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Comparison of different time-frequency analysis tools in a Slug Flow based on Slug Tracking model

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Abstract. *The aim of this work is to compare different time frequency analysis tools in a slug flow. The slug flow is the succession of long gas bubbles which are neither periodic in time nor in frequency. Experimental data mixed with Slug Tracking model, which is an analytical approach, are employed in order to obtain its time variant natural frequencies. The applicability of the Short Time Fourier Transform, Wavelet Transform, Wigner-Ville Distribution and Hilbert-Huang Transform are compared and discussed aiming at discussing advantages and disadvantages of each method in the analysis of the time variant behavior of the bubble natural frequency.*

Keywords: *Taylor bubble; Short Time Fourier Transform; Wavelet Transform; Wigner-Ville Distribution; Hilbert-Huang Transform; time-frequency analysis*

1. INTRODUCTION

Multiphase horizontal air water flow in pipes is common in many industrial activities such as the chemistry industry, power plants, and especially the oil industry. There are many multiphase horizontal flow patterns such as stratified, annular and slug, which is the mixture between liquid and long gas bubbles. Although there are many attempts to develop a full theory for slug flow patterns, there is little research on the oscillatory characteristics, which is time variant on a slug flow. Some examples of previous work are Vergniolle and Brandeis (1996), Polonsky et al. (1999), Liang and Ma (2004), Jame et al. (2004), Mazza and Rosa (2008), Madani et al. (2009).

Slug Tracking is used to describe long gas bubbles in horizontal slug flows and the oscillation characteristics is shown as variation of the bubble pressure with time. It is expected for the bubbles a time variant frequency since that the bubbles can change their geometry and stiffness along time. The signal analyzed in this work is obtained using the mathematical Taylor's bubble model and experimental velocity data for two Taylor's bubble pressure amplitudes varying wit time, and both can have time variant natural frequencies, as proposed by Mazza and Rosa (2008).

The aim of this work is to compare time-frequency analysis tools, namely the Short Time Fourier Transform, the Wavelet Transform, the Wigner-Ville Distribution and the Hilbert-Huang Transform, not only as single tools, but especially their combination to achieve the best results aiming at investigating the advantages and disadvantages of each method. Since the data represents two bubble models that have an analytical solution available, it can compared to the time-frequency analysis results.

This article is organized as follows. In Section 2 the Slug Tracking model is briefly presented. In Section 3 the Short-Time Fourier Transform (STFT), the Wigner-Ville Distribution (WVD) and the Hilbert-Huang Transform are presented as time-frequency analysis tools, and the Wavelet Transform is presented as a filtering approach. In Section 4 the obtained results are presented and discussed and, finally, Section 5 presents the concluding remarks.

2. Slug Tracking Model

2.1. Taylor Bubble

In two-phase horizontal flow patterns, increasing velocity of the gas phase has the effect of coalescence of the bubbles, and the new bubble diameters reach a size similar to the pipe size. When this occurs, it forms long gas bubbles known as Taylor bubbles, which may be present in a concentric or eccentric shape depending on the influence of gravity. Around the Taylor bubbles develop a film of liquid that can make the tiny gas bubbles disperse. The length of the film and the Taylor bubble are considered identical and their value will depend mainly on the flow of gas, since gas is preferably carried by the Taylor bubble.

2.2. Numerical Methods

Employing the slug-tracking model and considering a standard slug flow as horizontal, isothermal, one-dimensional, non-aerated, constant pressure on the bubble, and disregarding the friction of the piston, the interactions between bubbles and piston can be described by mass and motion conservation equations, (Mazza and Rosa, 2008), expressed as:

$$P_j = P_{j+1} + \rho_L.g.LS_j + \rho_L LS_j\left(\frac{dU_j}{dt}\right) \tag{1}$$

$$U_j = U_{j-1} - \frac{LB_j.\overline{RG}_j}{P_j}\left(\frac{dP_j}{dt}\right) \tag{2}$$

where U is the velocity of the liquid piston, LB and LS is the length of the bubble and the liquid piston, respectively, $\overline{RG}$ is the average void fraction in the bubble, P is the pressure in the bubble, D is the diameter of the pipe . The position of the front and rear of the bubble is represented by the coordinates Y_1 and X_0, respectively. Figure 1 shows, in simplified form, the variables involved.

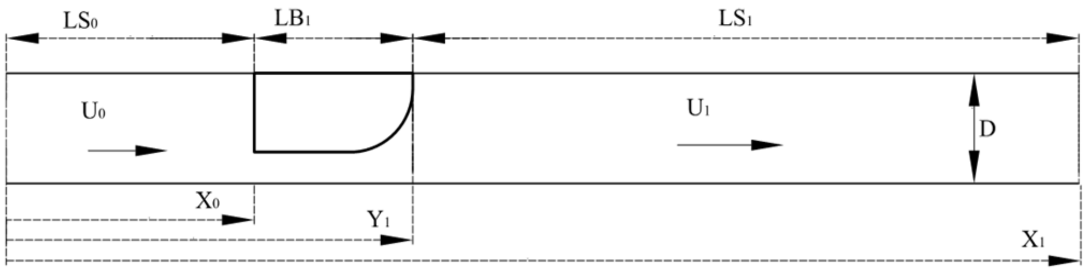


Figure 1. Diagram of Taylor bubble model

An analytical solution can be obtained by decomposition of terms: an average and a fluctuation. Analyzing the order of magnitude of the terms and their mean variations, the model is simplified and an approximate analytic solution can be obtained. The equation for two bubbles in the horizontal direction can be written in matrix form. (Mazza and Rosa, 2008).

$$[M]\ddot{\tilde{P}}+[K]\tilde{P}=E \tag{3}$$

where [M] is the mass matrix, defined as:

$$M=\begin{pmatrix} \frac{\rho_L\overline{LS_1}}{P_{atm}}(RG_1)\overline{LB_1} & 0 \\ \frac{\rho_L\overline{LS_2}}{P_{atm}}(RG_1)\overline{LB_1} & 0 \end{pmatrix} \tag{4}$$

and [K] is the stiffness matrix, defined as:

$$K=\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \tag{5}$$

In order to obtain the natural frequencies, it is sufficient to solve the following equation:

$$\text{Det}\left|M_{ij}^{-1}k_{ij}-\omega^2\delta_{ij}\right|=0 \tag{6}$$

where δ_{ij} is the Kronecker delta, representing the identity matrix elements. Considering a two-bubble flow, it is possible to define:

$$C_1 = \frac{1}{\left(\sqrt{2}\right)\pi}\sqrt{P_{atm} / \rho_L} \tag{7}$$

$$C_2 = \left(\overline{LS_1}.\overline{LB_1}.RG_1\right) + \left[\left(\overline{LS_2}.\overline{LB_1}.RG_1\right) + \left(\overline{LS_2}.\overline{LB_2}.RG_2\right)\right] \tag{8}$$

$$C_3 = \left(\overline{LS_1}.\overline{LB_1}.RG_1\right)\left(\overline{LS_2}.\overline{LB_2}.RG_2\right) \tag{9}$$

Then, the natural frequencies can be obtained as follows:

$$\omega_j = \left(C_1\right)\left[\sqrt{\frac{1}{\left(C_2\right) \pm \sqrt{\left(C_2\right)^2 - 4\left(C_3\right)}}}\right] \tag{10}$$

3. TIME-FREQUENCY ANALISYS TOOLS

In this section, the Short-Time Fourier Transform (STFT), the Wigner-Ville Distribution (WVD) and the Hilbert-Huang Transform are briefly presented as time-frequency analysis tools, and the Wavelet Transform is presented as a filtering approach in order to separate the time variant natural frequencies within the analyzed signal.

3.1. Short-Time Fourier Transform (STFT)

The Fourier Transform approach does not give any information about time variation of the frequency content of a signal. The Short-Time Fourier Transform (STFT) offers a straightforward approach to overcome this, by performing the Fourier Transform on a block by block basis windowing the signal with a sliding window $\gamma(t-\tau)$. For a discrete signal sampled at each Δt, one can aply the Discrete Fourier Transform (DFT) to each block, leading to a both discrete time and frequency implementation of STFT, given by (Quian, 2002):

$$STFT[n,m] = \sum_{k=0}^{L-1} s[k]\gamma[k-n]W_L^{-mk} \tag{11}$$

where $s[k]$ is the sampled signal, $\gamma[k-n]$ is the L-point window shifted by n, $W_L^{-mk} = e^{-j2\pi mk/L}$. The time and frequency resolution are given by $t = n\Delta t$ and $\omega = 2\pi m/(L\Delta t)$.

The main drawback of the STFT is related to its time-frequency resolution, limited by the uncertainty principle, which means that a good time resolution leads to a poor frequency resolution and vice versa (Quian, 2002).

3.2. Wigner-Ville Distribution - WVD

The Wigner-Ville Distribution (WVD) is a time-frequency representation of the time-frequency energy density in contrast to the conventional power spectral density; the time-frequency energy density function describes the signal's energy distribution in terms of both time and frequency. Compared to correlation-based methods, such as the STFT and Wavelets, Wigner-Ville Distribution can yield representations with a better time-frequency resolution since it is not determined by a corresponding set of elementary functions (or windows) and is not restricted by the uncertainty principle. However, its main drawback is related to the cross frequencies (Quian, 2002).

For a signal s(t), the Wigner-Ville distribution is given by:

$$WVDs(t,\omega) = \int_{-\infty}^{\infty} s\left(t+\frac{\tau}{2}\right)s^*\left(t-\frac{\tau}{2}\right)e^{-j\omega\tau}d\tau \tag{12}$$

where the product $s(t+\tau/2)s^*(t-\tau/2)$ presents Hermitian symmetry in τ, ω is the angular frequency and $j=\sqrt{-1}$.

For a given sampled signal and a given symmetric window, such that $w[n]=w[-n]$, it is straightforward to define the pseudo WVD:

$$\overline{PWVD}_s(m,\omega)=2\sum_{n=-\infty}^{\infty}w[n]s[m+n]s^*[m-n]e^{-j2\omega n} \tag{13}$$

Using DFT, one can write:

$$DWVD_s[m,k]=4\,\mathrm{Re}\left\{\sum_{n=0}^{L-1}w[n]s[m+n]s^*[m-n]e^{-j\frac{4\pi k}{L}n}\right\}-2w[0]s[m]s^*[m] \tag{14}$$

with $0\le k\le L$.

3.3. Hilbert-Huang Transform - HHT

The Empirical Mode Decomposition (EMD) is a method proposed by Huang et al. (1998) to decompose any signal in a set of functions, the so-called Intrinsic Mode Functions (IMF), obtained in an adaptive basis to represent the signal. This basis usually offers a physically meaningful representation of the underlying process. There are no *a priori* assumptions about the signal even evenly-spaced sampling is not necessary.

Given any sampled signal, $x[n]$, the EMD can be written as:

$$x[n]=IMF_1[n]+IMF_2[n]+IMF_3[n]+\ldots+res[n] \tag{15}$$

It is expected that this decomposition leads to a finite number of IMFs plus a residue. The IMFs are found by an iterative algorithm, consisting basically of (Huang et al., 1998, Huang and Shen, 2005):

1. Finding all the local maxima and minima of the signal;
2. Interpolating them in order to have lower and upper envelopes of the signal;
3. Computing the mean of the lower and upper envelopes;
4. Subtracting the mean from the signal;
5. Back to step 1 and iterating under the remaining signal until a given stoppage criterion;
6. Subtracting the IMF found from the signal and back to step 1 until there are no more IMFs in the signal.

Note that there are two stoppage criteria, one for stopping the iteration process, known as sifting process, and another to stop the search for more IMFs in the signal. Essentially, the former stops when there are no more changes in the iteration process and the latter stops when there is no more information in the signal to extract another IMF. These stoppage criteria are some of the drawbacks of the algorithm due to their degree of arbitrariness, as well as the search of extrema and their interpolation.

For each found IMF one can apply the Hilbert Transform. In order to perform that, it is possible to define the analytic signal:

$$z(t)=x(t)+jH\{x(t)\}=a(t)e^{i\theta(t)} \tag{16}$$

where $H\{x(t)\}$ is the Hilbert Transform of $x(t)$. The instantaneous frequency is defined by:

$$\omega=\frac{d\theta}{dt} \tag{17}$$

and the instantaneous amplitude can be obtained by:

$$a(t)=\sqrt{x(t)^2+H\{x(t)\}^2} \tag{18}$$

3.4. Wavelet Transform - WT

The continuous-time wavelet transform (CWT) of a given signal $x(t)$ can be written as:

$$CWT(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t)\psi^{*}\left(\frac{t-b}{a}\right)dt \quad a \neq 0 \tag{19}$$

where * denotes the complex conjugate and $\psi(t-b/a)$ is called the mother wavelet, shifted by b and scaled by a. Traditionally, one can sample $CWT(a,b)$ in a dyadic grid, *i.e.*, $a = 2^{-m}$ and $b = n2^{-m}$. The transition from continuous-time wavelet do discrete wavelet, unlike the Fourier transform, requires much more effort than just replacing t by $n\Delta t$. This is done by the Multi Resolution Analysis (MRA), which allows a digital implementation by filter banks (Quian, 2002).

In that sense, the wavelet decomposition produces detail and approximation coefficients. The former contain the finer information, through high pass filtering, while the latter contain the coarser information, through low pass filtering, as shown in Fig. (2). These approximation and detail coefficients can be decomposed into another level of detail and approximation coefficients again. It can be shown that the resulting signal obtained from each decomposition contains half the bandwidth of its previous level.

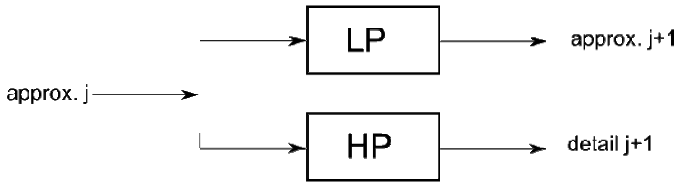


Figure 2. One-Dimensional DWT decomposition step. Low pass and high pass filtering lead to approximation and detail coefficients.

4. DISCUSSION AND RESULTS

For this analysis, we considered a two-bubble flow, because it allows an analytical response – Eq. (7), Eq. (8), Eq. (9), and Eq. (10) - in order to compare it with the results obtained with the time-frequency analysis method used. It is considered a horizontal pipe, under the following the inicial conditions based on experimental data: void fraction: 0.5, pipe length: 20 m, pipe diameter: 0.0265 m, atmospheric pressure: 500 kPa, surface water velocity: 0.4 m/s, surface air velocity: 0.6 m/s, bubble length: 1.3 m.

These conditions lead to a characteristic velocity signal evolution of the liquid throughout the time, considering standard slug flow. This signal, shown in Fig. (3), was numerically obtained, and its implementation and modeling details can be found in Mazza and Rosa (2008). It was sampled with Δt = 0.012 s and 1,196 points.

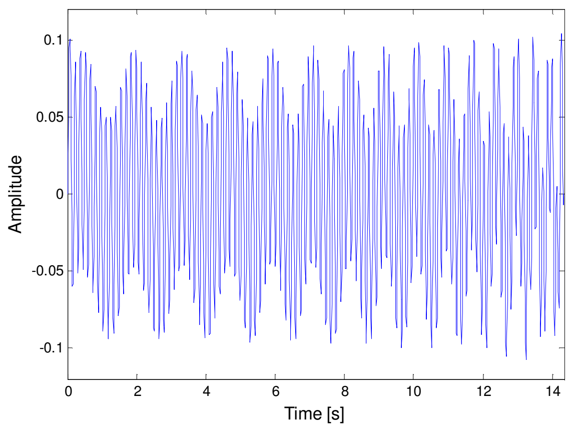


Figure 3. Analyzed signal.

Figure (4) shows the response obtained with the analytical approach - Eq. (7), Eq. (8), Eq. (9), and Eq. (10). Figure(5) shows the DFT (upper left), the spectrogram (upper right), the HHT (lower left), and the WVD (lower right) obtained when applied to the signal shown on Fig. (3). It can be noted that there exist two fundamental frequencies related to the two bubbles in the flow.

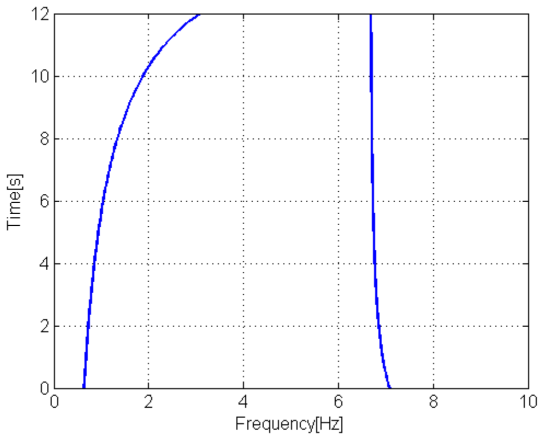


Figure 4. Analytical response for a two bubble Taylor model.

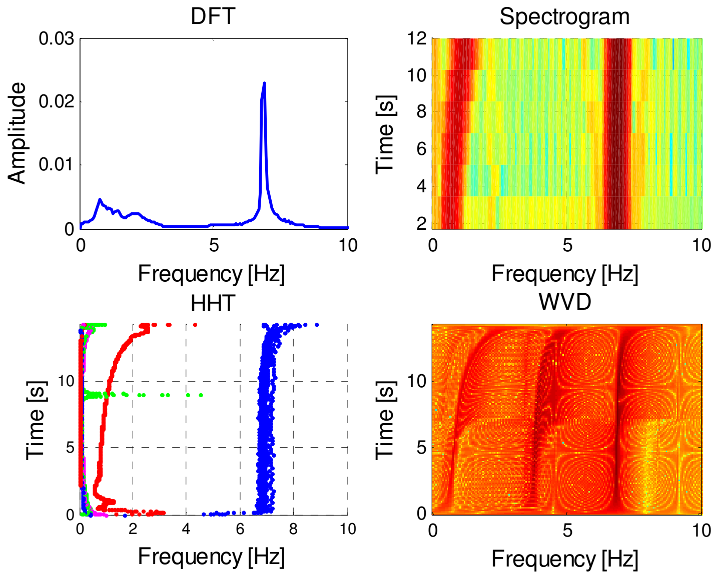


Figure 5. DFT (upper left), Spectrogram (upper right), HHT (lower left) and WVD (lower right) applied to the original signal.

There are two natural frequencies related to the two bubbles. The greater one has a relatively constant behavior, while the smaller one varies with time. Note that the DFT approach is not able to represent this time dependent behavior. The spectrogram can show it, however with a poor resolution. The HHT plot shows the frequency behavior for each obtained IMF from the EMD, Eq. (15). Note there were obtained seven IMF and only two of them are physically meaningful – related to each bubble. Although the WVD approach presents clearly both frequencies, it also shows a cross term, which can be misleading.

The usage of all these time-frequency tools can be complementary. In that sense, the spectrogram, despite of its poor resolution, can be used to detect the cross terms of the WVD approach, that offers a high resolution response, and to confirm the quality of the EMD, since it is an empirical approach.

It is very common the presence of several bubbles in a slug flow, and all of them in a very close bandwidth. It is possible to use the MRA approach to filter the signal in order to separate the frequency bands of interest. This is supposed to simplify the analysis. Appling the wavelet decomposition using the Dmeyer mother wavelet, it was possible to separate both frequencies and apply the time-frequency tools to each one of them. Before that, it was performed the signal reconstruction using the level 4 approximation coefficients, Fig. (6), and the detail coefficients of level 3, Fig. (7), both of them with the same mother wavelet. Note that with this approach the WVD presented no cross terms and the EMD has less IMFs.

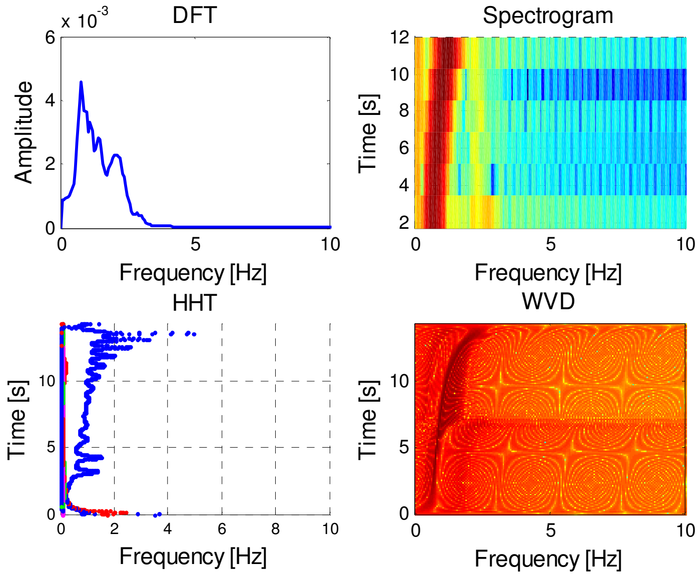


Figure 6. DFT (upper left), Spectrogram (upper right), HHT (lower left) and WVD (lower right) applied to the MRA. Reconstruction with approximation coefficients, level 4 using DMeyer mother wavelet.

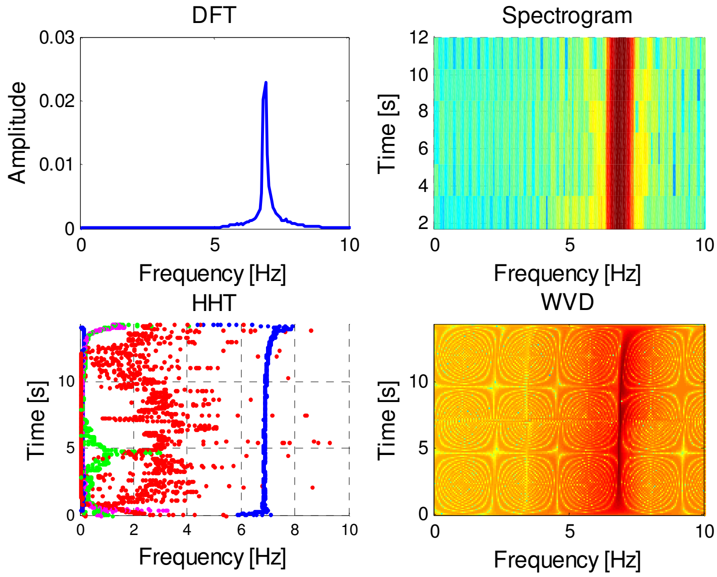
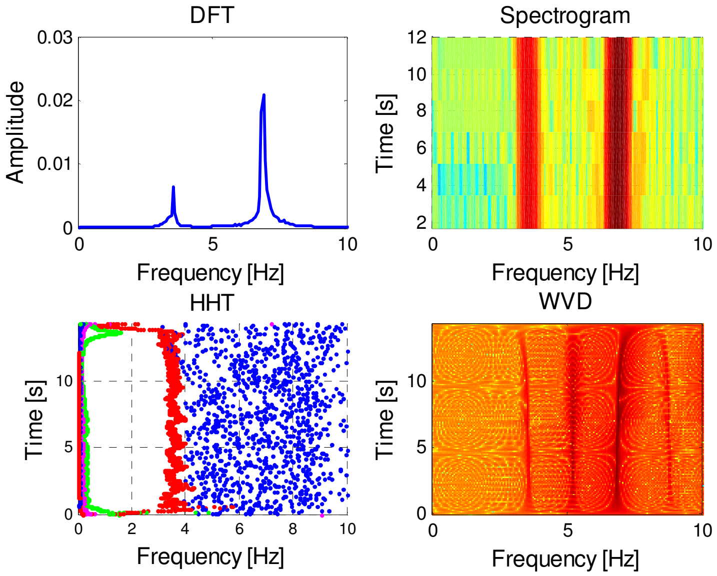


Figure 7. DFT (upper left), Spectrogram (upper right), HHT (lower left) and WVD (lower right) applied to the MRA. Reconstruction with detail coefficients, level 3 using DMeyer mother wavelet.

However this decomposition approach is very sensitive to the choice of the mother wavelet. For instance, the same processes was applied, but using the Daubechie's 5 as the mother wavelet instead of the Dmeyer's. Fig. (8) shows the reconstruction with detail coefficients of level 3. It was expected to obtain the same results shown in Fig. (7), but a spurious frequency appeared, with no physical meaning. There is a lack of understanding on how to choose the best mother wavelet, thus requiring *ad hoc*.



solutions.

Figure 8. DFT (upper left), Spectrogram (upper right), HHT (lower left) and WVD (lower right) applied to the MRA. Reconstruction with detail coefficients, level 3 using Daubechies5 mother wavelet.

5. CONCLUSIONS

This work has compared different time-frequency analysis strategies applied to the oscillation characteristics of long gas bubbles in horizontal slug flows. It was considered a two Taylor bubble physical model allowing to compare the obtained results with an available analytical response.

As a first approach, the Short Time Fourier Transform, Wigner-Ville Distribution and also Hilbert-Huang Transform were applied to the analyzed signal and their respective advantages and disadvantages were presented. In the second approach, the wavelet decomposition was used as a filtering approach in order to separate the time-variant frequencies related to both bubbles. This complementary step made the analysis more clear.

The usage of multiple time-frequency tools seems to lead to an insightful understanding of this kind of phenomenon.

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