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THE INFLUENCE OF THE PRESENCE OF AIR IN GRANULAR FLOW FROM A HOPPER

Alice Jordam Caserta
Hélio Aparecido Navarro
Department of Mechanical Engineering, São Carlos School of Engineering, University of São Paulo, Av. Trabalhador São-Carlense, 400, São Carlos, SP, 13566-590, Brazil
e-mails alicejordam@yahoo.com.br and han@sc.usp.br

Abstract. *The focus of this study is to analyze the granular material discharge from a hopper. In this work was used the modification of frictional-kinetic model for gas-particle flow proposed by Srivastava and Sundaresan to study the influence of the presence of the air in granular flow from a hopper. The source code MFIX (Multiphase Flow with Interphase eXchange) developed by NETL ("National Energy Technology Laboratory") was used to perform the numerical simulations. The discharge rate from a hopper in a vacuum and in the presence of the air was evaluated for monodisperse granular flow with particles diameter of 0.1 cm. The results showed that the discharge rate is significantly higher in the presence of the air than that in a vacuum. That was expected due to the increase of the mobility of the particles in the presence of the air.*

Keywords: *Granular flow, hopper, MFIX.*

1. INTRODUCTION

Hoppers are used widely in various industries, such as food, pharmaceuticals, agricultural and chemicals, for storage and flow of granular materials. The discharge of solids from hoppers has been studied since the 1960's, Beverloo et al. (1961) investigated the behavior of granular flow of through the circular orifices and found out an important correlation for the prediction of the discharge rate from hoppers and bin. Reisner (1968) studied experimentally the effect on discharge rate of the hopper shape and the size and form of the outlet and of the properties of the bulk material. Nedderman e Tüzün (1979) proposed a purely kinematic model involving only one experimental constant for the description of the velocity field in a granular material discharging from a hopper. Nedderman et al. (1982) reviewed the published literature in the discharge rates of such materials from hoppers and made some comments on the previous publications. Currently, understanding the behavior of granular flow is important for the optimization and application of hoppers. Thus studies about granular materials flow in hoppers are also relevant.

According to Yalamanchili et al. (1994) the Eulerian–Eulerian approach can be used to analyze granular materials flow under the action of gravity. Lyczkowski et al. (2000) also used the hydrodynamic model to analyze critical discharge flow from a non-aerated hopper containing a square obstacle, obtaining excellent correlations with that of the experimental data.

Comparison of the approach to model particulate flow systems has been carried out and published in the literature. Benyahia (2008) conducted a comparative study of two frictional flow theories and concluded that the S-S frictional model, proposed by Srivastava and Sundaresan (2003), is the better suited for dense frictional flows.

Despite the fact that the discrete element method (DEM) is widely used for the simulation of granular flow, in order to solve practical problems computationally several authors use the two-fluid model which treats the solids phase as a continuum medium, as can be found in Enwald et al. (1996), Ishii and Hibiki (2011), Lun et al. (1984), Gidaspow (1994), Agrawal et al. (2001), Goodman and Cowin (1972), Tuzun et al. (1982), Jenike (1987), Schaeffer (1987), Sun and Sundaresan (2011) and Campbell (2006). These works show good results comparing with data experimental.

In the present work was considered the two-fluid model. The Kinetic Theory for Granular Flows was adopted to model the solid stress in viscous regime (see Lun et al. (1984), Gidaspow (1994) and Agrawal et al. (2001)). The modification of a frictional model for gas-particle flow proposed by Srivastava and Sundaresan (2003), the S-S model, was used in the plastic flow regime, where the solid stress was described by applying soil mechanics theory (see Goodman and Cowin (1972), Tuzun et al. (1982), Jenike (1987) and Schaeffer (1987)).

The focus of this paper is to study the influence of the presence of air in granular flow from a hopper. Two different model to obtain the drag correlation were also compared, one proposed by Wen and Yu (1966) and the other presented by Syamlal et al. (1993), both in the presence of air. For these investigations, the discharge rate from a 2D rectangular bin in a vacuum and in the presence of the air was evaluated for monodisperse granular flow with particles diameter of 0.1 cm.

The numerical simulations were performed by using the open source code MFIX (Multiphase Flow with Interphase eXchange), Syamlal et al. (1993), developed at NETL ("National Energy Technology Laboratory").

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2. MODEL EQUATION

The model equation used in the numerical simulations is the Eulerian two-fluid model proposed by Gidaspow (1994) and Enwald et al. (1996). In Eulerian two-fluid model, the gas and solid phases are treated like continua. This model is usually obtained using the procedure of the averaging of Euler. The fundamental equations of mass conservation and momentum are formulated considering each phase separately. Since both phases interact together, it will appear terms due to this interaction in the equations. Therefore, the additional expressions are needed to obtain the closed system conservation equations. Assuming that the gas-solid flow is isothermal without chemical reactions, the governing equation are described follow.

The continuity equation for the gas phase is expressed as:

$$\frac{\partial}{\partial t}(\epsilon_g \rho_g) + \vec{\nabla}\cdot(\epsilon_g \rho_g \vec{v}_g) = 0 \tag{1}$$

where ϵ_g , ρ_g and $\vec{v}_g$ are, respectively, the void fraction, density and velocity of the gas phase.
The gas phase momentum balance equation is:

$$\frac{\partial}{\partial t}(\epsilon_g \rho_g \vec{v}_g) + \vec{\nabla}\cdot(\epsilon_g \rho_g \vec{v}_g \vec{v}_g) = \vec{\nabla}\cdot(\epsilon_g \bar{\bar{\sigma}}_g) + \epsilon_g \rho_g \vec{g} - \vec{I}_{gm} \tag{2}$$

here, $\vec{g}$ is the gravitational force, $\bar{\bar{\sigma}}_g$ and $\vec{I}_{gm}$ are respectively, the gas phase stress tensor and the momentum transfer between the gas phase and the solid phase. They are defined by:

$$\bar{\bar{\sigma}}_g = -p_g \bar{\bar{I}} + \mu_g \left[\vec{\nabla}\vec{v}_g + (\vec{\nabla}\vec{v}_g)^T - \frac{2}{3}\vec{\nabla}\cdot\vec{v}_g \bar{\bar{I}} \right] \tag{3}$$

where, p_g is the gas pressure, μ_g is the gas viscosity and $\bar{\bar{I}}$ is the unit tensor.

$$\vec{I}_{gm} = -\epsilon_s \vec{\nabla} p_g - \beta_{gm}(\vec{v}_s - \vec{v}_g) \tag{4}$$

Two different drag correlations β_{gm} were used in the simulation, the first proposed by Wen and Yu (1966), which is expressed as:

$$\beta_{gm} = \frac{3}{4} C_{Ds} \frac{|\vec{v}_g - \vec{v}_s|}{d_p} \rho_g \epsilon_s \epsilon_g^{-2.65} \tag{5}$$

where d_p is the particle diameter of the solid phase and the drag coefficient C_{DS} is:

$$C_{Ds} = \begin{cases} \frac{24}{Re_m}(1 + 0{,}15\, Re_m^{0.687}) & if \quad Re_m < 1000 \\ 0.44 & if \quad Re_m \geq 1000 \end{cases} \tag{6}$$

and Re_m is the Reynolds number of solid phase, given by:

$$Re_m = \frac{\epsilon_s \rho_g |\vec{v}_g - \vec{v}_s| d_p}{\mu_g} \tag{7}$$

here μ_g is the gas viscosity.

The second drag correlation β_{gm} used in the simulations was modeling according to Syamlal et al. (1993). The authors proposed a correlation based on measurements of terminal velocity of particles by the following relation:

$$\beta_{gm} = \frac{3}{4} \frac{\epsilon_s \epsilon_g \rho_g}{V_{rm}^2 d_p} C_{Ds}\left(\frac{Re_m}{V_{rm}}\right) |\vec{v}_g - \vec{v}_s| \tag{8}$$

where V_{rm} is the terminal velocity correlation for the solid phase. This velocity can be derived from a correlation developed by Garside and Al-Dibouni (1977):

$$V_{rm}=0.5\left[A-0.06\,Re_m+\sqrt{(0.06\,Re_m^2)+0.12\,Re_m(2B-A)+A^2}\right] \quad (9)$$

where, $A=\epsilon_g^{4.14}$, $B=\begin{cases}0.8\,\epsilon_g^{1.28} & if \quad \epsilon_g\leq 0.85\\ \epsilon_g^{2.65} & if \quad \epsilon_g>0.85\end{cases}$, the Reynolds number of solid phase is given by the Eq. (7) and $C_{Ds}\left(\frac{Re_m}{V_{rm}}\right)$ is a drag coefficient of spherical particle. This coefficient was proposed by Dalla Valle (1948), and can be expressed as: $C_{Ds}\left(\frac{Re_m}{V_{rm}}\right)=\left(0.63+4.8\sqrt{\frac{V_{rs}}{Re_s}}\right)^2$.

For the solid phase, the continuity equation is the following:

$$\frac{\partial}{\partial t}(\epsilon_s\rho_s)+\vec{\nabla}\cdot(\epsilon_s\rho_s\vec{v}_s)=0 \quad (10)$$

where ϵ_s , ρ_s and $\vec{v}_s$ are, respectively, the volume fraction, density and velocity of the solid phase.
The solid phase momentum balance equation is:

$$\frac{\partial}{\partial t}(\epsilon_s\rho_s\vec{v}_s)+\vec{\nabla}\cdot(\epsilon_s\rho_s\vec{v}_s\vec{v}_s)=\vec{\nabla}\cdot\bar{\bar{\sigma}}_s+\epsilon_s\rho_s\vec{g}+\vec{I}_{gm} \quad (11)$$

The solid phase stress tensor $\bar{\bar{\sigma}}_s$ is calculated as per Srivastava and Sundaresan (2003), who consider that the stress tensor is simply the sum of the kinetic stress tensor $\bar{\bar{\sigma}}_s^k$ and the frictional stress tensor $\bar{\bar{\sigma}}_s^f$ as:

$$\bar{\bar{\sigma}}_s=\bar{\bar{\sigma}}_s^k+\bar{\bar{\sigma}}_s^f \quad (12)$$

The kinetic stress tensor is based on the studies of Lun et al. (1984), Srivastava and Sundaresan (2003) and Agrawal et al. (2001) and is given by:

$$\bar{\bar{\sigma}}_s^k=-\left\{\epsilon_s\rho_s\theta\left(1+4\eta\,\epsilon_s g_0\right)-\eta\mu_b(\vec{\nabla}\cdot\vec{v}_s)\right\}\bar{\bar{I}}+\left(\frac{2+\alpha}{3}\right)\left\{\frac{2\mu^*}{g_0\eta(2-\eta)}\left[1+\frac{8}{5}\eta\epsilon_s g_0\right]\left[1+\frac{8}{5}\eta(3\eta-2)\epsilon_s g_0\right]+\frac{6}{5}\eta\mu_b\right\}\bar{\bar{S}}_s \quad (13)$$

where,

$$\eta=\frac{(1+e)}{2}\,,\ \mu_b=\frac{256}{5\pi}\mu\,\epsilon_s^{\,2}g_0\,,\ \alpha=1.6\,,\ \mu^*=\frac{\mu}{1+\frac{2\beta_{gm}\mu}{\rho_s^2\epsilon_s^2 g_0\theta}}\,,\ \mu=\frac{5}{96}\rho_s d_p\sqrt{\pi\theta}\ \text{and}$$
$$\bar{\bar{S}}_s=\frac{1}{2}\left[\vec{\nabla}\vec{v}_s+(\vec{\nabla}\vec{v}_s)^T\right]-\frac{1}{3}\vec{\nabla}\cdot\vec{v}_s\bar{\bar{I}} \quad (14)$$

and θ is the granular temperature of the solid phase, e is the coefficient of restitution of the solid phase and $\bar{\bar{I}}$ is the unit tensor.
The radial distribution function at contact g_0 is derived by Carnahan and Starling (1969):

$$g_0=\frac{1}{\epsilon_g}+\frac{1.5\,\epsilon_s}{\epsilon_g^2}+\frac{0.5\,\epsilon_s^2}{\epsilon_g^3} \quad (15)$$

The conservation of linear granular energy equation is:

$$\frac{3}{2}\left[\frac{\partial}{\partial t}(\epsilon_s\rho_m\theta)+\vec{\nabla}\cdot(\epsilon_s\rho_s\vec{v}_s\theta)\right]=-\vec{\nabla}\cdot\vec{q}-\bar{\bar{\sigma}}_s^k:\vec{\nabla}\vec{v}_s-J_{coll}-J_{vis} \quad (16)$$

here,

$$J_{vis}=3\beta\theta-\frac{81\epsilon_s\mu_g^2(\vec{v}_g-\vec{v}_s)^2}{g_0 d_p^3\rho_s\sqrt{\pi\theta}} \quad \text{and} \quad J_{coll}=\frac{48}{\sqrt{\pi}}\eta(1-\eta)\frac{\rho_s\epsilon_s^2}{d_p}g_0\theta^{\frac{3}{2}} \tag{17}$$

and the diffusive flux of granular energy, $\vec{q}$, is:

$$\vec{q}=-\frac{\lambda^*}{g_0}\left\{\left(1+\frac{12}{5}\eta\epsilon_s g_0\right)\left[1+\frac{12}{5}\eta^2(4\eta-3)\epsilon_s g_0\right]+\frac{64}{25\pi}(41-33\eta)\eta^2\epsilon_s^2 g_0^2\right\}\vec{\nabla}\theta \tag{18}$$

where,

$$\lambda^*=\frac{\lambda}{1+\left(\frac{6\beta\lambda}{5(\rho_s\epsilon_s)^2 g_0\theta}\right)} \quad \text{and} \quad \lambda=\frac{75\rho_s d_p\sqrt{\pi\theta}}{48\eta(41-33\eta)} \tag{19}$$

The frictional stress tensor is given by:

$$\bar{\bar{\sigma}}_s^f=-p_s^f\bar{\bar{I}}+\bar{\bar{\tau}}_s^f \tag{20}$$

Following the frictional model proposed by Srivastava and Sundaresan (2003), the S-S model, the terms in the Eq. (20) are defined as:

$$\frac{p_s^f}{p^*}=\left(1-\frac{\vec{\nabla}\cdot\vec{v}_s}{n\sqrt{2}\sin\varphi\sqrt{\bar{\bar{S}}_s:\bar{\bar{S}}_s+\theta/d_p^2}}\right)^{n-1} \tag{21}$$

here, φ is the angle of internal friction and p^* is the critical state pressure, given as:

$$p^*=\begin{cases} A(\epsilon_s-\epsilon_s^*)^{10} & if \quad \epsilon_s>\epsilon_s^* \\ F\frac{(\epsilon_s-\epsilon_s^{min})^r}{(\epsilon_s^*-\epsilon_s)^s} & if \quad \epsilon_s^*\ge\epsilon_s>\epsilon_s^{min} \\ 0 & if \quad \epsilon_s\le\epsilon_s^{min} \end{cases} \tag{22}$$

here, ϵ_s^* is the volume fraction at maximum packing, ϵ_s^{min} is the minimum solids' fraction, $A=10^{25}$, $F=0.5\text{dynes}/cm^2$, $r=2$, $s=5$ and,

$$n=\begin{cases} \frac{\sqrt{3}}{2}\sin\varphi & if \quad \vec{\nabla}\cdot\vec{v}_s\ge 0 \\ 1.03 & if \quad \vec{\nabla}\cdot\vec{v}_s<0 \end{cases} \tag{23}$$

The frictional stress, $\bar{\bar{\tau}}_s^f$, is expressed as:

$$\bar{\bar{\tau}}_s^f=2\mu_s^f\bar{\bar{S}}_s \tag{24}$$

where,

$$\mu_s^f=\frac{\sin\varphi}{\sqrt{2}}\frac{p_s^f}{\sqrt{\bar{\bar{S}}_s:\bar{\bar{S}}_s+\theta/d_p^2}}\left\{n-(n-1)\left(\frac{p_s^f}{p^*}\right)^{\frac{1}{n-1}}\right\} \tag{25}$$

2.1 The Boundary Conditions:

The wall boundary condition for the gas-phase is free-slip. For the solids phase this was taken from Johnson and Jackson (1987), and can be written as:

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$$\vec{n}\cdot(\bar{\bar{\sigma}}_s^k+\bar{\bar{\sigma}}_s^f)\cdot\frac{(\vec{v}_{sl})}{|\vec{v}_{sl}|}+(\vec{n}\cdot\bar{\bar{\sigma}}_s^f\cdot\vec{n})\tan\delta+\frac{\pi\sqrt{3}}{6\,\epsilon_s^*}\varphi'\rho_s\,\epsilon_s\,g_0\sqrt{\theta}\,\vec{v}_{sl}=0 \tag{26}$$

$$\vec{n}\cdot\vec{q}=\frac{\pi\sqrt{3}}{6\,\epsilon_s^*}\varphi'\rho_s\epsilon_s g_0\sqrt{\theta}(\vec{v}_{sl})^2-\frac{\pi\sqrt{3}}{4\,\epsilon_s^*}(1-e_w^2)\rho_s\epsilon_s g_0\theta^{\frac{3}{2}} \tag{27}$$

here, $\vec{n}$ is the unit vector normal to wall surface, $\vec{v}_{sl}$ is the solid velocity at wall, δ is the angle of wall friction for the particle, ϕ' is the specularity coefficient and e_w is the coefficient of restitution at the wall.

3. RESULTS AND DISCUSSION

All the applications of the models presented in this study are from a 2D rectangular bin, 8 cm wide by 100 cm high, open at the top and a 1.4 cm width of the central orifice at the bottom. A high 5 cm region below the bin was included, see Fig.1. The numerical grid resolution of 1 mm and 2 mm along the horizontal and vertical directions, respectively, was used.

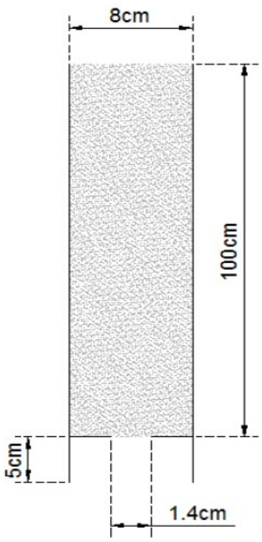

Figure 1. Hopper geometry

In the present work was applied an Eulerian–Eulerian approach coupled with the modification of a frictional model for gas-particle flow proposed by Srivastava and Sundaresan (2003), the S-S model, to study the influence of the presence of the air in granular flow from a hopper. The kinetic collisional stress was modeled applying the granular kinetic theory. Two different model to obtain the drag correlation were also studied, one proposed by Wen and Yu (1966) and the other presented by Syamlal et al. (1993), both in the presence of air. These investigations were realized with particle diameter of 0.1 cm. For numerical simulations, the open source code MFIX (Multiphase Flow with Interphase eXchange) Syamlal et al. (1993) developed at NETL ("National Energy Technology Laboratory") was used. This code describes the hydrodynamics, heat transfer and chemical reactions in fluid-solids systems.

The hopper geometry and all the parameters used in the simulations were the same presented by Srivastava and Sundaresan (2003).

The Tab. 1 shows the values of the parameters used in the simulations, both for in the presence and in an absence of the gas phase.

The Fig.2 shows the initial (Fig.2 (a)) and final (Fig.2 (b)) bulk density of the solid phase in a vacuum. The particles with a diameter of 0.1cm are represented in yellow. In the final instant can be observed that just some particles have been gone out from the hopper. It is important to emphasize that the same behavior happens with the presence of the air.

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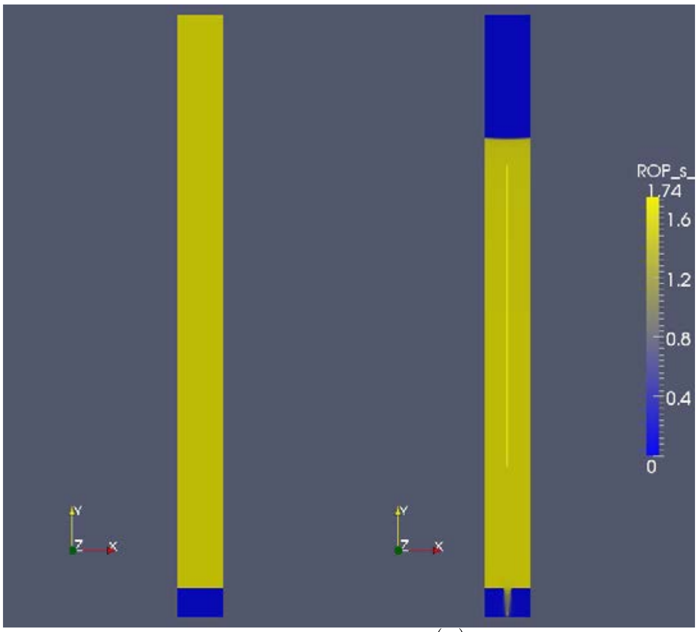
Table 1. Values of parameters used in the simulations.

Parameters	Values
ρ_g - gas density $[g/cm^3]$	1.3×10^{-3}
μ_g - gas viscosity $[g/cm.s]$	1.8×10^{-4}
ρ_{sm} - solids density $[g/cm^3]$	2.9
ϕ - angle of internal friction	28.5^o
δ - angle of wall friction	12.3^o
ϵ_{sm}^{*} - volume fraction at maximum packing	0.65
ϵ_{sm}^{min} - minimum solids fraction	0.5
ϕ' - specularity coefficient	0.25
e_{lm} - particle-particle coefficient of restitution	0.91
e_w - coefficient of restitution at wall	0.91
C_{flm} - coefficient of friction	0.1



$(a) t_i = 0s$ $(b) t_f = 3.5s$

Figure 2: Initial and final bulk density of the solid phase.

The discharge rate from a hopper in an absence and in the presence of the air was evaluated for monodisperse granular flow with particles diameter of 0.1 cm. The results from the Fig. 3 clearly indicates that the discharge rate is significantly higher in the presence of the air than that in an absence of one. That was expected due to the increase of the mobility of the particles in the presence of the air. Furthermore, this difference between the discharge rate is because of the influence of the drag force. In this comparison, the drag correlation used was proposed by Wen and Yu (1966).

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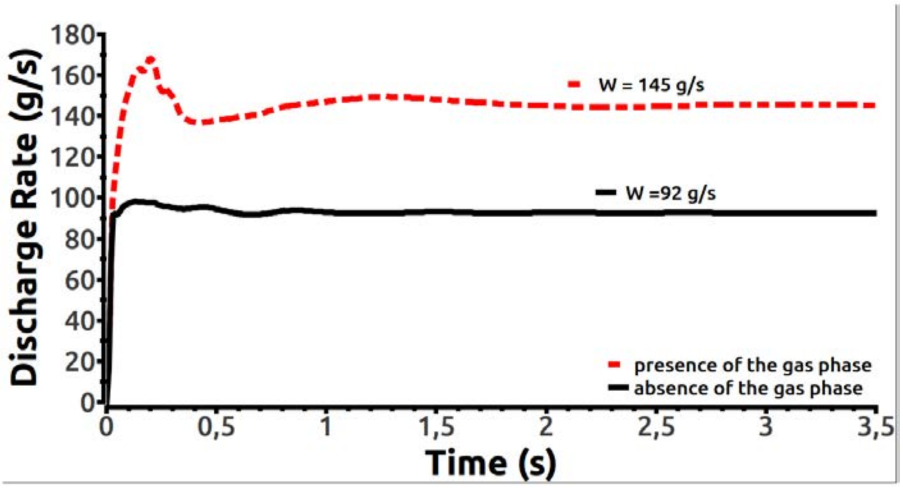

Figure 3: Discharge rate in a vacuum and in the presence of air.

In the Fig. 4 it was compared two different model to obtain the drag correlation, one proposed by Wen and Yu (1966) and the other presented by Syamlal et al. (1993), both in the presence of air. Observing the plots from this figure, there is not a great difference between these models. The discharge rate stability is practically the same in both of the models. Thus, in this simulation, the model used to calculate the drag correlation did not influence the discharge rate.

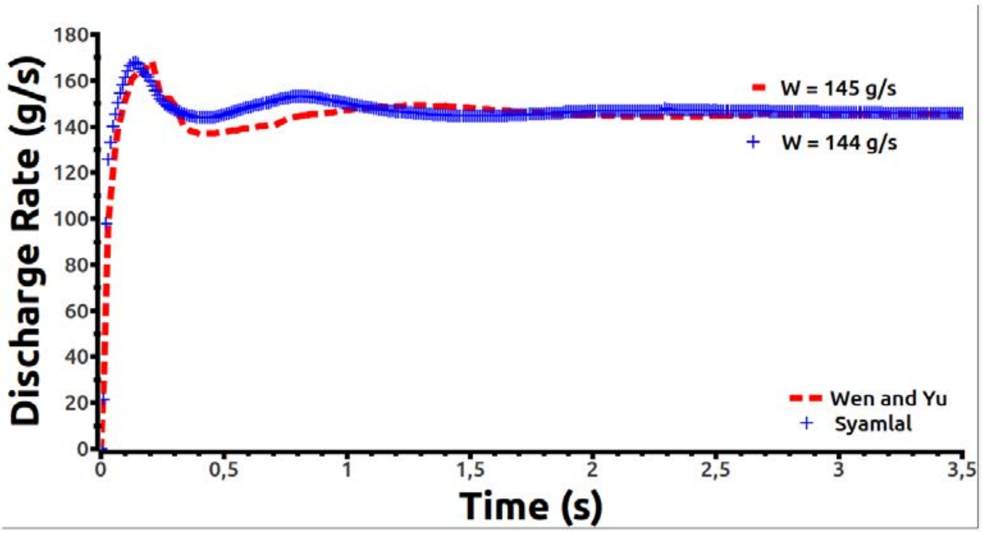

Figure 4: Discharge rate: Wen and Yu x Syamlal at al. drag correlations.

The numerical data achieved from the simulations was compared to the well-known Beverloo et al. (1961) correlation for the prediction of the discharge rate from hoppers and bin, which was simplified by Srivastava and Sundaresan (2003), for a 2D bin discharge as:

$$W = C \rho_i g^{1/2} D_o^{3/2} H \tag{28}$$

where, W is the discharge rate obtained from the simulations, C is an empirical constant denoted by Srivastava and Sundaresan (2003) in the range of $0.55 < C < 0.65$, $\rho_i = \rho_s \epsilon_s$ is the initial solids bulk density, g is the acceleration of the gravity, D_o is the width of the orifice and H is the thickness of the hopper, which in this study is taken as 1 cm.

In the Beverloo et al. (1961) correlation the values of W , ρ_i , g , D_o and H were used to calculate C .

For the presence of air, using the drag correlation presented by Wen and Yu (1966), the value of the empirical constant C are calculated as 1.93, when the drag correlation proposed by Syamlal et al. (1993) is used, $C = 1.91$. For the absence of air, the value of the empirical constant C are calculated as 1,22. In all these cases the values of C were computed larger than those proposed in the Beverloo et al. (1961) correlation, as a consequence of their bigger computed discharge rate.

4. CONCLUSION

In this study was analyzed the discharge rate from a hopper in a vacuum and in the presence of air for monodisperse granular flow with particles diameter of 0.1cm. For that, a frictional model for gas-particle flow proposed by Srivastava and Sundaresan (2003), the S-S model, was used. In additionally, two drag correlation were compared: one presented by Wen and Yu (1966) and the other proposed by Syamlal et al. (1993). The kinetic collisional stress was modeled applying the granular kinetic theory developed by Lun et al. (1984). For numerical simulations the open source code MFIX (Multiphase Flow with Interphase eXchange) Syamlal et al. (1993) developed by NETL ("National Energy Technology Laboratory") was used.

The results obtained from the comparison of the discharge rate in a vacuum and in the presence of air showed that the lower values of the discharge rate were computed in a vacuum as compared to the presence of air. That was expected due to the fundamental difference between these two situation, when the air is presented, there is an increase of the mobility of the particles, and consequently, the discharge rate are higher.

Two different model to obtain the drag correlation were compared, one proposed by Wen and Yu (1966) and the other presented by Syamlal et al. (1993), both in the presence of air. The discharge rate stability is practically the same in both of the models, which bring the conclusion that the model used to calculate the drag correlation does not influence the discharge rate.

These results were also compared with an empirical correlation proposed by Beverloo et al. (1961) and even though these were computed with larger values of the discharge rate, the comparison exhibited a tolerable agreement for all cases.

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7. RESPONSIBILITY NOTICE

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