddot{\theta} + (g/z) \sin \theta = 0 \quad (1)$$

It is known that there will be an envelope curve in generation of pendulum oscillations. The envelope curve occurs due to sinusoidal term in the equation of motion. In this work, first we made the analysis of simple pendulum oscillations considering the initial angle. Next, analyze the equation that describes the variation frequency of oscillation versus initial angle. Using the Fourier Transform we analyzed the oscillations for  $\theta_0=30^\circ$ . After analyze of oscillation curve, have been defined an envelope curve and using a low pass filter, show the FFT for harmonic curve. Then compare the results obtained from the harmonic analysis of simple pendulum curve with curve of oscillation of a simple harmonic motion, and the relationship with the envelope curve. Finally, analysis of results will be obtained for oil well operation equipped with PCP pump.

The Figure 2 presents a simplified diagram of simple pendulum, with weight  $m_0.g$  and length  $z$ , displaced a angle  $\theta$ . We used  $P_0 = m_0.g$ , knowing that for an arm length  $z$ , the angular displacement is given by  $s = \theta z$ , the speed by  $v = \dot{\theta} z$ , and the acceleration by  $a = \ddot{\theta} z$  (Meirovitch, 1990).

The torque of simple pendulum is shown in Eq. (2). Using  $T_0 = P_0.(z/2)$  and  $a = \ddot{\theta} z$ , obtains the differential equation that describes the angular displacement of pendulum, Eq. (3).

$$T_0 = P_0(z/2) \sin \theta_0 \quad (2)$$

and

$$m_0.a.z = m_0.\ddot{\theta}.z^2 = m_0.(z/2).g.\sin(\theta_0) \Rightarrow \ddot{\theta} = (g/2z) \sin(\theta_0) \quad (3)$$

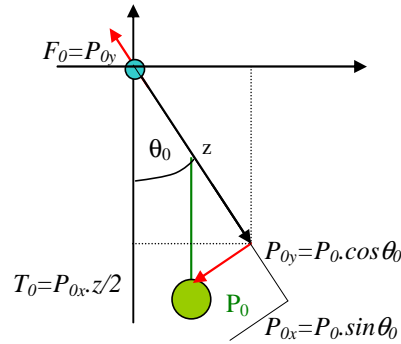


Figure 2. Diagram of simple pendulum.

The general solution of differential equation requires the use of elliptic functions (Meirovitch, 1990). The equation of period versus initial angle ( $\theta_0$ ) is shown in Eq. (4).

$$\tau = 4 \sqrt{\frac{z}{g} \int_0^{\pi/2} (1 - k^2 \sin^2 \beta)^{-1/2} d\beta} \quad (4)$$

where  $k = \sin(\theta_0 / 2)$  and  $\beta = \sin^{-1}(\sin(\theta_0 / 2) / k)$ . The oscillations of pendulum, due sinusoidal term, present a beat that can be called by envelope. The Figure 3 shows the frequency of pendulum as a function of initial angle obtained from the equation (4) which considers the term sinusoidal, with  $z = 0,25m$  and  $g = 10 m/s^2$ . From the results was collected a frequency of  $f = 1,066 Hz$  to  $\theta_0$  near to zero.

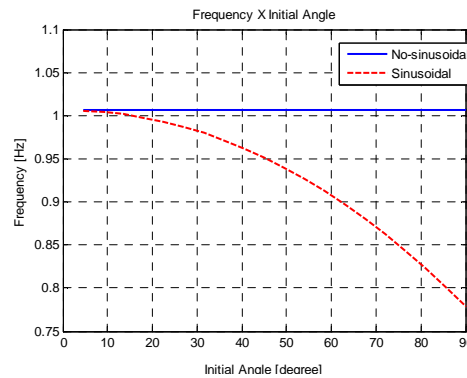


Figure 3. Frequency versus initial angle  $\theta_0$ .

In the Figure 3, show can see that frequency of oscillation of simple pendulum (stem) decreases when initial angle increases. The bending movements of stems cause a disturbance in the system due the frequency change.

The blue curve represents the pendulum frequency from the approach given by  $\sin(\theta) \cong \theta$  that is defined as

$$f = \left(\frac{1}{2\pi}\right) \sqrt{\frac{g}{z}} \quad [\text{Hz}] \quad (5)$$

where  $f$  is the pendulum frequency (stem),  $g$  is the gravity and  $z$  is the pendulum length. The red dotted curve represents the sinusoidal term that is defined as

$$f = \left(\frac{1}{2\pi}\right) 1/\tau \quad [\text{Hz}] \quad (6)$$

where  $\tau$  is the period defined by Eq. (4).

The Figure 4 presents the results of stem with length of 7.62 m. There is a natural frequency from the motion of a simple pendulum of 0,1823 Hz and sinusoidal variation of the term depending on angle of initial displacement.

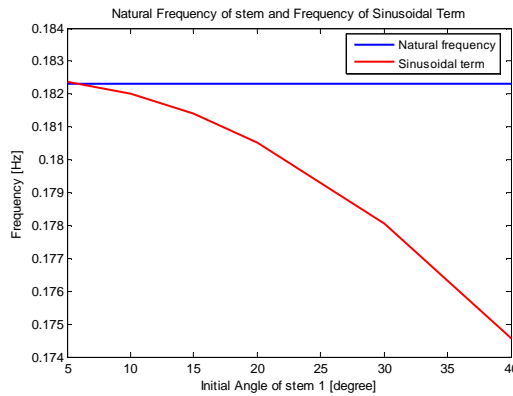


Figure 4. Natural frequency, Sinusoidal Term of stem versus Initial Angle.

Was observed that the sinusoidal term can be neglected because does not cause influence on the natural frequencies when the column of stems is considered like multiple pendulum. Next, the simulation for two pendulums likes the column of two stems.

### 3. SIMULATION FOR TWO PENDULUMS (TWO STEMS)

From Spong (1989), was studied the dynamics of two pendulums (stems) coupled as shown in Fig. 5. In Equation (7) shows the torque due to pendulum 2, and Eq. (8) present a differential equation of pendulum 2 oscillation. In Equation (9) shows the torque due to pendulum 1, and Eq. (10) present a differential equation of pendulum 1 oscillation.

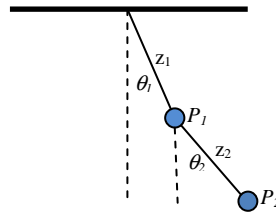


Figure 5. Configuration of Two pendulums

$$T_2 = (P_2 / 2) \sin(\theta_1 + \theta_2) \quad (7)$$

$$\ddot{\theta}_2 = \frac{P_2 / 2}{m_2 z} \sin(\theta_1 + \theta_2) \quad (8)$$

$$T_1 = \left( \frac{P_1}{2} + P_2 \right) \sin(\theta_1) + T_2 \quad (9)$$

$$\ddot{\theta}_1 = \left\{ \left( \frac{P_1}{2} + P_2 \right) \sin(\theta_1) + T_2 \right\} / m_1 z_1 \quad (10)$$

Comparing with differential equation of a single pendulum given below,

$$\ddot{\theta}_0 = \left\{ \left( \frac{P_0}{2} \right) \sin(\theta_0) \right\} / m_0 z \quad (11)$$

with equation (10), where differ in torque term  $T_2$ .

Now, can see in the Fig. 6 the stem simulator that was used for data collect (Rodrigues, 2008). Was used a length for two stems of 0,25 m.

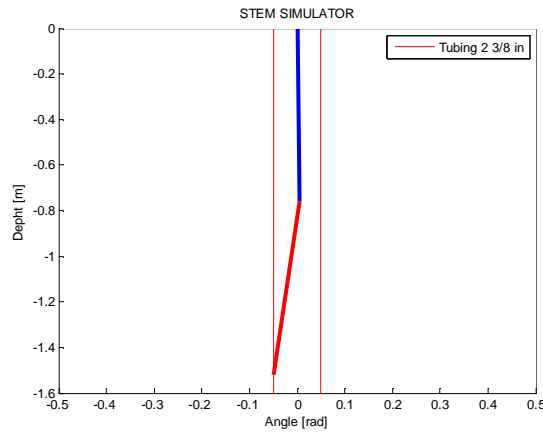


Figure 6. Stem Simulator for two stems.

We simulate the movements of the double pendulum which were considering two situations:

a) In the first situation, the pendulum 2 is fixed to pendulum 1. This way, the double pendulum works like simple pendulum, and,

b) In the second situation, the pendulum 2 is free.

The simulations were made in positioning the stem 1 in  $\theta_1=5^\circ$  and stem 2 in  $\theta_2=0$ . The Figure 7 shows the curve of time domain with stem 1 free and stem 2 fixed. Shows at the bottom of Fig. 9 the response in frequency domain for stem 1 ( $f_1 = 17 \text{ Hz}$ ).

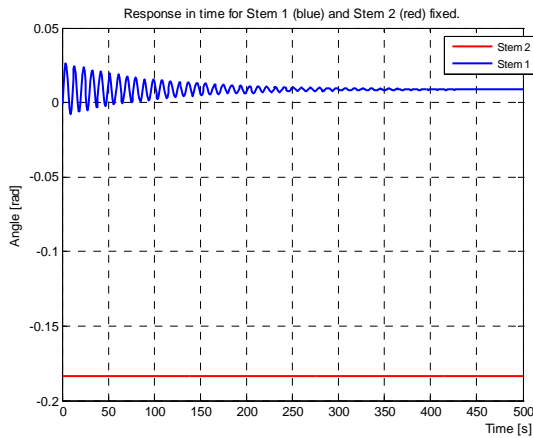


Figure 7. Response in time domain for stem 1 with stem 2 fixed.

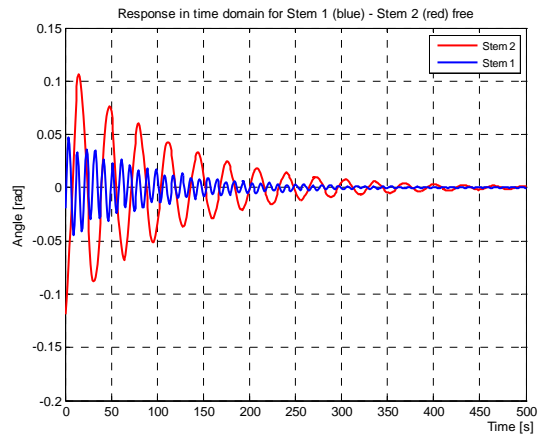


Figure 8. Response in time domain for stem 1 and stem 2 free.

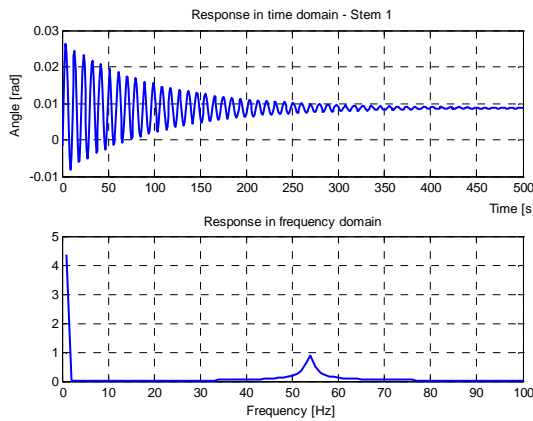


Figure 9. Response in time and frequency domain for stem 1 with stem 2 fixed.

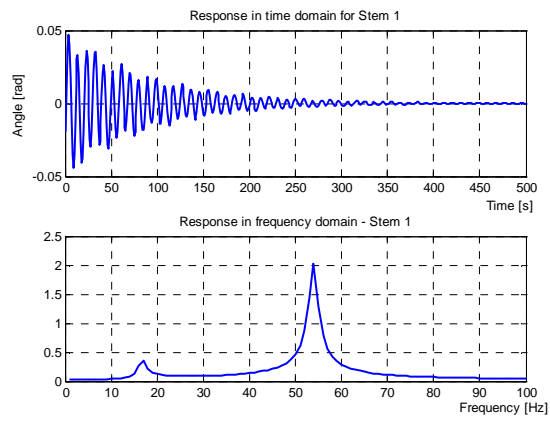


Figure 10. Response in time and frequency domain for stem 1 with stem 2 free.

The Figure 8 shows the response in time domain for stem 1 and stem 2 free when  $\theta_1=5^\circ$  and  $\theta_2=0$ . At the bottom of Figure 10 shows the response in frequency domain for stem 2 where was observed the two frequencies of stem 1 ( $f_1 = 54Hz$ ) and stem 2 ( $f_2 = 17Hz$ ).

#### 4. SIMULATION FOR ONE HUNDRED PENDULUMS (ONE HUNDRED STEMS)

The simulation is presented by interface developed in Matlab that can analyze the column with two, three, twenty and one hundred stems. The Figure 11 shows the interface for the simulator.

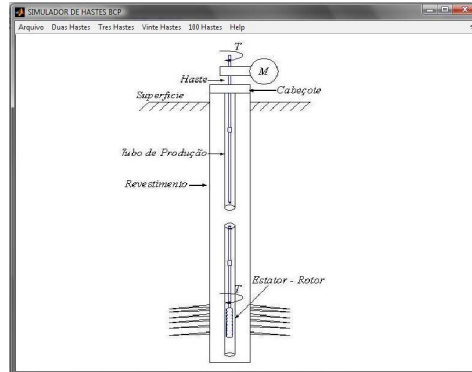


Figure 11. Interface for the simulator.

##### 4.1 Acting gravitation constant

The acting gravitation is constant like if actions of the drag caused by fluid within the tubing are constant. The results in the time domain and frequency domain are presented in the Fig. 12.

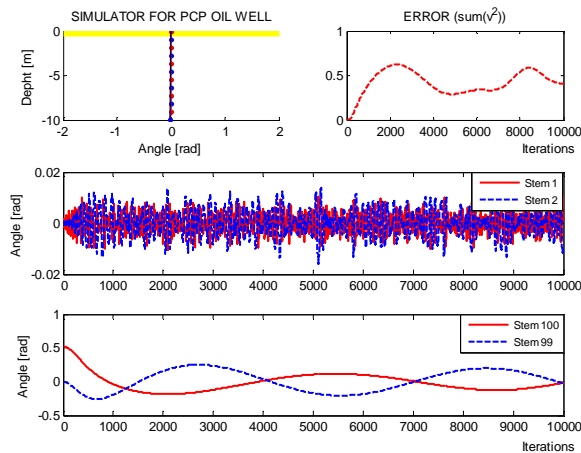


Figure 12. Simulation for column of stems and results in the time and frequency domain, and error.

The Figure 12 shows the displacements of the column of stems, the error, the displacement in the time domains for stem 1 and 2, and stem 99 and stem 100. Can see that the first stems have higher amplitudes than the last stems. This is important when thinking about developing a monitoring system.

##### 4.2 Acting gravitation variable

The variation of gravity means that the fluids exert opposing forces on the column of stems. The Figure 13 shows the variation for gravitation where the gravitation decreasing varies from stem 1 to stem 100, and shows the simulations of column and the displacement in the time domain for stem 1 and stem 2, stem 99 and 100 when acting gravitations is variable.

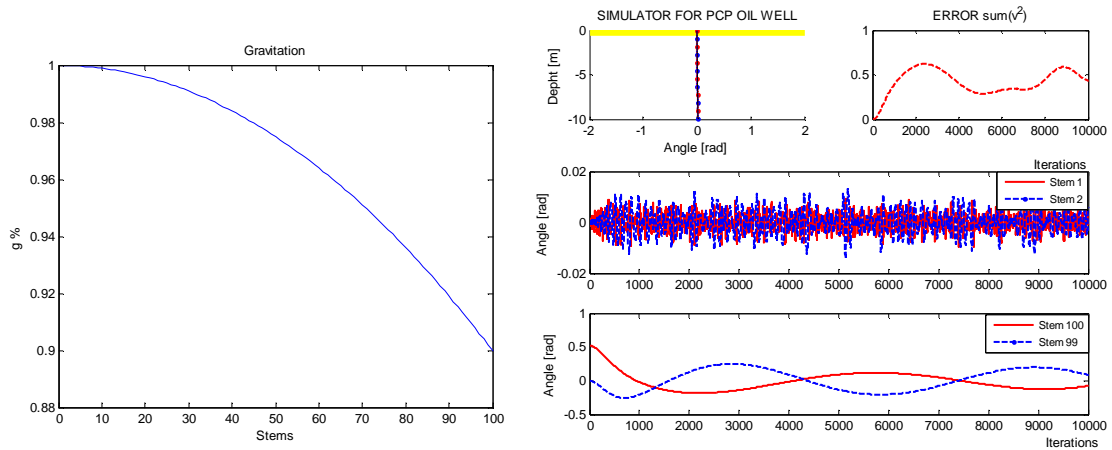


Figure 13. Gravitation variable 10% for each stem and simulation for column of stems and results in the time and frequency domain, and error.

The displacements in the two stems (1 and 2) have greater oscillations than the last stems (99 and 100) in the downhole. Can see that the first stems have more probability to break than others. This is consistent with what occurs in practice, the polished stem (first stem) has damage. The Figure 14 to 16 shows the results to stems 1, 2, 50, 99 and 100, when acting gravitation is constant and variable.

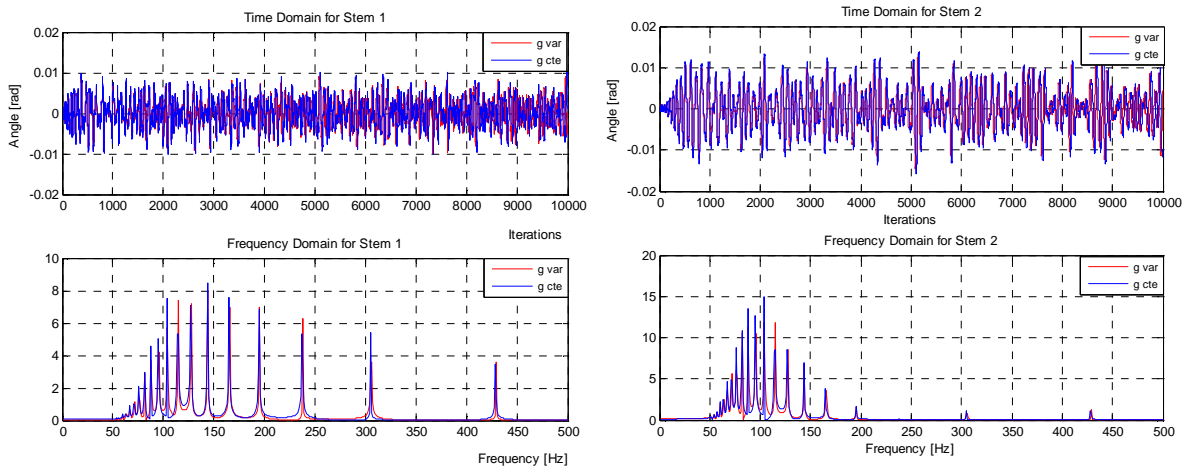


Figure 14. Results for time domain and frequency domain for stem 1 and stem 2 with acting gravitations is constant and variable.

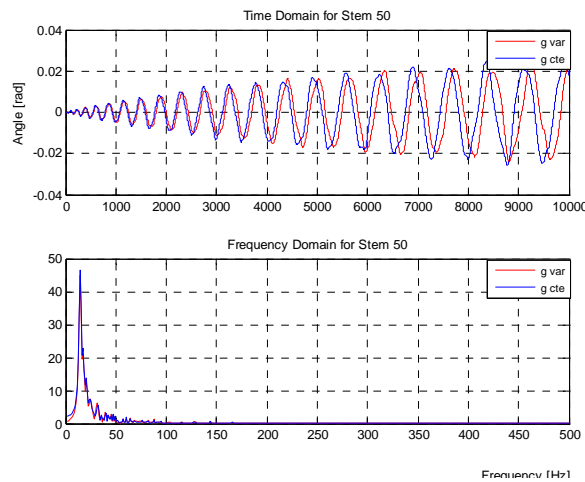


Figure 15. Results for time domain and frequency domain for stem 50 with acting gravitations is constant and variable.

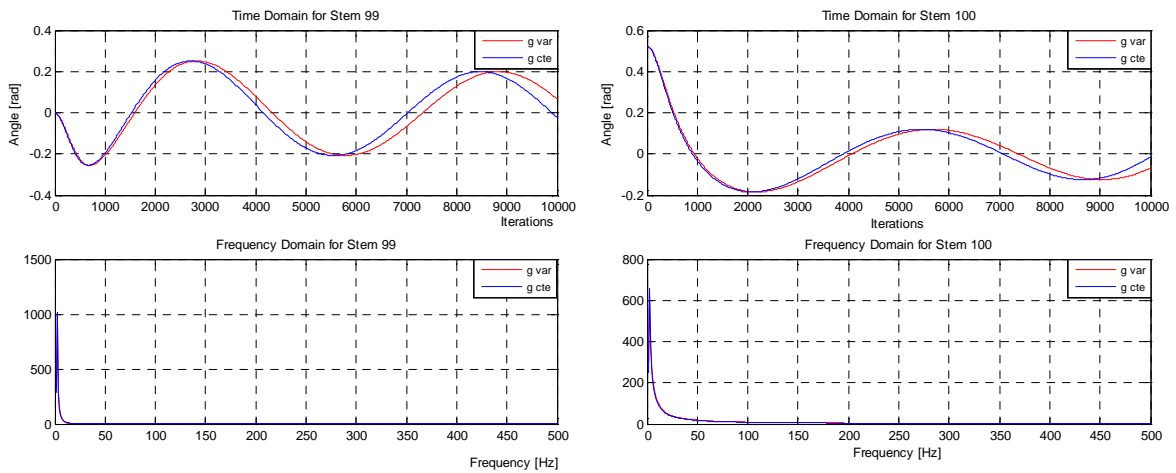


Figure 16. Results for time domain and frequency domain for stem 99 and stem 100.

In this case was showed that the gravitation variable around 10% not change the characteristics the column, then let us consider the gravitation variable around 50%. In the Fig. 17 presents the gravitation variable 50% and shows the simulator for PCP oil well. Figure 18 shows the time and frequency domain for stem 1 and 2 with the acting variable and constant gravitation.

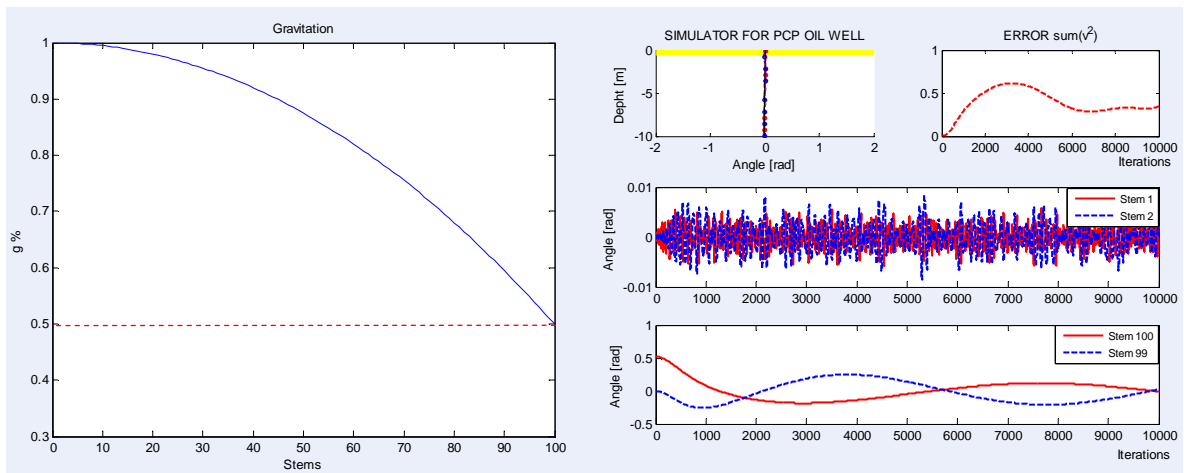


Figure 17. Gravitation variable 50% for each stem and simulation for column of stems and results in the time and frequency domain, and error.

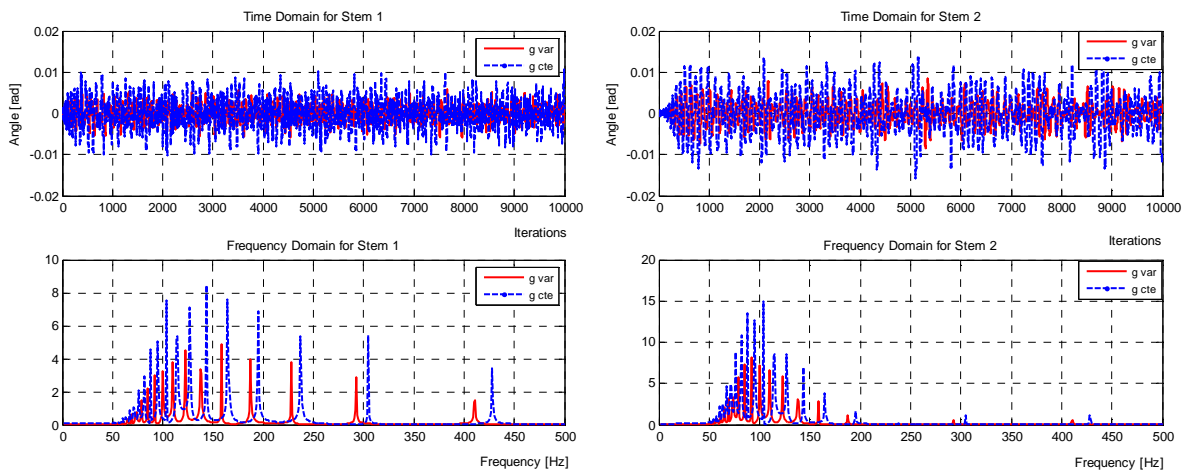


Figure 18. Results for time domain and frequency domain for stem 1 and stem 2 with acting gravitations is constant and variable 50%.



The Figure 19 shows the time and frequency domain for stem 50 (fifty) with acting variable and constant gravitation and Fig. 20 shows the results for stem 99 and 100.

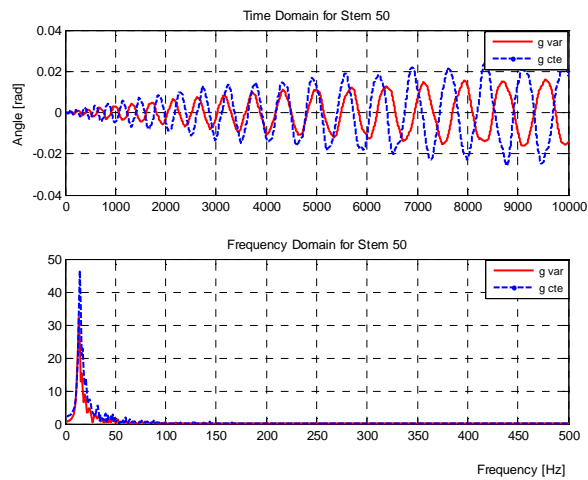


Figure 19. Results for time domain and frequency domain for stem 50 with acting gravitations is constant and variable 50%.

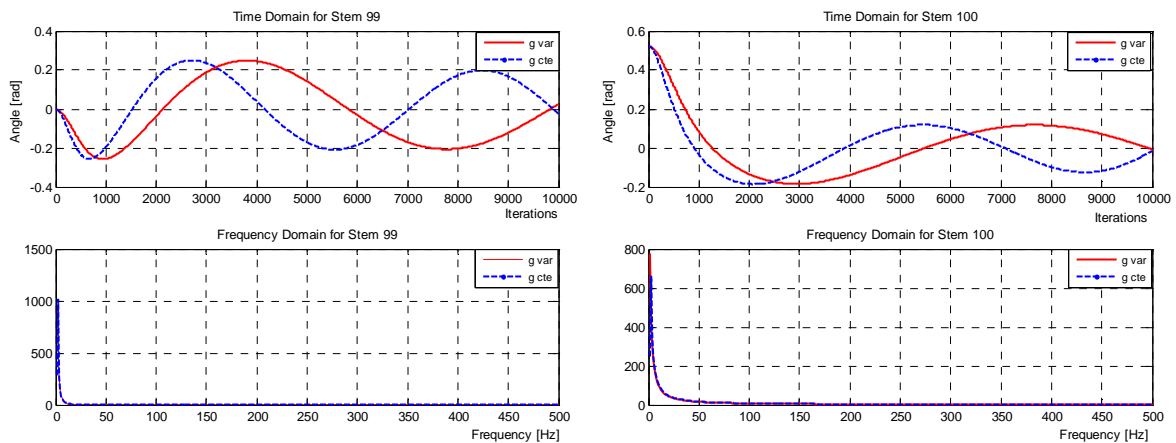


Figure 20. Results for time domain and frequency domain for stem 99 and stem 100 with acting gravitations is constant and variable 50%.

The variation of gravity cause change in the frequency domain that is more showed at the first stems, 1 and 2. To identify the damage in the any stems. The fluid when flows in the tubing provoke a drag force under the column of stems, that serving as a variation of gravity acting on the column. Recent searches indicates that the essential problem then encountered was the inability of a sensor to operate reliably downhole in a petroleum well and to send any information about the equipment’s performance. Initial findings of the study indicated the possibility of utilizing the equipment vibrations that were transmitted through the pipe that connects the pump to the surface.

The results show that the first stems show more signs of vibrations than other stems, which can be monitored because they are easiest to apply sensors.

## 5. DISCUSSION AND CONCLUSIONS

With the advance in research and development of systems for monitoring structural health, know the frequencies and the disturbances of the system is very important. For a monitoring system and collect data (vibrations) for oil well, should consider the torsional and bending vibrations that cause damage in the stems and provoke breaks and stops in the production oil and avoid high costs of maintenance.

The preliminary results showed that the frequencies of pendulum varies with initial angle, and the simulation of stems column for PCP pump presented the variation of frequencies to first stems.

These results help the correct choice about the diameter of stem and tubing and increase the data for system of monitoring structural health that will be developed.

In the future works, will analyze the differential equations of multiple pendulums to verify dependence on the initial angle and apply in the simulator. Knowing of the envelope curve of columns of stems PCP pump, the Supervisor and Petroleum Engineer can choice the correct equipment to equip the oil well and reduce the number of maintenance.

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