HEAT TRANFER IN A MOVING POROUS BED USING A TWO-ENERGY EQUATION CLOSURE

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Abstract: The objective of this work is to present simulations for heat transfer in a porous reactor, in which both the permeable bed and the working fluid moves with respect to the fixed bounding walls. For simulating the flow and heat transfer, a two-energy equation model is applied in addition to a mechanical model. Transport equations are discretized using the control-volume method and the system of algebraic equations are relaxed via the SIMPLE algorithm. For both the solid and the fluid phase, temperature distribution are presented as a function of the particle diameter, medium permeability, solid-to-fluid thermal conductivity ratio and solid-to-fluid velocity ratio.

Keywords: moving bed, porous media.

1. INTRODUCTION

There is an increasing interest in the use of a moving bed technology as the separation of chemical compounds, recuperation of petrochemical processes, drying of grains and seeds, channel of moving bed for the removal of organic matter in the affluent, etc. The advantages of using moving bed are low investment, low energy consumption, low maintenance with mechanical equipment and improvement process performance, such as reducing the moisture content in agricultural products, for example. Another important technology that can benefit from the studies herein is a moving bed composed of pellets. Pellet techniques are used for various purposes, among them one can mention the pelletization of iron ore and biomass use.

With respect to pelletization of iron ore, Parisi and Laborde (2004) and Negri et.al. (1991) presented a study about the direct reduction of iron oxide in a countercurrent reactor in a moving bed. Also within this context, Valipour et.al.(2006) developed a mathematical model to simulate grain kinetics and thermal behavior of a pellet of porous iron oxide. Their study considered chemical reactions with a mixture of hydrogen, carbon monoxide, carbon dioxide and water vapor. Further, Valipour and Saboohi (2007a) presented a mathematical model to simulate the multiple heterogeneous reactions with a countercurrent complex set of thermal phenomena and physical-chemical modeling in a moving bed of porous pellets on reactor. Valipour and Saboohi (2007b) described a model to predict countercurrent flow in a cylindrical reactor in which pellets of iron ore went through a gas mixture.

Henda and Falcioni (2006) described a thermal performance of a pre-heater that consists of a moving bed of pellets of nickel in concurrent flow with a gas, using both one and two equations energy models. Nakayama and Kuwahara et.al. (2001) in their paper presented the exact solution of energy equations for two fundamental cases, i.e. one-dimensional porous plate with internal heat generation within the solid and the thermally developing unidirectional flow through a semi-infinite porous medium, which was used to validate the problem of heat transfer between fluid and solid phases in a laminar flow channel.

The present study is concerned with simulation of energy transport in a moving bed though which a laminar flow occurs. Energy equations for both phases are applied.

2. MACROSCOPIC MODEL FOR MOVING BED

A macroscopic form of the governing equations is obtained by taking the volumetric average of the entire equation set. In this development, the porous medium is considered to be rigid, fixed and satured by the incompressible fluid. As mentioned, derivation of this equation set is already available in the literature (Pedras and de Lemos 2001a; Pedras and de Lemos 2001b; Saito and de Lemos 2005; de Lemos 2006) so that details need not to be repeated here. Nevertheless, for the sake of completeness, the final laminar form of the equations is here presented:

Continuity: $\nabla \cdot \mathbf{u}_D = 0$ (1)

Momentum:
$$\rho\left[\frac{\partial \mathbf{u}_D}{\partial t} + \nabla \cdot \left(\frac{\mathbf{u}_D \mathbf{u}_D}{\phi}\right)\right] = -\nabla(\phi\langle\overline{p}\rangle^i) + \mu\nabla^2\mathbf{u}_D - \underbrace{\left[\frac{\mu\phi}{K}\mathbf{u}_{rel} + \frac{c_F\phi\rho|\mathbf{u}_{rel}|\,\mathbf{u}_{rel}}{\sqrt{K}}\right]}_{Relative\,drag} \tag{2}$$

where the last two terms in Eq. (2) represent the Darcy and Forchheimer contributions. The symbol K is the porous medium permeability, c_F is the form drag or Forchheimer coefficient, $\langle\overline{p}\rangle^i$ is the intrinsic average pressure of the fluid, $\mathbf{u}_D$ is the Darcy velocity vector, $\mathbf{u}_{rel}$ is the relative velocity based on total volume, ρ is the density, μ is the fluid dynamic viscosity and ϕ is the porosity of the porous medium.

For a moving bed, only cases where the solid phase velocity is kept constant will be considered here, a moving bed crosses a fixed control volume in addition to a flowing fluid, which is not necessarily moving with a velocity aligned with the solid phase velocity. The steps below show first some basic definitions prior to presenting a proposal for a set of transport equations for analyzing such systems.

A general form for a volume-average of any property φ, distributed within a phase i that occupy volume ΔV_i, can be written as Gray and Lee (1977),

$$\langle\varphi\rangle^i = \frac{1}{\Delta V_i}\int_{\Delta V_i}\varphi\,dV_i \tag{3}$$

In the general case, the volume ratio occupied by phase i will be $\phi^i = \Delta V_i / \Delta V$.

If there are two phases, a solid $i = s$ and a fluid phase $i = f$, volume average can be established on both regions. Also,

$$\phi^s = \Delta V_s / \Delta V = 1 - \Delta V_f / \Delta V = 1 - \phi^f \tag{4}$$

and for simplicity of notation one can drop the superscript "f" to get $\phi^s = 1 - \phi$.

As such, calling the instantaneous local velocities for the solid and fluid phases, $\mathbf{u}_s$ and $\mathbf{u}$, respectively, one can obtain the average for the solid velocity, within the solid phase, as follows,

$$\langle\mathbf{u}\rangle^s = \frac{1}{\Delta V_s}\int_{\Delta V_s}\mathbf{u}_s\,dV_s \tag{5}$$

with, in turn, can be related to an average velocity referent to the entire REV as,

$$\mathbf{u}_s = \overbrace{\frac{\Delta V_s}{\Delta V}}^{(1-\phi)}\underbrace{\frac{1}{\Delta V_s}\int_{\Delta V_s}u_s\,dV_s}_{\langle\mathbf{u}\rangle^s} \tag{6}$$

A further approximation herein is that the porous bed is kept rigid and moves with a steady average velocity $\mathbf{u}_s$. Both velocities can then be written as,

$$\mathbf{u}_D = \phi\langle\mathbf{u}\rangle^i, \ \mathbf{u}_s = (1-\phi)\langle\mathbf{u}\rangle^s = const. \tag{7}$$

A relative velocity is then defined as,

$$\mathbf{u}_{rel} = \mathbf{u}_D - \mathbf{u}_s \tag{8}$$

Temperatures for the fluid and solid phase are governed by,

$$\left\{(\rho c_p)_f\,\phi\right\}\frac{\partial\langle T_f\rangle^i}{\partial t} + (\rho c_p)_f\,\nabla\cdot\left(\mathbf{u}_D\langle T_f\rangle^i\right) = \nabla\cdot\left\{\mathbf{K}_{eff,f}\cdot\nabla\langle T_f\rangle^i\right\} + h_i a_i\left(\langle T_s\rangle^i - \langle T_f\rangle^i\right) \tag{9}$$

$$\left\{(1-\phi)(\rho c_p)_s\right\}\frac{\partial\langle T_s\rangle^i}{\partial t} + (\rho c_p)_s\,\nabla\cdot\left(\mathbf{u}_s\langle T_s\rangle^i\right) = \nabla\cdot\left\{\mathbf{K}_{eff,s}\cdot\nabla\langle T_s\rangle^i\right\} - h_i a_i\left(\langle T_s\rangle^i - \langle T_f\rangle^i\right) \tag{10}$$

where c_p is the specific heat, and $\langle T_f \rangle^i$, $\langle T_s \rangle^i$, $\mathbf{K}`s$, a_i and h_i are the fluid and solid temperatures, the conductive tensor, the interfacial area and interfacial heat transfer coefficient, respectively see Saito and de Lemos (2006).

Non-dimensional temperatures for the solid and fluid are defined as:

$$\theta_{s,f} = \frac{\langle T_{s,f} \rangle^i - T_{\min}}{T_{\max} - T_{\min}} \tag{11}$$

where the subscripts s, f stands for the solid and fluid phases, respectively, and "max" and "min" refers to both temperature maximum and minimum of either phase.

3. RESULTS AND DISCUSSION

The problem under investigation is a laminar flow through a channel completely filled with a moving layer of a porous material, as shown in Fig. (1). The channel shown in the Fig. (1) has length and height given by L and H, respectively. The porous matrix moves with constant velocity $\mathbf{u}_s$.

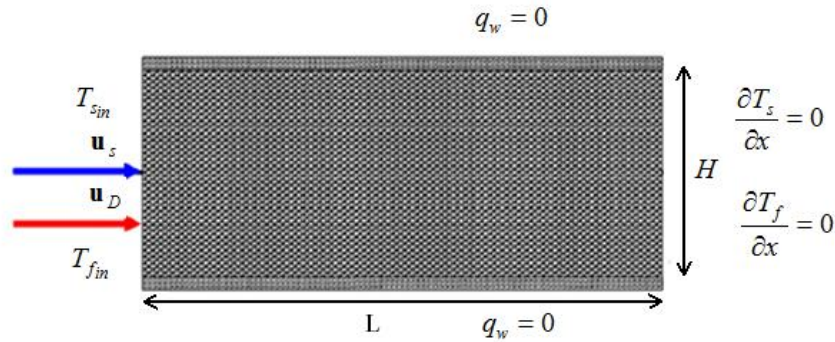The fluid and solid phases are given different temperatures at the inlet.

Figure 1. Porous bed reactor with a moving solid matrix.

3.1. Validation

Figure (2) shows values for the longitudinal non-dimensional temperature profiles compared with the analytical solution by Nakayama and Kuwahara et.al. (2001), written as,

$$\theta_f = \frac{\langle T \rangle^f - T_{\min}}{(T_{\max} - T_{\min})} = 1 + \left\{ \frac{\gamma\sigma(1 - \phi + (\sigma - 1)G)}{\left[Pe + \gamma\left(\frac{K_{eff} + K_{xx}}{k_f} \right) \right]} \right\} \times (\exp(-\gamma X) - 1) \tag{12}$$

$$\theta_s = \frac{\langle T \rangle^s - T_{\min}}{(T_{\max} - T_{\min})} = 1 + \left\{ \frac{\gamma\sigma(1 - \phi + (\sigma - 1)G)}{\left[Pe + \gamma\left(\frac{K_{eff} + K_{xx}}{k_f} \right) \right]} \right\} \times (\exp(-\gamma X) - 1) - \exp(-\gamma X) \tag{13}$$

with the dimensionless coordinate given by:

$$X = \frac{x}{\sqrt{\frac{k_f}{a_i h_i}}} \tag{14}$$

where, G is the tortuosity parameter, K_{xx} is the axial component of thermal dispersion tensor, K_{eff} is the axial component of effective thermal conductivity, γ is the negative real root, Pe is the Peclet number based on Darcian

velocity, σ is the thermal conductivity ratio of solid to fluid k_s / k_f. The values of parameters were calculated according to the expressions found in Nakayama and Kuwahara et.al. (2001), as shown in Tab. (1).

Table 1. Properties and parameters considered in the investigation

γ	σ	ϕ	G	K_{eff}/k_f	K_{xx}/k_f
0.625	40	0.4	-0.013219	4.29293	50

The parameters used by Nakayama and Kuwahara were: porosity $\phi = 0.4$, Peclet number $Pe_d = 100$, velocity ratio $\mathbf{u}_s / \mathbf{u}_D = 0$ and thermal conductivity ratio $\sigma = \dfrac{k_s}{k_f} = 40$.

The properties of solid and fluid such that $k_s / k_f = 40$ are presented in Tab. (2).

Table 2. Physical properties of solid and fluid

Fluid: Water Vapour				
$k_f(W/mK)$	$\rho_f(kg/m^3)$	$c_{pf}(J/kgK)$	$\mu(Ns/m^2)$	$T(K)$
0.0345	0.4345	1986.8	173.1×10^{-7}	507.5
Solid: Silicon Dioxide				
$k_s(W/mK)$	$\rho_s(kg/m^3)$	$c_{ps}(J/kgK)$	-	$T(K)$
1.38	2220	745	-	300

It is clearly seen from Fig. (2) an excellent agreement between the numerical solution of Eqns. (9)–(10) and the analytical solution given by Eqns. (12)–(13).
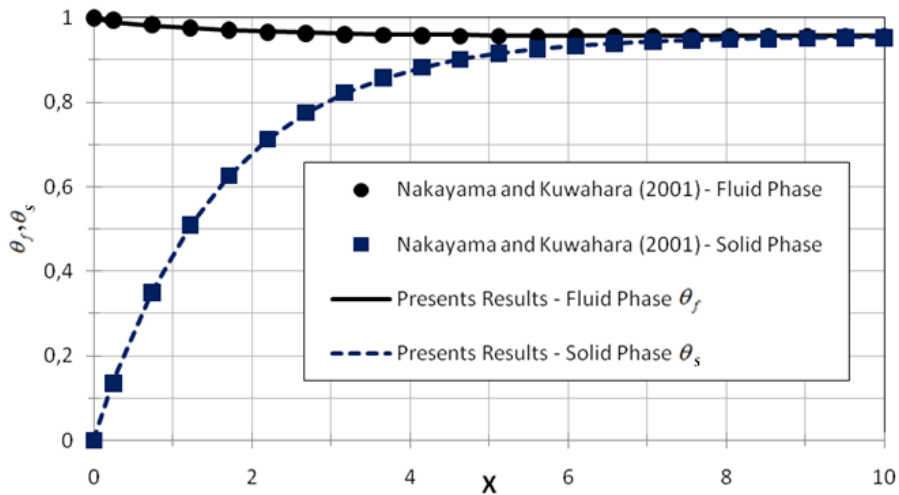

Figure 2. Comparison of dimensionless temperature profile θ_f and θ_s the present study with those presented in Nakayama and Kuwahara et.al. (2001).

3.2. Effect of Reynolds Number, Re_D

Figure (3) shows values for the longitudinal non-dimensional temperature profiles as a function of Re_D. Fig. (3) indicates that the cold porous structure is heated up as the hot fluid permeates through it. Also, the equilibrium temperature rises as more energy in convected into the system as a results of increasing the Reynolds number. The axial length needed for reaching the equilibrium value is further reduced as Re_D is increased due to the relative velocity between phases, which promotes heat transfer between the solid matrix and the fluid.
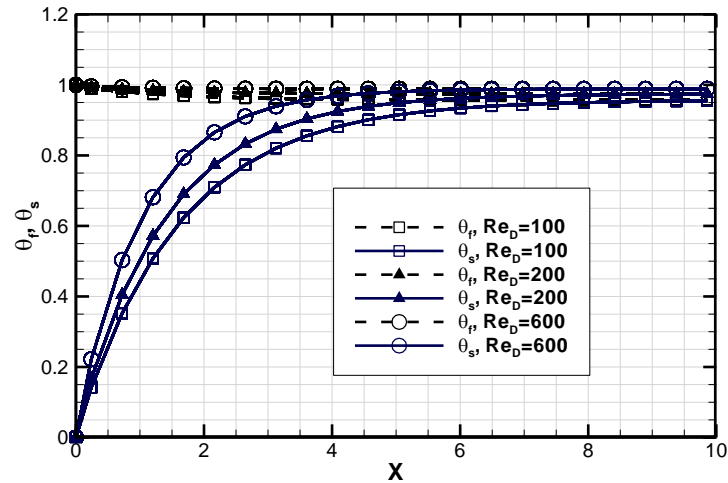
Figure 3. Non-dimensional temperatures as a function of Re_D **for** $Da = 2.5 \times 10^{-3}$**,** $\phi = 0.6$**,** $k_s / k_f = 40$**,** $\rho_s / \rho_f = 5109.3$**,** $(\rho c_p)_s / (\rho c_p)_f = 1915.86$**,** $\mathbf{u}_s / \mathbf{u}_D = 0$ **.**

3.3. Effect of Permeability K

Figure (4) presents the effect of particle diameter d_p on the axial temperature profiles. For a give particle diameter, permeability is given by,

$$K = \frac{d_p^2 \phi^2}{144(1 - \phi)^2}$$ (15)

leading to a Darcy number $Da = K / H^2$ where H is the height of channel. The Reynolds number and the porosity are kept constant for all curves. It is observed in Fig. (4) that for a small permeability, as a result of a decrease of particle diameter, a larger interfacial heat transfer area promotes heat transfer between phases and reduces the length necessary for thermal equilibrium to be reached.
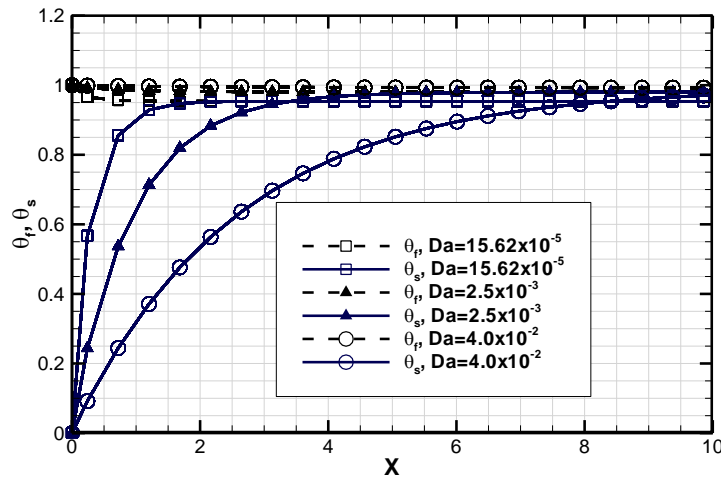

Figure 4. Non-dimensional temperatures as a function of Darcy number Da **for** $\phi = 0.6$**,** $Re_D = 500$ **,** $k_s / k_f = 40$**,** $\rho_s / \rho_f = 5109.3$**,** $(\rho c_p)_s / (\rho c_p)_f = 1915.86$**,** $\mathbf{u}_s / \mathbf{u}_D = 0$ **.**

3.4. Effect of Slip Ratio $\mathbf{u}_s / \mathbf{u}_D$

Figure (5) shows temperature profiles for a moving bed, as a function of $\mathbf{u}_s / \mathbf{u}_D$. It is observed that the higher the value of $\mathbf{u}_s / \mathbf{u}_D$, the greater is the temperature difference between fluid and solid phases. By decreasing the relative velocity between phases, more axial length is needed for the equilibrium temperature to be reached. Increasing the ratio $\mathbf{u}_s / \mathbf{u}_D$, the solid, with a lower temperature, brings down the equilibrium temperature of the system. When the solid

velocity approaches that of the fluid, $\mathbf{u}_s/\mathbf{u}_D = 0.95$, exchange of heat between phases is mostly governed by conduction, requiring then a longer axial length for thermal equilibrium to be established.
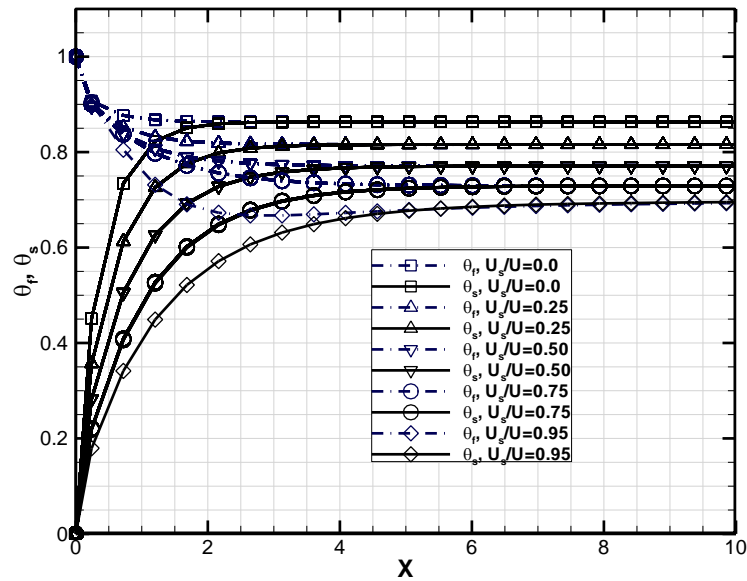

Figure 5. Non-dimensional temperatures as a function of slip ratio $\mathbf{u}_s/\mathbf{u}_D$ for $\phi = 0.6$, $Da = 1 \times 10^{-4}$, $Re_D = 50$, $k_s/k_f = 40$, $\rho_s/\rho_f = 1$, $(\rho c_p)_s/(\rho c_p)_f = 0.3749$.

3.5. Effect of Ratio k_s/k_f

Figure (6) shows the effect of ratio k_s/k_f on longitudinal non-dimensional temperature profiles for a moving bed. It is noted that the higher the ratio value k_s/k_f, the longer is the length needed for thermal development since heat is transported only by conduction within the solid causing its temperature distribution to be more connected to the inlet temperature. With increasing the ratio k_s/k_f, the lower solid temperature brings down the equilibrium temperature of the system. On the other hand, by decreasing the thermal conductivity ratio k_s/k_f between phases, less axial length is needed for the equilibrium temperature to be reached.
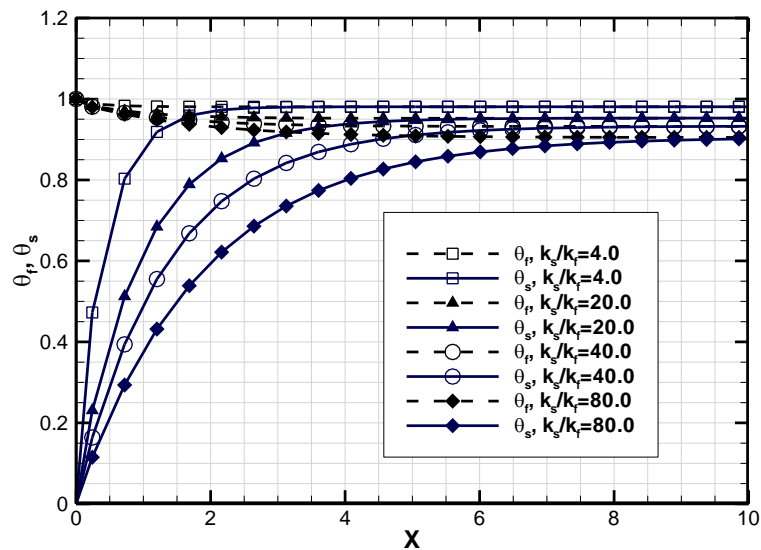

Figure 6. Non-dimensional temperatures as a function of slip ratio k_s/k_f for $\phi = 0.6$, $Da = 1 \times 10^{-4}$, $Re_D = 50$, $\mathbf{u}_s/\mathbf{u}_D = 0$, $\rho_s/\rho_f = 1$, $(\rho c_p)_s/(\rho c_p)_f = 0.3749$.

4. CONCLUSIONS

Numerical solutions for laminar flow in a moving bed porous were obtained for different Reynolds number, porosity, permeability and solid-to-fluid thermal velocity ratio. Governing equations were discretized and numerically solved. Good agreement was obtained when comparing the results herein with those obtained from Nakayama and Kuwahara et.al. (2001). For the same problem, varying parameters such as Reynolds, porosities, speed ratios and conductivity, show an excellent agreement with the results presented in de Lemos and Saito (2008), where was found that increasing the speed of the solid relative to the fluid speed reduces the interfacial drag forces and the transport of energy between the phases are mainly due to conduction. The results presented here have a wide application to problems involving engineering equipment that could be identified as a moving bed porous.

5. ACKNOWLEDGEMENTS

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