CONEM 2010
VI Congresso Nacional de Engenharia Mecânica
ABCM Associação Brasileira de Engenharia e Ciências Mecânicas — COMPROMETIDA COM A PROMOÇÃO DO DESENVOLVIMENTO DA ENGENHARIA E DAS CIÊNCIAS MECÂNICAS
VI CONGRESSO NACIONAL DE ENGENHARIA MECÂNICA
VI NATIONAL CONGRESS OF MECHANICAL ENGINEERING
18 a 21 de agosto de 2010 – Campina Grande – Paraíba - Brasil
August 18 – 21, 2010 – Campina Grande – Paraíba – Brazil

ANALYSIS AND OPTIMIZATION OF A TURBOPROP TRANSPORT AIRCRAFT OPERATING COST

Rafael dos Santos, rafaelds.eng@gmail.com[1]
Pedro Paglione, paglione@ita.br[2]

[1]Instituto Tecnológico de Aeronáutica, Divisão de Engenharia Aeronáutica, Departamento de Mecânica do Vôo – Praça Marechal Eduardo Gomes, 50 – Vila das Acácias – CEP 12228-900 – São José dos Campos, SP, Brazil
[2]Instituto Tecnológico de Aeronáutica, Divisão de Engenharia Aeronáutica, Departamento de Mecânica do Vôo – Praça Marechal Eduardo Gomes, 50 – Vila das Acácias – CEP 12228-900 – São José dos Campos, SP, Brazil

Abstract*. The purpose of this paper is to analyze the performance of a turboprop transport aircraft, aiming for the minimization of the operating costs. The direct operating cost is discussed, and a parameter called σ (sigma) that represents the relative importance of fuel and time in cost is defined. A mathematical model of the aircraft in flight (considered as a point-mass restricted to a vertical plane) is created, allowing for the determination of data such as distance, altitude, fuel and time by numerical integration. These results are then iterated in an optimization routine, obtaining climb, cruise and descent speeds and cruise altitudes that minimize the cost in a given scenario. These resulting optimum flight parameters are verified by simulating a sample mission, flown using speeds and altitudes obtained for solutions ranging from minimum time to minimum fuel. Finally, the resulting cost associated with each of these cases is obtained, the optimal case is determined and the impacts in term of cost increase for the off-optimal solutions are discussed.*

Keywords*: performance, cost, aircraft, turboprop, optimization*

1. INTRODUCTION

The global trend of rising fuel prices in the near future will unavoidably affect the aviation business. As the slice of the costs linked to fuel grows, aircraft that are more fuel-efficient will become increasingly attractive. Within the range of speeds and altitudes used in commercial aviation, the most efficient propulsion type is the turboprop engine (Roskam, 1997). Currently turboprop aircraft are mostly used in short-haul routes, but as fuel becomes more expensive there is a tendency that they will become more interesting in a broader range of missions.

This paper intends to present an operating cost oriented analysis of the performance of a hypothetical turboprop aircraft. A mathematical model of the aircraft is defined to calculate parameters such as flight time, distance and fuel burnt, which are input to optimization methods aimed at find minimum cost. The obtained data are then applied onto a sample set of missions, whose results are then presented and analyzed.

2. AIRCRAFT TECHNICAL SPECIFICATIONS

The aircraft studied in this paper is a FAR-23 basis certified light twin-engine turboprop transport aircraft, mainly directed at military operation. Its performance-related specifications are listed in Tab. 1:

Table 1. Aircraft specifications

Wing Area	38.0 m²	Maximum Payload Weight	3000 kg
Basic Operational Weight (BOW)	4632 kg	Maximum Fuel Weight	2000 kg
Maximum Takeoff Weight (MTOW)	8600 kg	Maximum Engine Power (1)	932 kW (1250 SHP)
Maximum Operating Speed (VMO)	120.9 m/s CAS (235 KCAS)	Operational Ceiling	8230 m (27000 ft)
Operating cost per fuel unit (C_F) (3)	1.15 US$/kg	Operating cost per time unit (C_T) (3)	0.0539 US$/s
Maximum Lift Coefficient (C_L) (2)	1.85		

(1) per engine
(2) clean configuration
(3) costs are valid only for a single hypothetical military operator

3. MATHEMATICAL MODEL

The four main forces relevant to flight are lift (L), drag (D), thrust (F) and weight ($W = m \cdot g$). Figure 1 illustrates how each of these forces acts on an aircraft moving in a vertical plane, where α is the angle of attack, γ the angle of trajectory, α_f the propulsive angle and V the aircraft's velocity vector.

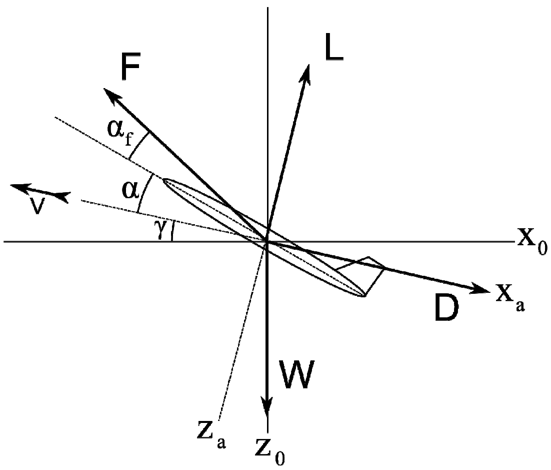

Figure 1. Forces acting on an aircraft in a 2-D plane

Assuming there is no wind, considering the aircraft as a point-mass entity and restricting the movement to a two-dimensional vertical plane, the equations that describe the motion are:

$$\dot{V} = \frac{F \cdot \cos(\alpha + \alpha_f) - D - m \cdot g \cdot \sin(\gamma)}{m} \tag{1}$$

$$\dot{\gamma} = \frac{F \cdot \sin(\alpha + \alpha_f) + L - m \cdot g \cdot \cos(\gamma)}{m \cdot V} \tag{2}$$

$$\dot{x} = V \cdot \cos(\gamma) \tag{3}$$

$$\dot{H} = V \cdot \sin(\gamma) \tag{4}$$

Also, it is necessary to define an expression to account for the variation of mass over time:

$$\dot{m} = -\dot{m}_f \tag{5}$$

The above defined expressions can be then numerically integrated to determine flight times and fuel masses. Lift and drag are given by Eq. (6) and Eq. (7), where ρ is the atmospheric density (in this paper, considered as defined by the International Standard Atmosphere), V the aircraft's true airspeed, S the reference area and C_L and C_D respectively the lift and drag coefficient. C_D is a function of C_L in a drag polar. Thrust (F) and fuel consumption ($\dot{m}_f$) are outputs from a specific computer program which simulates the engine/propeller combination, as a function of factors such as speed, altitude and throttle setting.

$$L = \tfrac{1}{2} \cdot \rho \cdot V^2 \cdot S \cdot C_L \tag{6}$$

$$D = \tfrac{1}{2} \cdot \rho \cdot V^2 \cdot S \cdot C_D \tag{7}$$

3.1. Special Case: Cruise

When cruising the aircraft flies in a horizontal trajectory and thus $\gamma = 0$ and $\dot{\gamma} = 0$. Also, it is safe to assume that speed is constant or varies very little ($\dot{V} \approx 0$) and the angle of attack is small ($(\alpha + \alpha_f) \approx 0$). With these considerations the equations of motion in cruise are reduced to:

$$F = D \tag{8}$$

$$L = m \cdot g \tag{9}$$

$$\dot{x} = V \tag{10}$$

3.2. Special Case: Climb/Descent

In a steady climb it can be assumed that the aircraft climbs at a near-constant angle, therefore $\dot{\gamma} \approx 0$. Also assuming that the angle of attack is small ($(\alpha + \alpha_f) \approx 0$), the following expression results:

$$m \cdot \dot{V} = F - D - W \cdot \sin(\gamma) \tag{11}$$

Knowing that $\dot{H} = V \cdot \sin(\gamma)$, an expression for the rate of climb can be obtained by an algebraic manipulation:

$$\dot{H} = \frac{\dfrac{(F - D) \cdot V}{W}}{1 + \dfrac{V}{g} \cdot \dfrac{dV}{dH}} \tag{12}$$

In the expression above the denominator is called SEP (Specific Excess Power) and the numerator is called AF (Acceleration Factor). Expressions for AF in commonly used flying techniques such as constant calibrated airspeed or Mach number are developed and presented in great detail in (ESDU, 1981).

4. OPERATING COST

The fraction of aircraft's operating cost related directly to the operation is called DOC (Direct Operating Cost). DOC is defined by the sum of the costs due to elapsed flight time and expended fuel, plus a fixed cost (C_0). The cost due to time is defined as the flight time (T) multiplied by the cost per time unit (C_T), while the cost due to fuel is given by the fuel spent (F) multiplied by the cost per fuel unit (C_F) (Airbus, 2002).

$$DOC = C_0 + C_F \cdot F + C_T \cdot T \tag{13}$$

Equation (13) can be expressed in the form of an integral on a time interval:

$$DOC - C_0 = \int_{t=0}^{t=T} \left(C_F \cdot \dot{m}_f \right) dt + \int_{t=0}^{t=T} (C_T) dt \equiv \int_{t=0}^{t=T} \left(C_F \cdot \dot{m}_f + C_T \right) dt \tag{14}$$

It is convenient to define a variable which represents the relative importance of the time and fuel related costs. In this paper a parameter called σ (sigma) expressed in Eq. (15) will be used.

$$\sigma = \frac{C_F / C_T}{1 + C_F / C_T} \tag{15}$$

Sigma (σ) is contained in a closed interval between 0 and 1. When C_F is much larger than C_T (that is, fuel costs are much more critical than time costs), σ tends to 1. Likewise, when C_T is much larger than C_F , σ tends to 0. By means of algebraically substituting Eq. (15) into Eq. (14), DOC can be expressed as a function of σ:

$$DOC = C_0 + \left[(C_F + C_T) \cdot \int_{t=0}^{t=T} \left(\sigma \cdot \dot{m}_f + (1-\sigma) \right) dt \right] \tag{16}$$

In the expression above, the integral is known as the cost function and is called J. Since C_0, C_F and C_T are constants, the value for DOC is minimized when J Eq. (17) reaches its minimum value.

$$J = \int_{t=0}^{t=T} \left(\sigma \cdot \dot{m}_f + (1-\sigma) \right) dt \tag{17}$$

It is of the utmost importance to have in mind that the expression for σ is not homogeneous in terms of units. Therefore, the units used for C_F and C_T directly influence the value of σ. A determined value for J will only yield the correct value for DOC if the same units were used for C_F and C_T both when calculating σ and $(C_F + C_T)$. Throughout this paper metric units are used, with C_F in dollars per kilogram and C_T in dollars per second. Since the studied aircraft has a C_F of 1.15 US$/kg and a C_T of 0.0539 US$/s, the associated value of σ that should result in the minimum cost is 0.9552.

5. OPTIMIZATION TECHNIQUES

The problem of optimizing a flight segment consists basically in selecting a combination of flight speed and altitude that yields a minimum value for J within the aircraft's operational envelope. The flight profile used in this paper is a very typical case, consisting of a climb at a constant calibrated airspeed (with engines set to maximum climb power), a cruise at a constant altitude and variable speed (set through adjustments in power) and a descent at a combination of constant vertical speed and calibrated airspeed flown with engines at idle power.

5.1. Climb Segment

The cost factor associated with an entire flight segment can be determined by solving Eq. (18) for the appropriate segment:

$$J = \sigma \cdot m_f + (1-\sigma) \cdot T \tag{18}$$

In order to minimize the cost associated with a climb it is necessary to minimize the value of J associated with it, by selecting an appropriate climb speed. It is important to note that climbs performed at different speeds will result in different flight distances – lower airspeeds tend to result in steeper and therefore shorter climbs. As a result, the values for J corresponding to each climb speed cannot be compared directly. For this purpose, it is necessary to append a cruise segment at the end of the analyzed climb in order to force the aircraft to fly the same distance, regardless of the climb length (as shown in Fig. 2) (Boeing, 1989).

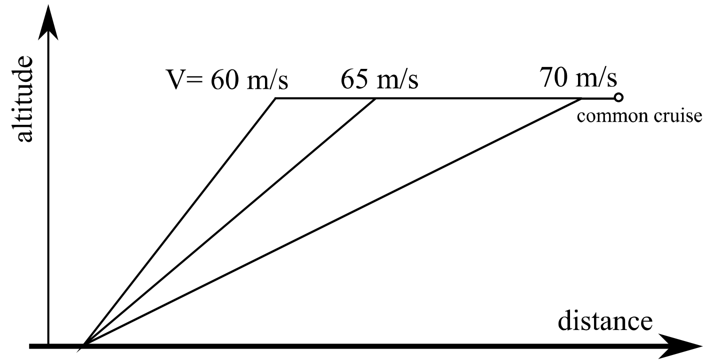

Figure 2. Analyzed climb profiles (speeds are for illustration purposes only)

Consequently, the goal is to find a climb speed that minimizes the sum of the cost factors associated with the climb itself and its matching cruise segment:

$$J = \left[\sigma \cdot m_f + (1-\sigma)\cdot T\right]_{CLIMB} + \left[\sigma \cdot m_f + (1-\sigma)\cdot T\right]_{CRUISE} \tag{19}$$

The speed used in climb has to be within a certain range. On the lower boundary there is the stall speed (V_S), dictated by maximum C_L. Typically a safety margin is used over that value, and in this paper the minimum speed is defined as $1.3 \cdot V_S$. The high boundary speed is also very important: because the goals of the optimization are basically reducing time and fuel, the process tends to converge to very large numbers, since more speed means less time and consequently less fuel burn. However, larger speeds also mean shallower climbs, and so it is necessary to define a maximum airspeed that actually allows the aircraft to reach the top of climb at an acceptable climb rate. In this paper the maximum climb speed is defined as the speed where at least 1.524 m/s (300 ft/min) can be sustained throughout the climb segment, limited by VMO.

5.2. Cruise Segment

Assuming that the aircraft flies a unitary distance, the flight time can be expressed by:

$$T = 1/V \tag{20}$$

The cost factor per unitary distance can then be found by integrating Eq. (17) and substituting Eq. (20) into it:

$$J = \frac{\left(\sigma \cdot \dot{m}_f + (1-\sigma)\right)}{V} \tag{21}$$

Then a given combination of altitude and speed that minimizes the value of J can be found. Assuming $L = m \cdot g$, C_L and consequently C_D can be determined. Since $F = D$, the propulsive model can be used to find the value of $\dot{m}_f$ corresponding to F.

The chosen value for cruise speed is also restricted by lower and upper boundaries. Besides the aerodynamic and structural limitations ($1.3 \cdot V_S$ and VMO), speeds should also be checked if they are physically feasible, since the powerplant has limitations on how much (and how little) thrust can be provided.

5.3. Descent Segment

The optimization of the descent is identical to the one for the climb. In practice, the results of the process tend to exhibit very steep descent profiles, since these take the least amount of time and consequently burn less fuel. However, the descent rate is normally limited either by pressurization systems limitation or human comfort (in case of unpressurized aircraft). As a consequence there is often little or no room of optimization in the descent phases, and it is customary to adopt a constant calibrated airspeed or vertical speed that is suitable for the aircraft.

6. RESULTS

The optimization methods previously described were applied onto a sample mission, consisting in flying a distance of 926 km (500 NM) carrying a 2000 kg payload.

The climb was performed at a constant calibrated airspeed from sea level to the cruise altitude. Cruising was performed at a constant altitude (with values constrained to multiples of 304.8 m (1000 ft)) and variable airspeed. The descent was flown at a constant rate of -15.24 m/s (-3000 ft/min) at altitudes above 3048 m (10000 ft) and at a constant calibrated airspeed of 97.7 m/s (190 KCAS) at and below 3048 m. Fuel reserves were considered for a two-hour holding pattern.

All simulations were repeated for values of σ spanning from 0.0 to 1.0 in intervals of 0.1, with addition of $\sigma = 0.955$ where the minimum cost was expected.

Table 2. Optimized speeds and altitudes

	climb speed		cruise altitude		average cruise speed	
σ	**(m/s CAS)**	**(KCAS)**	**(m)**	**(ft)**	**(m/s)**	**(KT)**
0.0	99.3	193	3353	11000	136.0	264.3
0.1	99.8	194	3658	12000	136.0	264.3
0.2	99.3	193	3658	12000	136.0	264.3
0.3	105.5	205	5182	17000	135.1	262.7
0.4	99.3	193	6096	20000	134.3	261.1
0.5	95.2	185	6706	22000	133.6	259.7
0.6	88.5	172	7620	25000	132.1	256.7
0.7	83.9	163	8230	27000	130.7	254.1
0.8	83.9	163	8230	27000	130.7	254.1
0.9	83.9	163	8230	27000	130.7	254.1
0.955	81.3	158	8230	27000	127.5	247.9
1.0	74.6	145	8230	27000	110.1	214.0

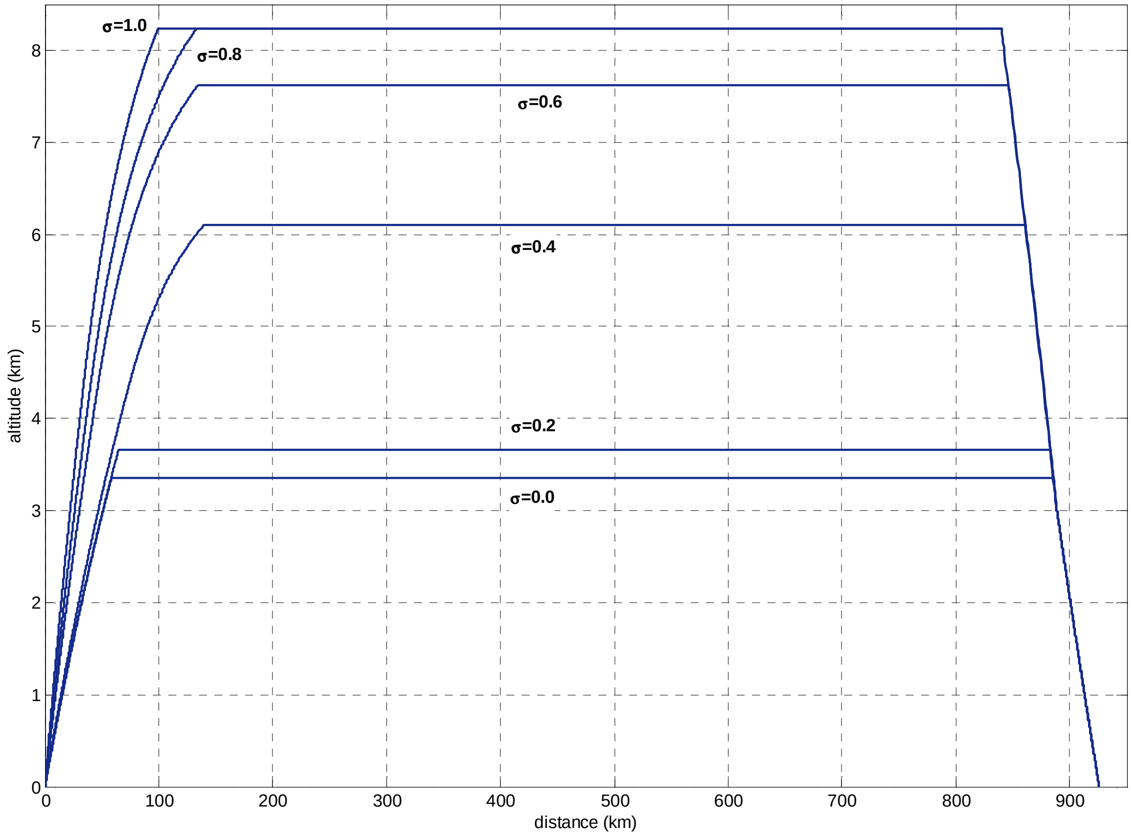

Figure 3. Optimized mission vertical profiles

Table 3. Optimized mission costs

	fuel (kg)	time (min)	cost (US$)		
σ			$C_F.F$	$C_T.T$	DOC
0.0	1022.9	116.7	1175.90	377.44	1553.34
0.1	994.8	116.9	1143.60	378.09	1521.69
0.2	994.7	116.9	1143.48	378.09	1521.57
0.3	878.0	117.8	1009.33	381.00	1390.33
0.4	804.7	118.6	925.06	383.59	1308.65
0.5	761.2	119.4	875.06	386.18	1261.23
0.6	702.9	121.0	808.04	391.35	1199.39
0.7	669.1	122.5	769.18	396.20	1165.38
0.8	669.1	122.5	769.18	396.20	1165.38
0.9	669.1	122.5	769.18	396.20	1165.38
0.955	660.0	125.1	758.72	404.61	1163.33
1.0	639.1	141.3	734.69	457.01	1191.70

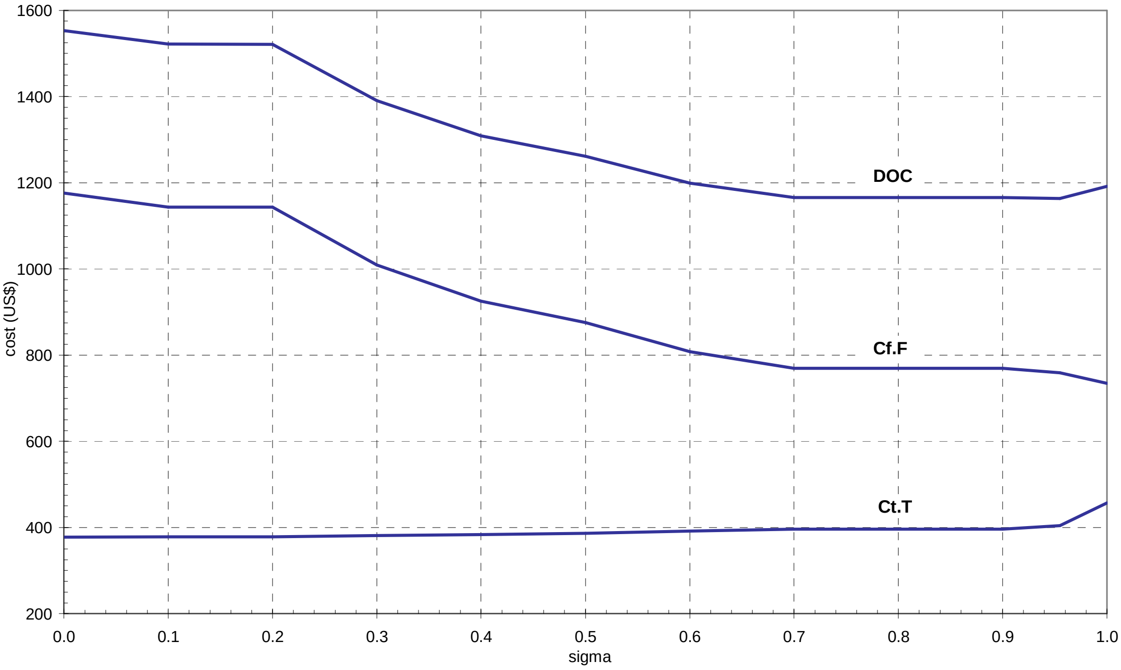

Figure 4. Optimized mission costs

7. CONCLUSIONS

Lower optimal cruise altitudes were found for lower values of σ. For $\sigma = 0.0$, where time is the critical cost-related variable, an altitude of 3353m was obtained – the maximum altitude where the engines can sustain maximum power. For higher values of σ, when fuel costs begin to play a bigger role, progressively higher altitudes were found, with the operational ceiling being reached for $\sigma \geq 0.7$.

Cruise speeds tended to be higher when σ nears zero. With the exception of $\sigma = 0.995$ and $\sigma = 1.0$, cruise was performed at maximum engine power, hence making speed a function of altitude. With $\sigma = 1.0$ a much lower speed was obtained: cruise was performed at the maximum range speed, where fuel use is minimized.

Higher values of σ resulted in steeper climbs: when fuel is more critical it is better to start cruising sooner. For σ closer to 0.0 climbs tended to be less steep, since this allows for the aircraft to develop higher speeds.

Operating costs due to fuel decreased from US$1175.90 to US$734.69 as the considered value for σ spanned from 0.0 to 1.0. Costs due to time had the exact opposite behavior, increasing from US$377.44 to US$457.01. The direct operating cost, which is the sum of both costs due to fuel and time, as expected reached its minimum when $\sigma = 0.995$ with a value of US$1163.33. Flying with a profile that minimizes only time ($\sigma = 0.0$) or fuel ($\sigma = 1.0$) cost respectively US$1553.34 (33.52% more) and US$1191.70 (2.44% more).

The presented results emphasize the importance of adopting an optimized flight profile considering the actual importance of the fuel and time related costs. While the substantial increase in operating costs generated by flying at the maximum speed regime does not surprise, it is important to acknowledge that the added cost by flying in minimum fuel regime is by no means negligible: in fact, numbers as small as one percent are very representative in modern airline operations. It is convenient to remind that the studied aircraft is meant for military operations and thus features a relative small importance of time on total costs – in other types of operation time is usually a bigger factor – and this helped to bring the optimal minimum cost solution closer to minimum fuel. However, the tendency of rise in fuel costs should bring the relative importance of time and fuel closer to the values used in this paper.

8. REFERENCES

Airbus, 2002, "Getting to grips with aircraft performance", Blagnac, France.

Boeing, 1989, "Jet Transport Performance Methods", Flight Operations Engineering, Seattle, USA.

Embraer, 2007, course notes: "Desempenho Básico de Aeronaves", São José dos Campos, Brazil.

ESDU, 1969, "Data Item 69026: Airspeed data for performance calculations", ESDU Intl plc, London, UK.

ESDU, 1981, "Data Item 81046: Acceleration factors for climb and descent rates at constant EAS, CAS, M", ESDU Intl plc, London, UK.

Instituto Tecnológico de Aeronáutica, 2009, course notes: "MVO-10 – Desempenho de Aeronaves", São José dos Campos, Brazil.

Pradines, L., 2006, "Métodos para a redução de custos operacionais em empresas aéreas", São José dos Campos, Brazil.

Roskam, J. and Lan, C. T., 1997, "Airplane Aerodynamics and Performance", DARcorporation, Lawrence - KS, USA

9. RESPONSIBILITY NOTICE