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MODELLING AND IMPLEMENTATION OF A TENDON BASED STWEART PLATFORM

Jorge Audrin Morgado de Gois, audrin@ime.eb.br
Alexandre Back e Travi, eng.travi@hotmail.com
Instituto Militar de Engenharia, Praça Gen Tibúrcio, 80, Praia Vermelha – Rio de Janeiro, RJ, Brasil

Abstract. *Stweart-platforms are six-degree-of-freedom parallel manipulators, usually hydraulically or electrically actuated. This work presents the modeling of such a platform, but instead of conventional actuators, here cables - or tendons - are used to drive it, what means that only traction forces can be imposed on the platform. This characteristic leads to the appearance of non-holonomic constraints in the model, what leads a non-standard procedure to model the system. Besides, a simple prototype is constructed based on step-motors and only four tendons, what makes the system under-actuated, which requires a special procedure to the determination of the traction on the tendons.*

Keywords: *Stweart-Platform,tendon-based,kinematics, dynamics*

1. INTRODUCTION

In recent years, there has been much interest in parallel-link kinematical structures (also known as parallel manipulators – PM) as the basis for six degree-of-freedom machine tools, which are even available commercially (Figure 1 – left). Interest in parallel-link kinematical structures is motivated by their high rigidity and excellent positioning capability if compared to those of serial-link kinematical structures. Moreover, due to placement of actuators on the base, each actuator does not need to support or drive the mass of other actuators. As a result, they can handle heavy loads and are energy-efficient.

Based on the idea of parallel manipulators, tendon-based parallel manipulators (TBPM, also Known as tendon-based Stewart platform) have been proposed and attract more and more attention in the last decades (Figure 2 – right). Here, a movable platform is connected to the fixed base by tendons instead of rigid links as in conventional parallel manipulators. The tendons are rolled up on winches attached to the base; hence the on1y moving parts are the tendons, winches and the platform. They are very energy-efficient because the mass of the moving parts is extremely low in comparison to that of a manipulator with rigid 1inks. Therefore, they are appropriated for handling heavy loads like cranes and can achieve high acceleration and velocity. They can be designed in extremely large scale as well as in micro-scale applications. Further merits of this type of manipulators are their low weight, flexibility and maneuverability. They can be applied in diverse fields such as shipbuilding, telescopes positioning and camera systems for stadiums where robot arms with heavy rigid links are not preferable and accuracy is not of primary importance.

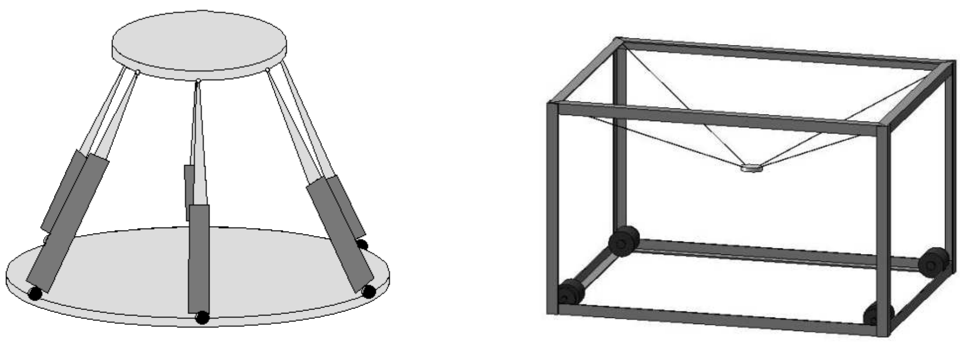

Figure 1. Parallel manipulators: with rigid links (left) and a tendon-based parallel manipulator (right)

2. THE SYSTEM

The main problem in applications of PM is the restricted workspace if compared to that of manipulators with serial links. This disadvantage appears in TBPM too, and becomes more complicated due to the fact that tendons can only pull. Unlike the PM with rigid links, where the workspace is restricted by limits of joints and singularity, the technically usable workspace of TBPM is mainly restricted by the condition that tendon forces must be positive. In general, the technically usable workspace of TBPM is relatively small and depends highly on its geometrical configuration. Therefore, workspace is an important issue for TBPM applications. To design a TBPM, it is necessary to evaluate the form and size of the workspace. Since the workspace of TBPM cannot be described in a closed-form, we can only use some conditions (criterion) to determine whether a posture belongs to the controllable workspace (Verhoeven and Hiller, 1998).

2.1. TBPM Classification

Based on the definition of *redundancy* in kinematics of robotics, TBPM can be classified into three types:

a) Incompletely Kinematic Restrained Manipulators (IKRM). In this case, the number of tendons m is less than the number of degree of freedom (DOF) of the platform n, i.e. $m < n$. The position of the platform is not completely specified kinematically by the tendon lengths (Maier and Woernle, 1999).
b) Completely Kinematic Restrained Manipulators (CKRM). Here, the number of tendons satisfies $m=n$, the position of the platform is kinematically determined by the tendon lengths. But because a tendon can only pull but not push an object, additional dynamic conditions are required to place the platform. (Albus, et al., 1992). .
c) Redundantly Actuated Manipulators (RAMP). In this case, the number of tendons m satisfies $m>= n+1$, the manipulators have actuation redundancy. The position of the platform is completely specified by the lengths of tendons without dynamic conditions. A system with actuation redundancy can improve the manipulability due to possibly more advantageous geometries; meanwhile redundancy improves safety during breaking, what is of primary importance for TBPM applications where human transportation is involved. The disadvantage of this type is the possibility of auto collision of tendons (Verhoeven, 2004).

This paper shows an IKRM, where four tendons connect the platform to the base. To drive all the degrees-of-freedom of the platform, the weight is treated as an external force on the system. Therefore, an overview of the mechanism´s work is given, where the kinematics and dynamics of the problem is presented.

(a) (b) (c)

Figure 2. Classes of TBPM: a) IKRM ($m > n$), b) CKRM ($m = n$), c) RAMP ($m \geq n$+1)

A TBPM, where the platform is directly driven by tendons, represents a parallel kinematical mechanism. If only the mechanical system is considered, the system can be treated as a multi-body system, which consists of three basic components: rigid bodies, joints and forces (Hiller and Kecskeméthy, 1987, 1994). In comparison with parallel kinematical manipulators with rigid 1inks the unique characteristics of TBPM are the flexible tendons carrying forces in only one direction.

In order to solve the kinematical problem, we treat TBPM as a conventional parallel manipulator with rigid 1inks. In other words, we treat the tendons as prismatic joints. With this model the motion of the platform is determined by the motion prescribed by the drivers. We note that the mass of tendons is neglected in this paper, since it is much smaller than that of the platform.

3. KINEMATICS

For a TBPM, the kinematical problem refers to the determination of relations between platform posture and actuator variables (tendon lengths or revolute angles of motors). In fact, the solutions of the kinematical problem can only give relations between platform posture and the lengths of vectors, which connect base points and connection points of respective tendons (Fang, 2004). If tendons are tensional, the lengths of vectors are just the lengths of tendons. In this section we assume that the tendons are always tensional so that we can treat tendons as two-side constraint and the whole system is regarded as a general multi-body system for solving the kinematics problem. The posture of the platform or actuator variables can be selected as independent input variables of the system.

Since the tendons are assumed to be massless due to their very small weight, the tendon profile deflected by gravity is ignored in the following discussion. In large TBPM, where long and heavy tendons are used, the tendon profile deflected by gravity must be considered. In the following sections, the inverse kinematics is discussed with the tendon lengths chosen as actuator variables.

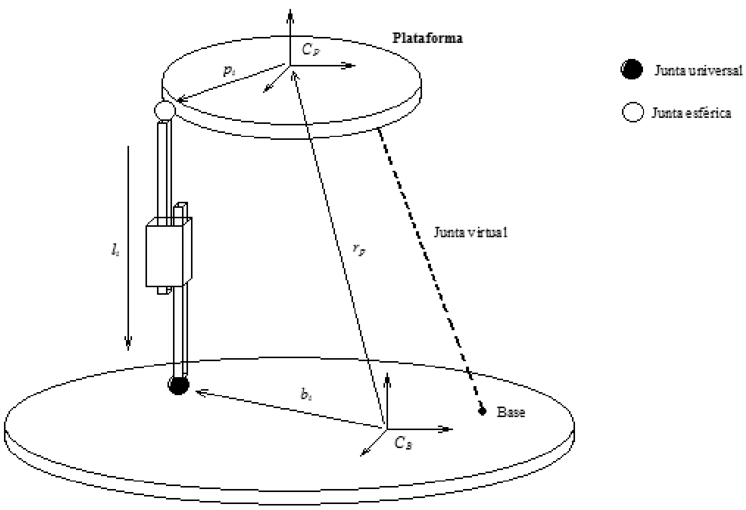

Figure 3. Kinematics of a TBPM.

3.1. Inverse Kinematics

The inverse kinematics of TBPM consists of the determination of the tendon lengths or actuator motion corresponding to a given platform position and orientation. This allows the tendon lengths or motor angle to be expressed as a function of the platform position and orientation. The solution to the inverse kinematics problem is of fundamental importance in transforming motion specifications, assigned to the platform in the operational space, into corresponding actuator space motion that allows execution of the desired motion (motion control in joint space).

Referring to Figure 3, an inertial frame *CB* is fixed on the base and another moving coordinate system *Cp* is fixed at center of gravity of the platform. The location of the origin of body- fixed frame is described by

$$r_p = \left[r_x, r_y, r_z\right]^T \tag{1}$$

The orientation is defined by the CARDAN (BRYANT) angles, $\theta_p = [\psi, \upsilon, \varphi]^T$ which is obtained by rotating the frame sequences about the Z-axis by angle , about the Y -axis by angle ϑ, and about the X -axis by angle φ. Sequentially, the position and orientation (posture) of the platform is specified by Cartesian coordinates:

$$x = \left[{r_p}^T, {\theta_p}^T\right]^T \tag{2}$$

Thus, m tendon lengths are defined as dependent variables $q = [q_1, q_2, \dots, q_m]^T$, which are described by vectors vetores $l_i = [l_{i,x}, l_{i,y}, l_{i,z}]^T$, *i*=1,...,*m*. The vector b_i is the vector of base point of *ith* tendon in the inertial system C_B, the vector p_i denotes the vector of the connection point of *ith* tendon at the platform with respect to body- fixed coordinate system *Cp*. By means of vector analysis we can easily obtain the explicit constraint equations for determining the dependent coordinates. To determine the tendon lengths, the following equations can be employed:.

$$l_i = b_i - r_p - p_i \quad \textit{where } i = 1, \dots, m. \tag{3}$$

The above equations can be described in the inertial system CB as:

$${}^B l_i = {}^B b_i - {}^B r_p - {}^B R_p \; {}^P p_i \tag{4}$$

The rotation matrix ${}^B R_p$ is:

$${}^B R_p = \begin{bmatrix} C\psi C\vartheta & C\psi S\vartheta S\varphi - S\psi C\varphi & C\psi S\vartheta C\varphi + S\psi S\varphi \\ S\psi C\vartheta & S\psi S\vartheta S\varphi + C\psi C\varphi & S\psi S\vartheta C\varphi - C\psi S\varphi \\ -S\vartheta & C\vartheta S\varphi & C\vartheta C\varphi \end{bmatrix} \tag{5}$$

where Cψ = cosψ and Sψ = sinψ

Then the length of the i-th tendon can be calculated from components of l_i.

$$q_i = |l_i| = \sqrt{l_{x,i}^2 + l_{y,i}^2 l_{z,i}^2} \quad where \;\; i=1,...,m \, . \tag{6}$$

Considering the above expression, comes:

$$q = \phi_x(x) \tag{7}$$

The kinematical velocity and acceleration of tendons can be obtained through time derivative of Eq. (7) and can be expressed form:

$$\dot{q} = \underbrace{\frac{\partial \phi_x}{\partial x}}_{J_x} \dot{x} \tag{8}$$

and

$$\ddot{q} = J_x \ddot{x} + j_x \dot{x} \;, \tag{9}$$

where J_x is a $(m \times n)$ Jacobian-matrix.

With the angular velocity of the platform in the inertial system C_B represented by $\omega = [\omega_x, \omega_y, \omega_z]^T$, from rigid body kinematics it is:

$$\omega = H_B^{-1} \dot{\theta} \tag{10}$$

where $H_B^{-1} = \begin{bmatrix} 0 & -S\psi & C\psi C\vartheta \\ 0 & C\psi & S\psi C\vartheta \\ 1 & 0 & -S\vartheta \end{bmatrix}, and\; \dot{\theta} = \begin{bmatrix} \dot{\psi} \\ \dot{\vartheta} \\ \dot{\varphi} \end{bmatrix}.$

4. DYNAMICS OF COMPONENTS

The dynamic model is very useful for simulation of motion, analysis of manipulator structure and design of control algorithms. Simulating manipulator motion allows testing control strategies and motion planning techniques without the need to have a physically available system. The computation of forces and torques required for the execution of typical motions (inverse dynamics) provides useful information for choice of actuators.

In general, a TBPM consists of a parallel mechanism and driving units. The parallel mechanism consists of a movable platform and a number of tendons that connect the platform to a fixed base. Considering very small mass of tendons in comparison with that of the platform, tendon mass will be neglected in the dynamic model. Therefore, the parallel mechanism is treated as a simple free-body system, which consists only of a platform. The tendons are considered as elastic elements that act on the platform along respective orientation of the tendons.

Driving units are mounted on the frame. Generally, each of them consists of pulley, drum, reduction gear and a motor. Without considering elasticity of driving unit and friction on the drum and reduction gear, the dynamic behaviors is only related to motor, reduction gear and drum which can be dealt with together (Hiller and Keskeméthy, 1994).

4.1. Platform

Hence the posture of platform $x = \left[r_x, r_y, r_z, \psi, \theta, \vartheta\right]^T$ and its time derivative are employed to describe the state of system, the resultant wrench of all tendon tensions applied on the platform, is computed by:

$$w_{tendon} = \begin{bmatrix} f_{sum} \\ \tau_{sum} \end{bmatrix} = A^T f, \tag{11}$$

where f_{sum}, τ_{sum} are the resultant force and torque, respectively; A^T denotes the structure-matrix and f denotes the tensions of all m tendons. Using Newton's laws, the equations of motion for the translation and rotation can be written in the following form:

$$m_p \ddot{r}_p = \begin{bmatrix} 0 \\ 0 \\ m_p g \end{bmatrix} + f_{sum} \tag{12}$$

$$I\dot{\omega} + \omega(I\omega) = \tau_{sum}. \tag{13}$$

where:

m_p: mass of platform
I: inertia tensor defined with respect to the inertial system C_B, which is an expression of rotation angles,
r_p:: vector of the location of the origin of platform with respect to the inertial system C_B,
ω vector of the angular velocity of platform in the inertial system C_B.

From Eq. (10) the angular velocity of platform in the inertial systemC_B is

$$\dot{\omega} = H_B^{-1}\ddot{\theta} + \dot{H}_B^{-1}\dot{\theta}\ , \tag{14}$$

Considering Eq. (10), Eq. (11), Eq. (12), Eq. (13) and Eq. (14) we can obtain the equations of motion of the platform

$$\begin{bmatrix} m & 0 \\ 0 & H_B^{-T}\, I\, H_B^{-1} \end{bmatrix} \begin{bmatrix} \ddot{r}_p \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \left(\left(H_B^{-T}\, I\, H_B^{-1}\right) + \left(H_B^{-T} H_B^{-1} \tilde{\Theta}\, I\, H_B^{-1}\right) - \left(H_B^{-T} \tilde{\Theta}\, H_B^{-1}\, I\, H_B^{-1}\right)\right)\theta \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = A^T f \tag{15}$$

4.2. Motor

From the modeling viewpoint, a DC motor equipped with a position sensor can be described by differential equations. The electric balance of the armature in the i[th] motor is usually described by the equations

$$L_{a,i}\dot{I}_{a,i}(t) + R_{a,i}I_{a,i}(t) = U_{a,i} - U_{g,i} \quad , \ \text{i=1, ... , m} \tag{16}$$

$$U_{g,i} = K_{g,i}\dot{\theta}_i \quad , \ \text{i=1, ... , m} \tag{17}$$

where $U_{a.i}$ and $1_{a,i}$ denote the armature voltage and current, respectively, parameters $R_{a,i}$ and $L_{a,i}$ are respectively the armature resistance and inductance and $U_{g.1}$ denotes the back electromotive force which is proportional to the angular velocity θ_i through the voltage constant $k_{v,i}$ that depends on the construction details of motor as well as on the magnetic flux of coil (Sciavicco and Siciliano, 2000).

4.3. Integration of Subsystems

In general, the state of a dynamic system can be described by state variables *z*, consisting of state variables of subsystems:

$$z = [z_1{}^T, \dots, z_s{}^T]^T \ \ \text{sendo}\ z_i = \left[z_{i,1}, \dots, z_{i,n_i}\right]^T \quad , \quad i = 1\ , \dots\ , s \tag{18}$$

where *s* denotes the number of subsystems and *n* the number of state variables of the subsystem *i*. The dynamics *of* the complete system is described by nonlinear differential equations of first order in terms of state variables:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \vdots \\ \dot{z}_s \end{bmatrix} = \begin{bmatrix} f_1(z_1, \dots, z_s) \\ f_2(z_1, \dots, z_s) \\ \vdots \\ f_s(z_1, \dots, z_s) \end{bmatrix} \tag{19}$$

To describe the state of complex mechanical system, the state variables can be chosen from the posture of platform and the motor angles, since solving inverse kinematics problem is relative easy comparing to its forward kinematics for TBPM:

$$z_{mec} = \left[x^T, \theta^T, \dot{x}^T \dot{\theta}^T\right]^T. \tag{20}$$

The electric dynamic behavior of motor system can be treated as a electronic subsystem and is closely coupled with the mechanical subsystem of the motor, it is given by armature current I_a.

$$z_{eletr} = I_a \tag{21}$$

Writing the motion equations of respective mechanical and electrical subsystems in a state form, with

$$z = \begin{bmatrix} z_{mec} \\ z_{eletr} \end{bmatrix} \tag{22}$$

We can describe the dynamic behavior of the complete system by coupled nonlinear differential equations of first order:

$$\dot{z} = (z, u) \tag{23}$$

5. COCLUSIONS

In this work Stewart platforms, tendon-based platforms, were discussed and different types of platform according to the amount of degrees of freedom and actuation were presented.

The geometry shows as a fundamental factor in defining the kinematics of the manipulator been sometimes necessary the using of forcing to a complete definition of the movement. Since this was discussed briefly the dynamics of TBPM, doing the equivalent of the tendons with rigid actuator. Was also discussed a simple model of actuator (electric motor and its integration to the system).

There was the complexity of the systems actuate by tendons, which is necessary a careful design of the kinematics so that the desired movement became possible as well as modeling dynamic of the integrated electrical and mechanical parts. In spite of the difficulties such manipulators shows a very promising due to various configurations and dimensions of the workspace.

6. ACKNOWLEDGMENTS

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