A NEW TECHNIQUE FOR STRUCTURAL MONITORING BASED ON STATE OBSERVERS IN MODAL DOMAIN

Aldemir Aparecido Cavalini Jr, aacjunior@aluno.feis.unesp.br
Gilberto Pechoto de Melo, gilberto@dem.feis.unesp.br
Vitor Ramos Franco, vrfranco@aluno.feis.unesp.br
Camila Gianini Gonsalez, cggonsalez@aluno.feis.unesp.br
Vicente Lopes Jr, vicente@dem.feis.unesp.br

GMSINT – Grupo de Materiais e Sistemas Inteligentes, Mechanical Engineering Department, Faculdade de Engenharia de Ilha Solteira – UNESP, Av. Brasil 56, Ilha Solteira, SP, Brazil, ZIP CODE 15385000, www.dem.feis.unesp.br/gmsint

Abstract. *The aim of this paper is to evaluate a new structural monitoring technique based on state observer in modal domain. This technique allows to find the vibration mode os the system more affected by damage presence. The paper concludes with an experimental application in a clamped-free-free-free aluminum plate with PZT actuators coupled. In order to analyze the new approach, the excitation frequency and the measurement positions are changed. The results lead to the conclusion that the Modal State Observer is a potential useful SHM (Structural Health Monitoring) tool. The plate model considers the electromechanical coupling between the host structure and the PZT actuator.*

Keywords*: Structural Health Monitoring, PZT actuator and Modal State Observer.*

1. INTRODUCTION

Many aerospace and civil infrastructure systems are working beyond their design life. However, it is envisioned that they will remain in service for an extended period. SHM is one of the enabling technologies that will make this possible and involve a large number of non destructive inspection techniques for detecting local damage and incipient failure in critical structures. Not surprisingly, the SHM techniques have recently received increased attention (Inman *et al.*, 2005).

Rytter (1993) classified the various Structural Health Monitoring techniques based on four damage detection levels: detection of the damage presence (level 1), geometric location of the damage (level 2), quantification of the severity of the damage (level 3) and prediction of the remaining service life of the structure (level 4). This paper deals with the level 1 of SHM using the state observer approach.

Different techniques can be applied for structural monitoring, as for instance, that one using methodologies in the modal domain based on modal forms of the structure (Maia *et al.*, 2003). Recently, a new SHM technique was proposed: the Modal State Observer (Cavalini *et al.*, 2008). This observer associate the already know state observers with features obtained in the modal domain. Thereby, the new technique is capable detects the vibration modes that are more affected by damage presence. This is possible because unlike the traditional state observers, the Modal State Observer estimates the modal state vector of the structure. Additionally, following a worldwide trend in SHM area (Carden and Fanning, 2004), in this approach only the undamaged condition structural model is used. The dynamic model was identified by subspace method.

Great part of the identification methods concerns with computing polynomial models, which, typically, give rise to numerically ill-conditioned mathematical problems, especially for Multi Input Multi Output systems (Van Overschee and De Moor, 1996). Numerical algorithms for subspace state space system identification (N4SID) are then viewed as optimal alternatives. This approach is advantageous, especially for high order multivariable systems, where the parameterization is not trivial. With N4SID algorithms only the order of the systems is needed and it can be determined through inspection of the dominant singular values.

2. STRUCTURAL MODELING

The approach is demonstrated theoretically through an analytical model. It is possible to describe the dynamical behaviour of the structure as (Gawronski, 1998):

$$\ddot{q}(t) + M^{-1}D_a\dot{q}(t) + M^{-1}Kq(t) = M^{-1}B_0u(t) \quad (1)$$

$$y(t) = Cq(t) \quad (2)$$

where q(t) is the displacement vector, u(t) is the input vector, y(t) is the output vector, M is the n x n mass matrix, D_a is the n x n damping matrix, and K is the n x n stiffness matrix. B_0 is the n x s input matrix and C is the r x n output matrix. The mass matrix is positive definite, and the stiffness and damping matrices are positive semi-definite, n is the number of degrees of freedom of the system (linearly independent coordinates describing the finite-dimensional structure), *r* is the number of outputs and *s* is the number of inputs.

Using the classic procedure of modal analysis (Maia *et al.*, 1997), it is possible to write the equations of motion in modal coordinates, $q_m(t)$. Thus, the second order modal model is given by:

$$q(t) = \Phi q_m(t) \tag{3}$$

$$\ddot{q}_m(t) + 2Z\Omega\dot{q}_m(t) + \Omega q_m(t) = B_m u(t) \tag{4}$$

$$y(t) = C_m q_m(t) \tag{5}$$

where Φ is the modal matrix, $\Omega = (M_m^{-1}K_m)^{1/2}$ is the matrix of natural frequencies and Z is the matrix of damping coefficients (ζ_i), given by:

$$Z = \frac{M_m^{-1}D_m\Omega^{-1}}{2} \tag{6}$$

The matrices M_m, K_m and D_m are diagonal matrices of modal mass, stiffness and damping, respectively, which are given by:

$$M_m = \Phi^T M\Phi \tag{7}$$

$$K_m = \Phi^T K\Phi \tag{8}$$

$$D_m = \Phi^T D_a\Phi \tag{9}$$

The matrix D_a is assumed to be proportional to mass and stiffness matrices, so that:

$$D_a = \alpha M + \beta K \tag{10}$$

with α and β constants. The matrix B_m in Eq. (4) is the input modal matrix or participation modal matrix given by:

$$B_m = M_m^{-1}\Phi^T B_0 \tag{11}$$

C_m is the output modal matrix given by:

$$C_m = C\Phi \tag{12}$$

The motion equations, Eq. (4) and Eq. (5), can be written in state space form by vector-matrix format through the triple A_m, B_m and C_m (Gawronski, 1998). It allows the equations to be manipulated more easily.

$$\dot{q}_m(t) = A_m q_m(t) + B_m Q(t) \tag{13}$$

$$y_m(t) = C_m q_m(t) \tag{14}$$

where, Q(t) is the modal input vector defined by $Q(t)=B_m u(t)$ and $y_m(t)$ is the modal output vector. A_m is the modal dynamic matrix, B_m is the modal input matrix and C_m is the modal output matrix.

The modal state space representation is characterized by the block diagonal dynamic, input and output matrices A_m, B_m and C_m, respectively (Gawronski, 1998).

$$A_m = \text{diag}(A_{mi}) \tag{15}$$

$$B_m = \begin{bmatrix} B_{m1} \\ B_{m2} \\ \vdots \\ B_{mn} \end{bmatrix} \tag{16}$$

$$C_m = \begin{bmatrix} C_{m1} & C_{m2} & \cdots & C_{mn} \end{bmatrix} \tag{17}$$

where A_{mi}, B_{mi} and C_{mi} are 2 x 2, 2 x *s* and *r* x 2 blocks, respectively, and i = 1, 2,…, n are the vibration modes of the system. These blocks can take several different forms and also it is possible to convert from one form to another by a linear transformation.

3. STATE OBSERVER

The state observer concept for a dynamic system was introduced by Luenberger in 1964 with the demonstration of how the known inputs and outputs of a system can be used to construct an estimative of the system state vector. Its dispositive of state reconstruction was called Luenberger Observer. The complete demonstration of the state vector reconstruction, for a linear system, is presented in Luenberger 1966.

However, an state observer for the observable physical system S(q, y, u) with state q, output y and input u, is an numerical dynamic system S′ (q′, y, u) with the following property: the estimated output q′ converges to state q of the S system, independently of the input u and state q, as shown in Fig. 1.

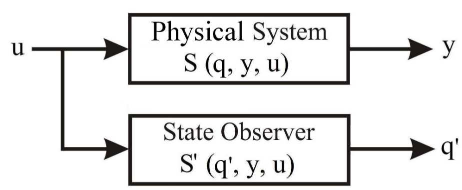

Figure 1. State Observer.

The Eq. (18) and (19) show the mathematical definition of the state observer (Luenberger, 1966). The linear and invariant time state space system of the Eq. (14) and Eq. (15) is considered.

$$\dot{q}'(t) = (A - LC)q'(t) + Bu(t) + Ly(t) \tag{18}$$

$$y(t) = Cq(t) \tag{19}$$

where q′(t) is the estimated state vector, A – LC is called observer matrix and L is the observer gain matrix. In this work, Kalman Filter was used to obtain the gain matrix L (Welch and Bishop, 2006).

4. THE PROPOSED SHM TECHNIQUE

A simple explanation about the proposed SHM technique is shown in the diagram of the Fig. 2. One can see that the structural model, in modal domain and block diagonal representation, is identified only to structural undamaged condition. However, the on–line condition is informed to the Modal State Observer by the output response y(t) measured directly in the structure.

The modal state vectors estimated by the observer, states for undamaged and unknown conditions, are compared in different time periods. With any deviation, the vibration mode more affected by the damage is detected. In practice, the technique detects that through comparing modal displacements and velocities of each vibration mode, both referents to the output measurement structural point. The mathematical definition of the Modal State Observer is shown in Eq. (20) and Eq. (21).

$$\dot{q}_m'(t) = (A_m - LC_m)q_m'(t) + B_m u(t) + Ly(t) \tag{20}$$

$$y(t) = Cx(t) \tag{21}$$

where $q_m'(t)$ is the estimated modal state vector. One can see that the variables u(t) and y(t) are in time domain. It can be demonstrated that the comparison between the states for undamaged and unknown conditions gives compatibility between these variables and the Modal State Observer equation.

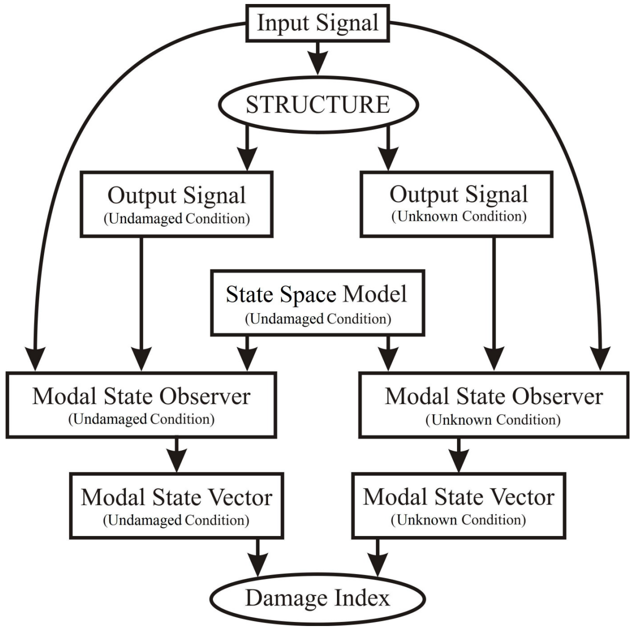

Figure 2. Schematic representation of the damage detection technique using the modal state observer.

5. EXPERIMENTAL APPLICATION

The proposed SHM technique was experimentally applied in a clamped–free–free–free aluminum plate as show the Fig. 3. The physics and geometric properties are shown in Tab. 1. The output signals were obtained with accelerometers model 352C22 PCB Piezotronics®. The input excitation was applied by PZTs actuators (properties in Tab. 2) from 100 Hz and 500 Hz sinusoidal electrical current (20v and 50mA maximum), separately. In the Fig. 3, the position of the accelerometers (ACCEL 1 and ACCEL 2) and PZTs actuators (PZT 1 and PZT 2) are shown. The plate model was identified from an average of five measurements performed by impulsive voltage (200v and 50mA maximum) in PZT1 and PZT 2, separately. The dSpace® DS1103 CONTROL BOARD was used for data acquisition.

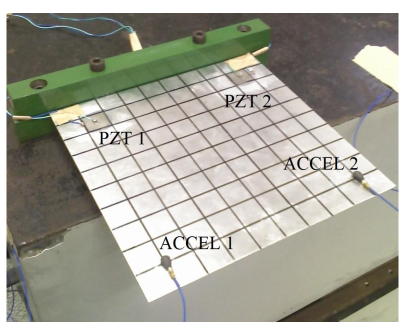

Figure 3. Aluminum plate in a clamped–free–free–free condition.

Table 1. Physics and geometric properties of host structure.

Property	Value
Length	0.2 m
Width	0.2 m
Thickness	0.0015 m
Young's Modulus	70 GPa
Density	2710 Kg m^{-3}

Table 2. Physics and geometric properties of the PZT actuators based on material designation PSI-5H4E (*Piezo Systems®, Inc.*).

Property	Value
Length	0.2 m
Width	0.2 m
Thickness	0.00027 m
Young's Modulus	62 GPa
Density	7800 Kg m^{-3}
Dielectric Constant	650e-12 m V^{-1}
Dielectric Permittivity	19e3 C m^{-2}

Damage cases were evaluated by loosening screws of the clamp (Fig. 4). In the first case (FD), the screw 2 was totally loosed, the screws 2 and 3 were loosed in the second one (SD). Due to the system observability, only the first and the second vibration modes (31 Hz and 73 Hz, respectively) were considered.

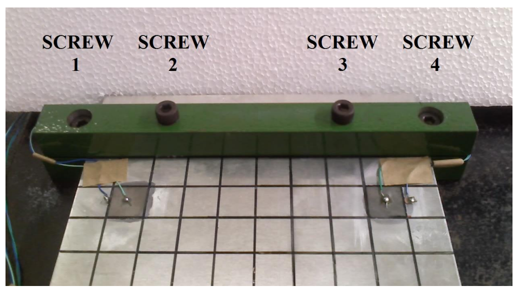

Figure 4. Screws of the clamp.

For 100Hz electrical current frequency in PZT 1 and the ACCEL 1, Fig. 5 shows the probability densities of the output signals for the undamaged plate (UD) and for the two cases of damage. One can see that the effect of both damages, FD and SD, are recognized in the outputs with the FD influence being more expressive than SD.

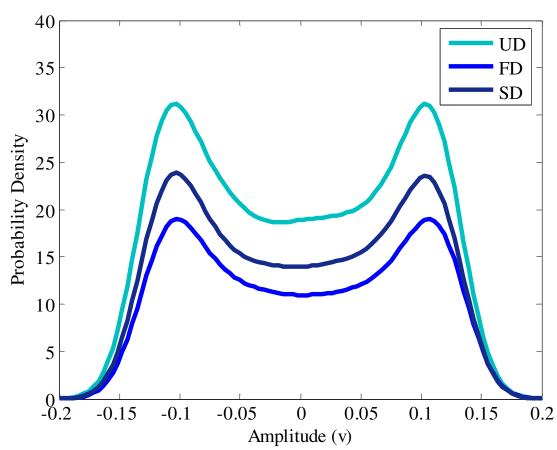

Figure 5. Probability densities of the output signals (100Hz – PZT 1 – ACCEL 1).

Figure 6 shows the difference between the RMS (Root Mean Square) values of the UD and the damages FD and SD, evaluating the two first vibration modes of the plate through the modal displacement estimated signals. Figure 7 shows similar results for velocity modal estimated signals. One can see that the second mode is more affected by both damages considering the modal displacement and velocity. As expected, the difference between the RMS values for the first damage is bigger than for the second one (see Fig. 5).

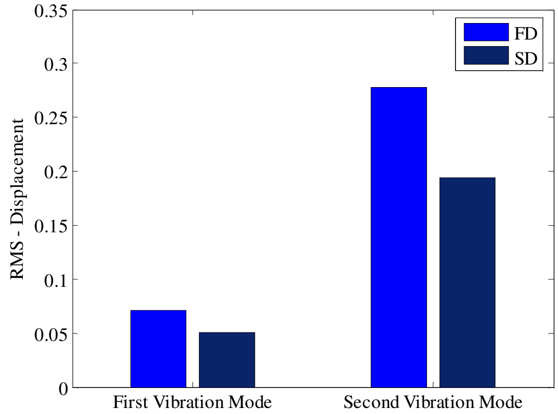

Figure 6. Vibration modes more affected evaluated by modal displacements (100Hz – PZT 1 – ACCEL 1).

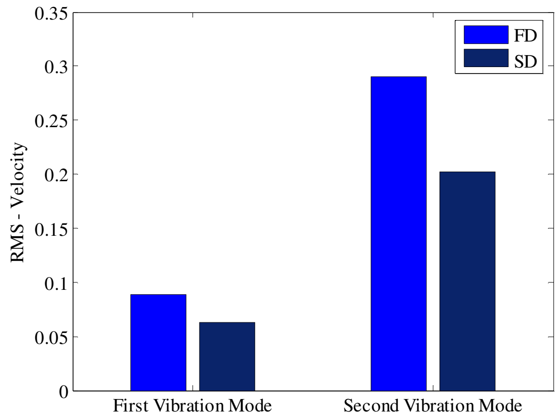

Figure 7. Vibration modes more affected evaluated by modal velocities (100Hz – PZT 1 – ACCEL 1).

Figure 8 shows probability densities of the output signals for the UD, FD and SD conditions, using PZT 1 with 100Hz frequency and ACCEL 2. In this evaluation, the effect of both damages is observed in the outputs. One can see that the FD influence is more expressive than SD in the output signal.

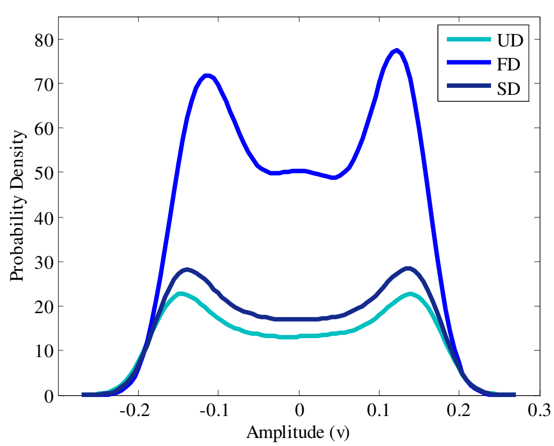

Figure 8. Probability densities of the output signals (100Hz – PZT 1 – ACCEL 2).

Figure 9 shows the difference between the RMS values of the UD and the damages FD and SD, for the two first vibration modes of the plate, comparing the modal displacement. Modal velocity results are shown in Fig. 10. We can see that the second mode is more affected by both damages using the ACCEL 2. As expected, the index related with the first damage is bigger than the second one (see Fig. 8).

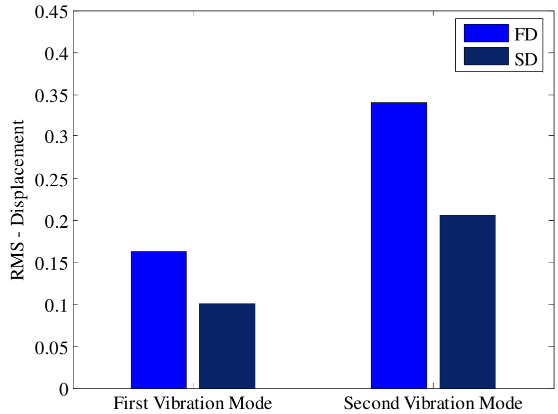

Figure 9. Vibration modes more affected evaluated by modal displacements (100Hz – PZT 1 – ACCEL 2).

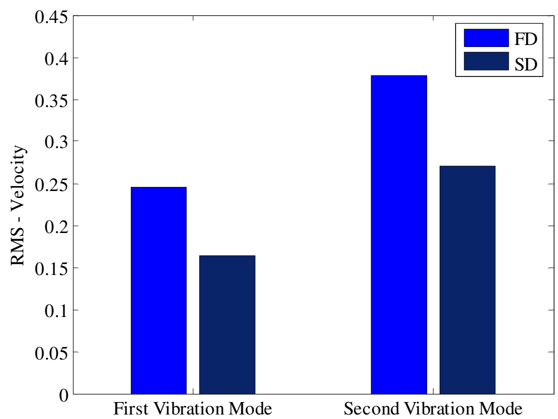

Figure 10. Vibration modes more affected evaluated by modal velocities (100Hz – PZT 1 – ACCEL 2).

Using PZT 1 with 500 Hz frequency and the ACCEL 1, Fig. 11 shows the probability densities of the output signals for the UD, FD and SD conditions. It is possible to observe that the effect of both damages, FD and SD, are recognized with the FD influence being more expressive than SD. Similar results were found for 100 Hz frequency (see Fig. 5).

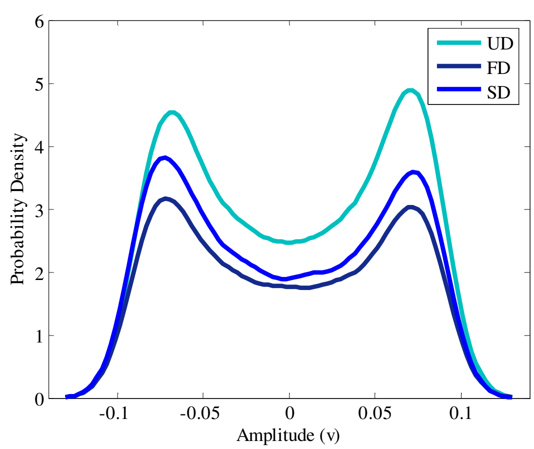

Figure 11. Probability densities of the output signals (500Hz – PZT 1 – ACCEL 1).

Figure 12 shows, for modal displacement, the difference between the RMS values of the UD and the damages FD and SD, for the two first vibration modes of the plate. For modal velocity signals, similar results are found (Fig. 13). It is possible to observe that the second mode is more affected by both damages considering the modal displacement and velocity. The same result was found for 100 Hz sinusoidal force (see Fig. 6 and Fig. 7). As expected, the difference between the RMS values for the first damage is bigger than for the second one (see Fig. 11).

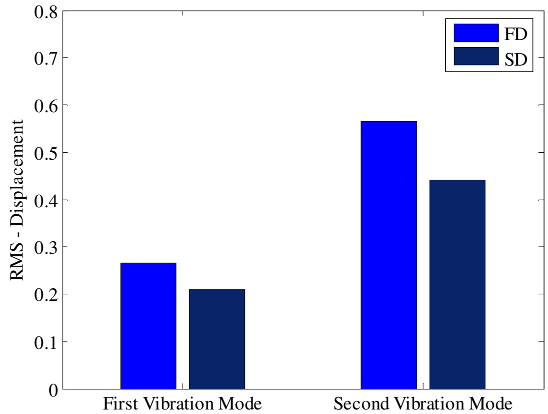

Figure 12. Vibration modes more affected evaluated by modal displacements (500Hz – PZT 1 – ACCEL 1).

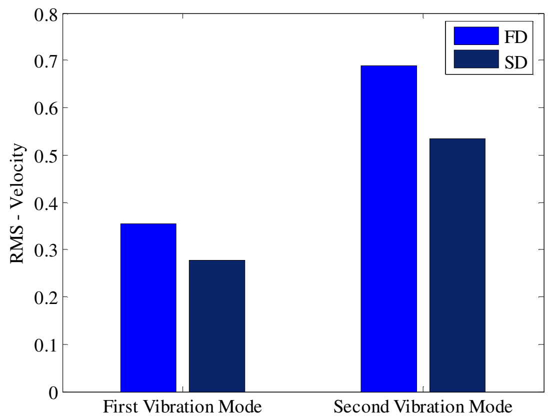

Figure 13. Vibration modes more affected evaluated by modal velocities (500Hz – PZT 1 – ACCEL 1).

Figure 14 shows the probability densities of the output signals for the UD, FD and SD conditions, using the ACCEL 2. Observe that now the FD and SD have similar behaviour.

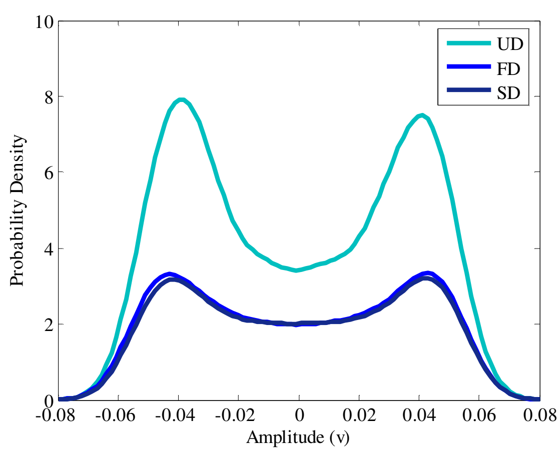

Figure 14. Probability densities of the output signals (500Hz – PZT 1 – ACCEL 2).

Figure 15 shows the difference between the RMS values of the UD and the damages FD and SD, for the two first vibration modes of the plate, comparing the modal displacement. The modal velocity signals are compared in Fig. 16. It is possible to observe that the first mode is more affected by both damages when analyzed with the modal displacement. The results are similar with the modal velocity. As expected, the index related with the first and second damage is practically the same (see Fig. 14).

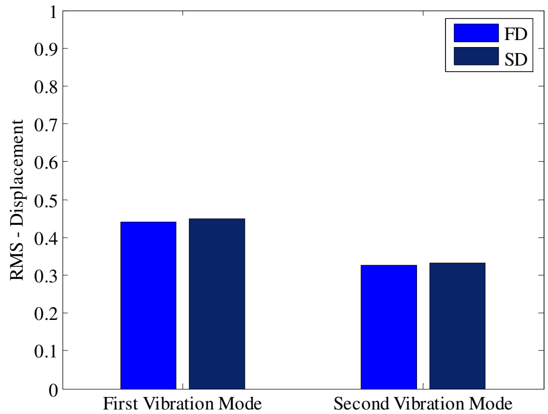

Figure 15. Vibration modes more affected evaluated by modal displacements (500Hz – PZT 1 – ACCEL 2).

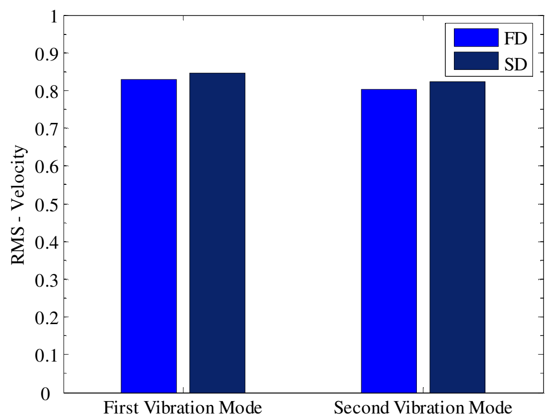

Figure 16. Vibration modes more affected evaluated by modal velocities (500Hz – PZT 1 – ACCEL 2).

The results found with PZT 2 will not be shown because of the plate building. The plate is symmetric, so the results found with PZT 1 and PZT 2 are similar.

6. FINAL REMARKS

In this paper was evaluated a new SHM technique based on state observer and the modal domain. The new approach presented excellent results in all analysis. It was changed the input frequency (100 Hz and 500 Hz) and the measurement position (ACCEL 1 and ACCEL 2). Also, we can see that the vibration mode output more affected by the damage can change with the measurement position (see Fig. 12 and Fig. 15) and the input frequency (see Fig. 10 and Fig. 15). These results agree with benchmark problems related in the literature. Thus, the results lead to the conclusion that the new approach is a useful tool in the SHM area. Possible extension of this work includes the evaluation of other vibration modes and additionally, the development of a modal control in the vibration mode output more affected by damage to extend the work life of the structure until the repair.

7. ACKNOWLEDGEMENTS

The authors thank Prof. Dr. Gustavo L. M. de Abreu for their critical comments and suggestions, and CAPES (Coordenação de Aperfeiçoamento de Pessoal do Nível Superior) that provided the financial support to this study.

8. REFERENCES

Carden, E. P. and Fanning P., 2004, "Vibration Based Condition Monitoring: A Review", Structural Health Monitoring, vol. 3, pp. 355-377.

Cavalini Jr., A. A., Franco, V. R., Gonsalez, C. G., Lopes Jr., V. and Melo, G. P., 2008, "Noise Influence on Damage Detection Through Modal State Observers Methodology, TEMA-Tendências em Matemática Aplicada e Computacional, vol. 9, No. 2, pp. 195-204.

Gawronski, W., 1998, "Dynamics and Control of Structures: A Modal Approach", 1 Ed., New York, Springer Verlag.

Inman, D. J., Farrar, C. R., Lopes Jr, V. and Steffen Jr, V., 2005, "Damage Prognosis: For Aerospace, Civil and Mechanical Systems", First Edition, Wiley.

Luenberger, D. G., 1966, "Observers for Multivariable Systems", IEEE Transactions on Automatic Control, vol. AC 11, pp. 190-197.

Luenberger, D. G., 1964, "Observing the State of a Linear System", IEEE Military Electronics, vol. MIL-8, pp. 74-80.

Maia, N. M. M., Silva, J. M. M., Almas, E. A. M. and Sampaio, R. P. C., 2003, "Damage Detection in Structures: from Mode Shape to Frequency Response Function Methods", Mech Syst Signal Process, vol. 17 i3, pp. 489-498.

Maia, N. M. M., Silva, J. M. M., He, J., Lieven, N. A. J., Lin, R. M., Skingle, G. W., To, W. and Urgueira, A. P. V., 1997, "Theoretical and Experimental Modal Analysis", Research Studies Press Ltd., Baldock, Hertfordshire, England.

Rytter, A., 1993, "Vibration Based Inspection of Civil Engineering Structures", Ph.D. Dissertation, Department of Building Technology and Structural Engineering, Aalborg University, Denmark.

Van Overschee, P. and De Moor, B., 1996, "N4SID: Subspace Algorithms for the Identification of Combined Deterministic-Stochastic Systems", Automatica, Special Issue on Statistical Processing and Control.

Welch, G. and Bishop, G., 2006, "An Introduction to Kalman Filter", UNC-Chapel Hill, Technical Report: TR 95- 041.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.