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ESTIMATION OF FORCES ON A COLD DRAWING METAL FORMING PROCESS

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Abstract: *The drawing process is used for manufacturing parts in many industries. The subject has been studied with different methods of analysis. Many researchers developed analytical relationships to estimate the drawing forces. Some experimental investigations were done to support the estimations of analytical studies. The numerical simulations of computational models rely mainly on finite element technique. In this paper, a computational model is simulated to estimate the drawing force for the production of a tube sinking. The contact problem between the tube and die presents difficulties related with the choice of element type, convergence rate and number of iterations. The lubricant quality, die semi-angle influence the drawing forces. The fem simulation numerical results are compared to analytical upper bound solutions for a cold drawing quasi-static process.*

Keywords: *plasticity; contact; finite element method; analytical approach; drawing*

1. INTRODUCTION

The cold drawing process is extensively used for the production of cylindrical parts, auto-parts, panels, etc… A large portion of automotive parts are produced via metal pressing processes. This important metal manufacturing process has been studied in different opportunities. Researchers have employed analytical, experimental and numerical methods to estimate the drawing forces. However, the number of factors affecting the process is large. Complete modeling is very hard as defects such as wrinkling, tearing, etc…are of difficult handling by analytical approaches. Moreover lubricants, grade of the metal blank, adjustment of drawing parameters constitute other factors influencing the quality of the final work piece. Better lubricants and better understanding of the effect of the friction on the drawing forces are important on production costs.

2. FORMULATION

2.1 Analytical Approach

The geometric and kinematics parameters of the process are represented on Figure 1. The external radius at the admission section A is $\mathrm{R_i}$ whereas the external radius at the exit section E is $\mathrm{R_0}$. Thickness in these sections is $\mathrm{t_i}$ and $\mathrm{t_0}$. The die semi-angle is α . The contact between the tube and die extends from section B to section D and the length of contact measured on the symmetry axis is L. The velocity at the admission section is $\mathrm{V_i}$ and the velocity at the exit is $\mathrm{V_0}$. Along the contact region it is assumed the existence of Tresca friction stresses between the surfaces of the tube and die. The radial coordinate along the process is ρ . $\mathrm{P} = \int_{\mathrm{A_0}} \sigma_a \mathrm{dA}$ is the drawing force on the exit section.

There are several simplifying assumptions about the material, stress distributions and rigidity of the operation tools, concerning the analytical relations. The main simplifications are (Marciniak and Duncan, 1992):

- The material of the blank is considered to be isotropic, perfectly plastic;
- During the deformation process, the blank thickness remains constant;
- Plane strain conditions are assumed together with incompressibility during plastic deformation.

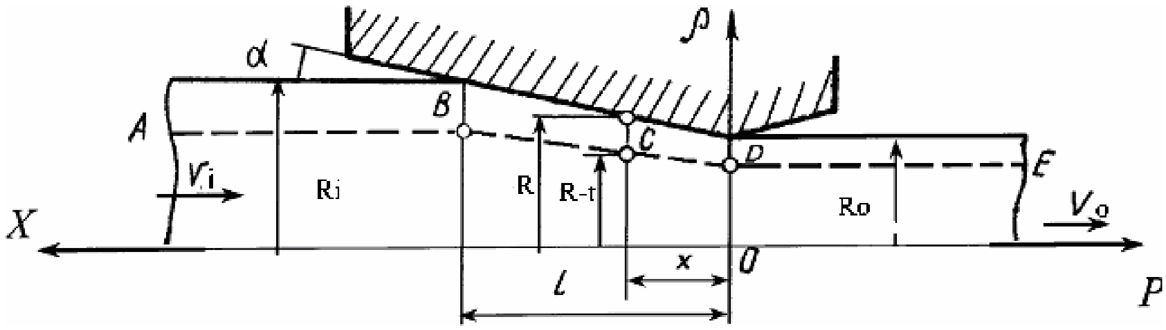

Figure 1 Geometrical and kinematics parameters of the drawing process

Equilibrium of axial forces in the longitudinal direction over the slab between positions x, with axial stress σ_a, and $x + dx$, with axial stress $\sigma_a + d\sigma_a$, leads to:

$$\frac{d\sigma_a}{dx} R(1 - \frac{t}{2R}) + \sigma_a \tan\alpha + \sigma_a(\frac{R}{t} - 1)\frac{dt}{dx} + \frac{R}{t}(\tau + p\tan\alpha) = 0 \tag{2}$$

where p is the lateral contact pressure and τ the corresponding frictional stress. Function $R(x) = R_0 + \tan\alpha\, x;\ \alpha = \tan^{-1}(R_i - R_0 / L);R_0 = R(0)$ describes the evolution of internal surface of the die, in this case, a conical section.

In the same form, equilibrium in the circumferential direction, with internal compressive stresses denoted as σ_θ, gives:

$$\sigma_\theta = \frac{\pi R}{2t}[p - \tau\tan\alpha] \tag{3}$$

Evidently the larger the thickness of the tubular, the smaller the circumferential stress.

Likewise, equilibrium in the radial direction, taking the middle surface as reference, and denoting the radial component as σ_r leads to the equation:

$$\sigma_r = \frac{2R}{2R - t}[p - \tau\tan\alpha] \tag{4}$$

Comparison of the last two results, Eqs. (3) and Eq. (4), shows that:

$$\sigma_\theta \cong \frac{\pi(2R - t)}{4t}\sigma_r \tag{5}$$

what shows that the extreme stresses are σ_a on the traction side, and σ_θ on the compression side. Tresca yield criterion of elastoplasticity for an elastic-perfectly-plastic material, with flow stress σ_0, writes as:

$$\sigma_a + \sigma_\theta = \sigma_0 \tag{6}$$

Tresca friction model is chosen to model interface contact under inelastic conditions. Interface stresses are related to flow stress by means of a coefficient of friction m, particular to each interface-material set (Zimmermain and Avitzur, 1970):

$$\tau = m\frac{\sigma_0}{\sqrt{3}} \tag{7}$$

Introduction of Eq. (5) into Eq. (6), and considering Eq. (7), gives contact pressures in terms of the axial stresses:

$$p = \frac{2t}{\pi R}(\sigma_0 - \sigma_a) - \frac{m\sigma_0}{\sqrt{3}}\tan\alpha \tag{8}$$

and therefore, allows us to write axial equilibrium, Eq. (2), with disregard to the thickness variations, in the form:

$$\frac{d\sigma_a}{dx}R(1-\frac{t}{2R}) + a\sigma_a + bR + c = 0 \tag{9}$$

where coefficients are defined as:

$$a = \tan\alpha(1-\frac{2}{\pi}) \tag{10}$$

$$b = \frac{1}{t}(1+\tan^2\alpha)\frac{m\sigma_0}{\sqrt{3}} \tag{11}$$

$$c = \frac{2}{\pi}\tan\alpha\,\sigma_0 \tag{12}$$

This equation may be solved numerically. When b term is small, an analytical approximate close form solution may be obtained. From Eq. (9):

$$\frac{\frac{d\sigma_a}{dx}}{a\sigma_a + c} = -\cot\alpha\frac{dR}{R(1-\frac{t}{2R})} \tag{14}$$

whose integration produces (Luis, León and Luri, 2005):

$$\sigma_a(x) = \sigma_a(0)\varsigma + \frac{c}{a}(\varsigma - 1); \qquad \varsigma = \left(\frac{[\frac{t}{2} - R(x)]}{[\frac{t}{2} - R(0)]}\right)^{a\cot\alpha} \tag{15}$$

with $\sigma_a(L) = 0$.

An energy method can be considered for the steady state process. The work rate for the drawing $\dot{W}$ can be calculated as the sum of the plastic deformation work rate $\dot{W}_p$ and the work rate dissipated $\dot{W}_f$ to overcome the friction against the die. Total work rate can be expressed by:

$$\dot{W} = \dot{W}_p + \dot{W}_f \tag{16}$$

The axial component of the plastic work rate $\dot{W}_a = PV$, with drawing force computed from integration of the axial stresses σ_a at the section, whose flow velocity is V. Friction dissipation computed from the frictional stresses and tangential components of velocity, computed supposing mass conservation along the process.

The volume of material passing the admission section A and the volume of material at the exit section E per unit of time are the same. Therefore we can establish for any section:

$$2\pi R_i V_i t_i = 2\pi RVt \tag{17}$$

where t_i is the the tube thickness at the admission and $V_0 = {}^{R_i}\!/_{R_0} V_i$ for constant thickness.

The rate, at which plastic work is done on the material passing between the initial contact section B and the end of the contact region D, as represented in Fig. 1, corresponds to:

$$\dot{W}_p = 2\pi RtV \int \sigma' d\varepsilon' \tag{18}$$

where $\sigma' = \sqrt{\frac{3}{2}[\sigma_a^2 + \sigma_r^2 + \sigma_\theta^2]}$ stands for the flow stress and $d\varepsilon' = \sqrt{\frac{2}{3}[d\varepsilon_a^2 + d\varepsilon_r^2 + d\varepsilon_\theta^2]}$ is the effective strain increment. The integral represents the plastic work per unit of volume of material.

At the contact between the tube external surface and the die internal surface, the process starts approximately as a circumferential compression. The thickness of the tube increases from the contact entrance section B until an intermediate section is reached. Beyond this point the thickness starts to decrease until the exit section of the contact region between tube and die. However in many cases it is sufficiently accurate to consider that the drawing process happens with an approximate thickness, what conducts to a plane strain condition. The strains in circumferential and axial directions, $\varepsilon_\theta = (R - R_0)/R_0$ and $\varepsilon_a = du/dx$, lead to the circumferential increment:

$$d\varepsilon_\theta = dR/R \tag{19}$$

Under this assumption and the incompressibility assumption, it can be considered that the circumferential strain increment and the axial strain satisfy the following condition $d\varepsilon_\theta = -d\varepsilon_a$, and then:

$$\varepsilon' \approx \frac{2}{\sqrt{3}} \ln \frac{R_i}{R} \tag{20}$$

2.2 Finite Element Approach

Solution of nonlinear mechanical problems involving contact with large deformations and rotations, discretized with finite elements, may be constructed from the principle of virtual work, in rate form:

$$G = \int_{V_t} \boldsymbol{\sigma} : \delta\mathbf{L} dV + \int_{S_c} \boldsymbol{\lambda}.\delta\mathbf{v} dA - \int_{S_t} \mathbf{t}.\delta\mathbf{v} dS - \int_{V_t} \mathbf{f}.\delta\mathbf{v} dV \tag{21}$$

where $\boldsymbol{\sigma}$ are Cauchy stresses, $\mathbf{t}$ are surface tractions, present on S_T surfaces, $\boldsymbol{\lambda}$ are the contact stresses present on S_c surfaces and $\mathbf{f}$ are the applied forces. Velocity gradient is $\mathbf{L} = \partial_{,\mathbf{x}} \mathbf{v}$, being $\mathbf{v}$ the velocities and $\mathbf{x} = \hat{\mathbf{x}}(\mathbf{X}, t)$ the displacement mapping, from which the deformation gradient $\mathbf{F} = \partial_{,\mathbf{X}} \mathbf{x}$.

In an incremental Lagrange procedure, discrete configurations are verified during processing. So, if at time t, equilibrium is attained, $G_t = 0$, and the value at t+1 is sought, $G_{t+1} = G_t + \Delta G$, with the increment ΔG computed in an implicit manner. It results:

$$\Delta G = \Delta I + \Delta C - \Delta E \tag{22}$$

where:

$$\Delta \mathrm{I} = \int_{V_t} \delta\mathbf{L} : (-\boldsymbol{\sigma}\Delta\mathbf{l}^{T} + \Delta\boldsymbol{\sigma} + \boldsymbol{\sigma}\mathrm{tr}\Delta\mathbf{l}) dV; \quad \Delta\mathbf{l} = \Delta\mathbf{F}\mathbf{F}^{-1} \tag{23}$$

$$\Delta \mathrm{C} = \int_{S_c} \delta\mathbf{v}.\Delta\boldsymbol{\lambda}^{\nabla} dA; \quad \Delta\boldsymbol{\lambda}^{\nabla} = \Delta\lambda_n^{\nabla}\hat{\mathbf{n}} + \Delta\lambda_i^{\nabla t}\hat{\mathbf{t}}_{i;} i = 1,2 \tag{24}$$

$$\Delta \mathrm{E} = \Delta \int_{S_t} \mathbf{t}.\delta\mathbf{v} dS + \Delta \int_{V_t} \mathbf{f}.\delta\mathbf{v} dV \tag{25}$$

where the normal and tangential components of friction depend upon the model used.

Upon using finite element discretization, velocities may be interpolated as $\mathbf{v} = \mathbf{N}\mathbf{v}^N$ so that:

$$\Delta G = \delta \mathbf{v}^T \Delta \mathbf{F} \quad \Delta \mathbf{F} = \Delta \mathbf{F}_i + \Delta \mathbf{F}_\mathbf{c} - \Delta \mathbf{F}_e \tag{26}$$

If the internal and contact vectors increments are factored in terms of a stiffness matrix and increment of displacements, it turns out that:

$$(K_m + K_c)\Delta u = \Delta F \tag{27}$$

being K_m the mechanical matrix and K_c the contact matrix (Belytschko, Liu and Moran 2000). These matrices depend upon the constitutive elasto-plastic matrices of the conforming material, as well as the constitutive equation for the interface interaction.

2.3 Finite Element Results

Both approaches to the solving of the problem were used in the modeling of drawing process, under quasi-static conditions of a tubular element. In the problem, as no mandrel is present, internal surface of the tube as well as the entrance surface, are S_t surfaces, with zero traction applied. Contact occurs with the rigid die, with constant inclination, on a S_c surface, whereas drawing is applied, with prescribed velocity at the exiting surface, S_v. In particular, the case of very thick tubing was considered. Material properties are included in Table 1, for an commercial type of aluminum, whose modulus of elasticity $\mathrm{E} = 71.7\mathrm{GPa}$ and Poisson's ratio , at a constant strain rate of $0.10\ \mathrm{s}^{-1}$.

Table 1. Material Data Sheet

Equivalent Plastic deformation ε'_p	Yield stress S_y [MPa]
0.00	60
0.125	90
0.250	113
0.375	124
0.500	133
1.00	165
2.00	166

Finite element modeling of the drawing process of a slab moving through a rigid die of length 300 mm, with radius of 100 mm, semi-angle 10 degrees, under a constant strain rate, using processor of program Abaqus (Hibbitt and Sorensen, 2002) was used to verify the approximate analytical model presented above. In Fig. 2 the deformed configuration of the material is shown. Observe the large amounts of straining, mostly due to shear, present around the processing tool. Residual deformations appear at the exiting sections.

Analysis was performed in a multi-step approach for the steady-state case. It consisted of an initial lateral squeeze, to impose initial contact of the blank against the conical die. Axisymmmetric elements were used. The interface was assumed to behave as a Coulomb-Tresca, with friction plastic coefficient $\mathrm{m} = 0.03$. Heat generation effects were disregarded. Symmetricity of boundary conditions was considered in the problem in order to reduce size. Loading was applied by means of displacements at the exit section of the blank. As an implicit method of solution was used, simulation variables had to be dealt with care, as in regions of high shearing, as the entrance region for example, conducted to convergence problems. For the free surfaces, zero traction was assigned. Drawing force was computed from the exit axial stresses.

The interface normal and tangential stress distributions acting along the contact zone were also computed and compared with the analytical formula. As predicted thickness direction stresses are small. High stresses in the transition zones, the conical regions, were circumvented using smoothing of the rigid die geometry.

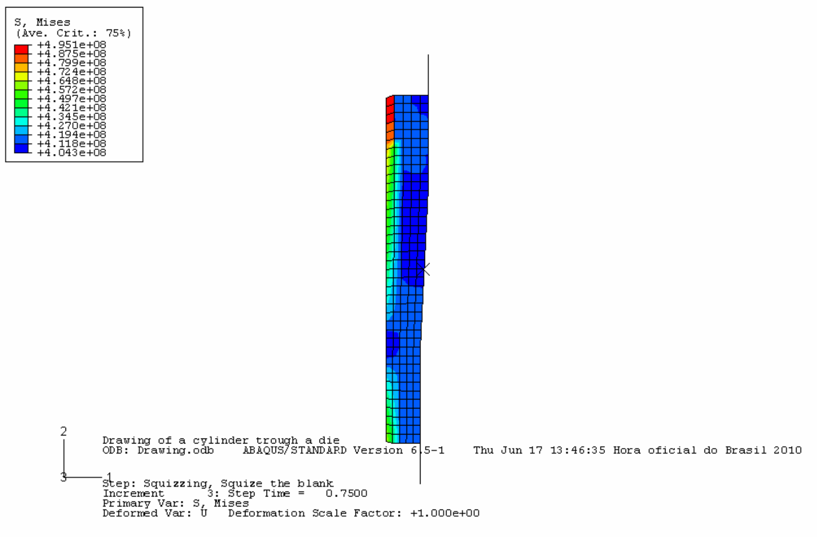

Figure 2. Intermediate deformed shape of material during process

2.4 Comparison

Approximate analytical and numerical results, derived from the procedures discussed above, may be compared as shown in the table 2. An acceptable, in engineering terms, error appears, even though flow stresses corresponding to average degrees of hardening of the material in the process were used. Other wise errors would be larger. (Karnezis and Farrugia, 1998)

Table 2. Maximum drawing forces

Analysis Type	Drawing Force, (N)
Aproximate analytical solution	19250
Finite element approach	18046

3. CONCLUSIONS

Model presented here may be extended, with the development of the complete close form solution. Furthermore isotropic hardening of the material may be considered, with use of a Datsko power function. As it is, the model shows an acceptable agreement with the finite element results, so that potentially effects of the die angle, coefficient of friction, rate and temperature of the blank may be analyzed with it.

4. REFERENCES

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5. RESPONSABILITY NOTICE

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