FINITE ELEMENT MODELING OF COMPOSITE SANDWICHE PLATES WITH VISCOELASTIC LAYERS

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Abstract. *Composite materials have been regarded as a convenient strategy in various types of engineering systems such as aeronautical and space structures, as well as architecture and light industry products. It is due to its advantages to the traditional engineering materials, characterized by its low density associated with high strength/stiffness relation characteristics, and its anti-corrosion properties. However, the great variety of materials properties and structural configurations makes the numerical modeling of the mechanical behavior of composite structures a complex task. Moreover, applications to the case of vibration and damping analysis of composite sandwich plates incorporating viscoelastic materials are not numerous, which motivates the study reported herein. This is a reason for which in the last decades, a great deal of effort has been devoted to the development of finite element models for characterizing the mechanical behavior of composite sandwich plates with viscoelastic layers, accounting for its typical variations of constructions, various orientations possibilites and the damping effects. In this paper the finite element modeling of composite sandwich plates incorporating viscoelastic materials is presented. The incorporation of the viscoelastic behavior into the composite sandwich finite element models is made by using the complex modulus approach and the numerical resolution of the resulting equations of motion are particularly relevant aspects of the modeling procedures since the viscoelastic stiffness matrix is frequency- and temperature-dependent. After the discussion of various theoretical aspects, the frequency responses functions are calculated for a rectangular composite sandwich plate with viscoelastic layers. Moreover, in this paper the formulation of first-order sensitivity analysis of complex frequency response functions is developed for composite sandwich plates with viscoelastic damping. The results obtained are compared with the corresponding obtained for a composite plate without damping, and the usefulness of the sensitivity modeling methodology in various types of analyses and design of composite sandwich plates incorporating viscoelastic damping is highlighted.*

Keywords*: Composite materials, finite elements, viscoelastic damping*

1. INTRODUCTION

Composite materials have been regarded as a convenient strategy in various types of engineering systems such as aeronautical and space structures, as well as architecture and light industry products. It is due to its advantages to the traditional engineering materials, characterized by its low density associated with high strength/stiffness relation characteristics, and its anti-corrosion properties. However, the great variety of materials properties and structural configurations makes the numerical modeling of the mechanical behavior of composite structures a complex task. This is a reason for which in the last decades, a great deal of effort has been devoted to the development of finite element models for characterizing the mechanical behavior of such materials, accounting for its typical variations of constructions and various orientations possibilities. Much of the knowledge available to date is compiled in the works by Reddy (1997) and Chee *et al.* (2000) and in some papers such as those by Lo *et al.* (1997), Meunier and Shenoi (2001), Cugnoni and Gmür (2004) and Berthelot (2006).

For the purposes of this paper, the well-known *Higher-order Shear Deformation Theory – HSDT* that enables to model both in-plane and out of plane modes of deformation, by using the cubic displacement functions relative to the transverse dimension, proposed by Lo *et al*. (1997) and Chee (2000) is retained. The main advantages of this displacement field are: (i) it can model both thin as well as thick laminated composite plates; (ii) it is not necessary to introduce any correction factors such as those required for the *First-order Shear Deformation Theory – FSDT*; (iii) it can model transverse shear effects and predicts a parabolic transverse shear strains; (iv) it is also able to model transverse normal strain. This method has been developed in order to study the dynamic behavior of undamped fibre reinforced plastic sandwich plates, where the energy dissipation mechanisms in the structures are neglected. However, most of the studied fibre reinforced plastic sandwich plates are composed of, among other things, a resin system and a PVC core, which exhibit viscoelastic damping characteristics (Meunier and Shenoi, 2001). Since the energy dissipation mechanism in a structure plays an important role in vibration control and fatigue, it is important to take into account the contribution of the viscoelastic damping when the dynamic behavior of composite sandwich structures is investigated.

In the context of analysis and design of composite structures, an important topic to be addressed is the so-called *sensitivity analysis*, which enables to evaluate the degree of influence of variations of physical and/or geometrical parameters on the mechanical behavior. Sensitivity analysis constitutes an important step in various types of problems such as model updating, analysis of modified structures, optimal design, system identification, control and stochastic reliability assessment (Lima *et. al.,* 2006). Sensitivity analysis is generally based on the evaluation of the derivatives (most frequently limited to the first-order) of the system response with respect to a set of parameters of interest (Haug *et al.,* 1986). It can be associated to different kinds of mechanical responses: static displacements, eigenvalues and eigenvectors, frequency response functions and time responses. According to Murthy and Haftka (1988), the optimal design structural systems has a narrow connection with sensitivity analysis, since a significant part of typical optimization algorithms generally perform a large number of evaluations of the system response for different values of the design variables. Derivatives can be used to approximate the response of modified systems, thus reducing the cost of re-analysis, especially for structures representative for industrial test cases. Several approaches have been developed for performing sensitivity analysis of dynamic responses, as reported in reference (Haug *et al.,* 1986). However, applications to the case of vibration and damping analysis of composite sandwich plates incorporating viscoelastic materials are not numerous, which motivates the study reported herein. The incorporation of the viscoelastic behaviour into the composite sandwich finite element models is made by using the complex modulus approach and the numerical resolution of the resulting equations of motion are particularly relevant aspects of the modeling procedures since the viscoelastic stiffness matrix is frequency- and temperature-dependent.

In the remainder, after the discussion of various theoretical aspects, first-order response derivatives are calculated for a rectangular composite sandwich plate with viscoelastic material. The results obtained are compared with first-order finite-difference approximations, and the usefulness of the sensitivity modeling methodology in various types of analyses and design of composite sandwich plates incorporating viscoelastic damping is highlighted.

2. BACKGOUND ON FINITE ELEMENT FORMULATION OF COMPOSITE PLATES

In this section the formulation of a composite plate finite element is summarized, based on the original developments made by Lo *et al*. (1997) and Chee (2000). Fig. 1 depicts the principal components of the composite plate reported herein, whose dimensions in directions x and y are denoted by a and b, respectively.

The mechanical behavior of the composite structure can be model by using the Higher-order Shear Deformation Theory, in which the displacements at an arbitrary point in such a composite is expressed as follows:

$$\boldsymbol{U}(x,y,z) = \boldsymbol{A}_u(z)\boldsymbol{u}(x,y) \tag{1}$$

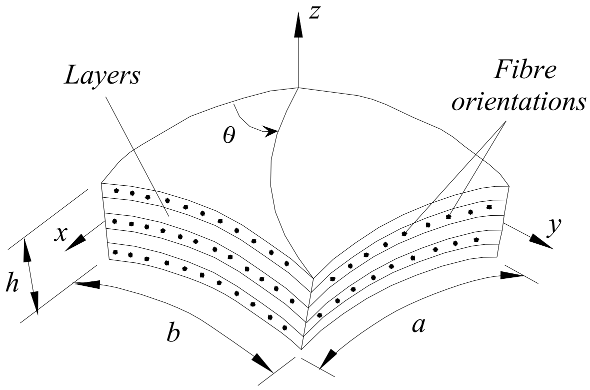


Figure 1. Illustration of the laminated composite plate components of thickness h .

where $\boldsymbol{A}_u(z) = \begin{bmatrix} 1 & 0 & 0 & z & 0 & 0 & z^2 & 0 & 0 & z^3 & 0 \\ 0 & 1 & 0 & 0 & z & 0 & 0 & z^2 & 0 & 0 & z^3 \\ 0 & 0 & 1 & 0 & 0 & z & 0 & 0 & z^2 & 0 & 0 \end{bmatrix}$, $\boldsymbol{u}(x,y) = \left[u_0 v_0 w_0 \psi_x \psi_y \psi_z \zeta_x \zeta_y \zeta_z \Phi_x \Phi_y\right]^T$ and $\boldsymbol{U}(x,y,z) = [u\ v\ w]^T$.

(u_0, v_0, w_0) and (ψ_x, ψ_y, ψ_z) are, respectively, the mid-plane displacements and the rotations due to shear deformations about x, y and z directions. The higher order terms, ζ_x, ζ_y, ζ_z, Φ_x and Φ_y, are analogous to the change in curvature of displacements.

The finite element discretization for the general composite plate is made based on the displacements fields (1) for which the thickness variable z is separated from the 11 in-plane functions. The mechanical strains-displacements relations are defined in the standard manner, and grouped as bending and transverse shear strains, $\boldsymbol{\varepsilon}_b$ and $\boldsymbol{\varepsilon}_s$, respectively, as follows:

$$\boldsymbol{\varepsilon}_b(x,y,z) = \left[\boldsymbol{D}_0 + z\boldsymbol{D}_1 + z^2\boldsymbol{D}_2 + z^3\boldsymbol{D}_3\right] \boldsymbol{u}(x,y) = \boldsymbol{Db}(z)\boldsymbol{u}(x,y) \tag{2.a}$$

$$\boldsymbol{\varepsilon}_s(x,y,z) = \left[\boldsymbol{D}_4 + z\boldsymbol{D}_5 + z^2\boldsymbol{D}_6\right] \boldsymbol{u}(x,y) = \boldsymbol{Ds}(z)\boldsymbol{u}(x,y) \tag{2.b}$$

where $\boldsymbol{\varepsilon}_b(x,y,z) = \left[\varepsilon_{xx}\ \varepsilon_{yy}\ \varepsilon_{zz}\ \gamma_{xy}\right]^T$ and $\boldsymbol{\varepsilon}_s(x,y,z) = \left[\gamma_{yz}\ \gamma_{zx}\right]^T$. $\varepsilon_{xx} = \partial u/\partial x$, $\varepsilon_{yy} = \partial v/\partial y$, $\varepsilon_{zz} = \partial w/\partial z$, $\gamma_{xy} = (\partial u/\partial y + \partial v/\partial x)$, $\gamma_{yz} = (\partial v/\partial z + \partial w/\partial y)$ and $\gamma_{zx} = (\partial u/\partial z + \partial w/\partial x)$ are the strain-displacement relations. Matrices $\boldsymbol{D}_i$ $(i = 0,\ldots,6)$ are formed by derivatives of the shape functions according to the differential operators appearing in the strains-displacements relations. Note that it is an important step that will result in much convenience in the parameterization process.

Space discretization of the mechanical variables based on the third-order displacement theory converts Eqs. (1) and (2) to a finite element using appropriate shape functions and mechanical nodal variables. Hence, for 8-node rectangular plate element, the 11-mechanical variables represented by vector $\boldsymbol{u}(x,y)$ are related to their corresponding 88-mechanical nodal variables appearing in vector $\boldsymbol{u}_e$ through the following relation:

$$\boldsymbol{u}(\xi,\eta) = \boldsymbol{N}_u(\xi,\eta)\boldsymbol{u}_e \tag{3}$$

where $\boldsymbol{u}_e^i = \left[u_i\ v_i\ w_i\ \psi_{xi}\ \psi_{yi}\ \psi_{zi}\ \zeta_{xi}\ \zeta_{yi}\ \zeta_{zi}\ \Phi_{xi}\ \Phi_{yi}\right]^T$ $(i = 1,\ldots,8)$ is the vector composed by the mechanical nodal variables appearing in vector $\boldsymbol{u}_e$. $N_u^i(\xi,\eta)$ $(i = 1,\ldots,8)$ found in matrix $\boldsymbol{N}_u(\xi,\eta)$ of dimensions 11×88, represents the standard serendipity 8-node shape functions formulated in local coordinates (ξ,η) that must correspond properly with the global coordinates numbering (x,y) (Reddy, 1997).

By considering Eqs. (1) to (3), the displacements field and bending and shear strain relations in the finite element local coordinates can be formulated as follows:

$$\boldsymbol{U}(\xi,\eta,z) = \boldsymbol{A}_u(z)\boldsymbol{N}_u(\xi,\eta)\boldsymbol{u}_e \tag{4}$$

$$\boldsymbol{\varepsilon}_b(\xi,\eta,z) = \boldsymbol{Db}(z)\boldsymbol{N}_u(\xi,\eta)\boldsymbol{u}_e = \boldsymbol{Bb}_u(\xi,\eta,z)\boldsymbol{u}_e \tag{5.a}$$

$$\boldsymbol{\varepsilon}_s(\xi,\eta,z) = \boldsymbol{Ds}(z)\boldsymbol{N}_u(\xi,\eta)\boldsymbol{u}_e = \boldsymbol{Bs}_u(\xi,\eta,z)\boldsymbol{u}_e \tag{5.b}$$

Based on the stress-strain relations, the strain and kinetic energies of the composite plate element can be formulated in terms of the natural variables of strain field and the mechanical material properties. After, Lagrange's equations are used, considering the nodal displacements and rotations as generalized coordinates, to obtain the following elementary mass and stiffnesses matrices, respectively:

$$\boldsymbol{M}^{(e)} = \int_{\xi=-1}^{\xi=+1}\int_{\eta=-1}^{\eta=+1}\int_{z_{inf}}^{z_{sup}} \rho\, \boldsymbol{N}_u^T(\xi,\eta)\boldsymbol{A}_u(z)\boldsymbol{N}_u(\xi,\eta)\, J\, dz\, d\eta\, d\xi \tag{6.a}$$

$$\boldsymbol{K}_b^{(e)} = \int_{\xi=-1}^{\xi=+1}\int_{\eta=-1}^{\eta=+1}\int_{z_{inf}}^{z_{sup}} \boldsymbol{Bb}_u^T(\xi,\eta,z)\boldsymbol{C}_b(\theta)\boldsymbol{Bb}_u(\xi,\eta,z)\, J\, dz\, d\eta\, d\xi \tag{6.b}$$

$$\boldsymbol{K}_s^{(e)} = \int_{\xi=-1}^{\xi=+1}\int_{\eta=-1}^{\eta=+1}\int_{z_{inf}}^{z_{sup}} \boldsymbol{Bs}_u^T(\xi,\eta,z)\boldsymbol{C}_s(\theta)\boldsymbol{Bs}_u(\xi,\eta,z)\, J\, dz\, d\eta\, d\xi \tag{6.c}$$

where $\xi = [-1, +1]$ and $\eta = [-1, +1]$ are the local coordinates intervals for the standard serendipity 8-node shape functions, and $z = [z_{inf}, z_{sup}]$ is the layer thickness interval.

The integrals appearing in Eqs. (6) are performed for a rectangular element in global coordinates by using the Jacobian J transformation. Moreover, one can note that the matrices $\boldsymbol{C}_b(\theta)$ and $\boldsymbol{C}_s(\theta)$ represent the transformed orthotropic bending and shear elastic material property matrices for a rotation θ about the z-axis (see Fig. 1), to consider the various orientations possibilities:

$$\boldsymbol{C}_b(\theta) = \boldsymbol{T}_b(\theta)\boldsymbol{C}_b\boldsymbol{T}_b^T(\theta) \tag{7.a}$$

$$\boldsymbol{C}_s(\theta) = \boldsymbol{T}_s(\theta)\boldsymbol{C}_s\boldsymbol{T}_s^T(\theta) \tag{7.b}$$

where $\boldsymbol{C}_b$ and $\boldsymbol{C}_s$ are the classical orthotropic bending and shear elastic material property matrices; $\boldsymbol{T}_b(\theta)$ and $\boldsymbol{T}_s(\theta)$ are, respectively, the bending and shear transformation matrices depending on the rotation about the z (Reddy, 1997).

3. COMPOSITE SANDWICH PLATES WITH VISCOELASTIC LAYERS

For a composite sandwich structure configuration incorporating viscoelastic layers between laminated composite plates, the viscoelastic material property matrix must be taking into account the frequency and temperature dependence behavior of the viscoelastic material. By considering that the viscoelastic material properties reported herein are assumed to be isotropic, the Eqs. (6.b) and (6.c) assume the following forms:

$$\boldsymbol{K}_b^{(v)}(\omega,T) = \int_{\xi=-1}^{\xi=+1}\int_{\eta=-1}^{\eta=+1}\int_{z_{inf}}^{z_{sup}} \boldsymbol{B}_{bu}^T(\xi,\eta,z)\boldsymbol{C}_b(\omega,T)\boldsymbol{B}_{bu}(\xi,\eta,z)\, J\, dz\, d\eta\, d\xi \tag{8.a}$$

$$\boldsymbol{K}_s^{(v)}(\omega,T) = \int_{\xi=-1}^{\xi=+1}\int_{\eta=-1}^{\eta=+1}\int_{z_{inf}}^{z_{sup}} \boldsymbol{B}_{su}^T(\xi,\eta,z)\boldsymbol{C}_s(\omega,T)\boldsymbol{B}_{su}(\xi,\eta,z)\, J\, dz\, d\eta\, d\xi \tag{8.b}$$

where the subscript (v) indicates the elementary viscoelastic zones. $\boldsymbol{C}_b(\omega,T)$ and $\boldsymbol{C}_s(\omega,T)$ are frequency- and temperature-dependent viscoelastic material property matrices.

The inclusion of the frequency- and temperature-dependent behaviour of the viscoelastic material can be made by using the so-called *Elastic-Viscoelastic Correspondence Principle* (Christensen, 1982), according to which, for a given temperature, matrices $\boldsymbol{K}_b^{(v)}(\omega,T)$ and $\boldsymbol{K}_s^{(v)}(\omega,T)$ can be first generated for the plate element assuming that the longitudinal modulus and/or the shear modulus appearing in matrices $\boldsymbol{C}_b(\omega,T)$ and $\boldsymbol{C}_s(\omega,T)$ (according to the stress-state) are constant. Then, after the FE matrices are constructed, the frequency-temperature dependency of those *moduli* is introduced according to the complex modulus approach combined with the *Frequency-Temperature Superposition Principle – FTSP* (Nashif *et al.*, 1985). By assuming the widely accepted hypothesis of a constant (frequency-independent) Poisson ratio for the thermorheologically-simple viscoelastic materials, $E(\omega,T)$ becomes proportional to $G(\omega,T)$ through the relation $G(\omega,T) = E(\omega,T)/2(1+\nu)$. Then, one of the two *moduli* can be factored-out of the viscoelastic stiffnesses matrices as follows:

$${}_{\alpha}\boldsymbol{K}_b^{(v)}(\omega,T) = G\,(\omega,T)\overline{\boldsymbol{K}}_b^{(v)} \tag{9.a}$$

$${}_{\alpha}\boldsymbol{K}_s^{(v)}(\omega,T) = G(\omega,T)\overline{\boldsymbol{K}}_s^{(v)} \tag{9.b}$$

where $\overline{\boldsymbol{K}}_b^{(v)}$ and $\overline{\boldsymbol{K}}_s^{(v)}$ are the frequency-independent stiffnesses matrices of the viscoelastic layers, which are combined with the stiffnesses matrices associated with the elastic parts represented by the Eqs. (6.b) and (6.c), to produce the following complex stiffness matrix:

$$\boldsymbol{K}(\omega,T) = \boldsymbol{K}_e + G(\omega,T)\overline{\boldsymbol{K}}_v \tag{10}$$

Based on the formulation presented in the previous sections, and neglecting other forms of damping, the finite element equations of motion in the frequency domain of the composite sandwich plate incorporating viscoelastic materials, can be expressed as follows:

$$\left[\boldsymbol{K}_e + G(\omega,T)\overline{\boldsymbol{K}}_v - \omega^2 \boldsymbol{M}\right] \boldsymbol{Q}(\omega,T) = \boldsymbol{F}(\omega) \tag{11}$$

where N is the number of degrees-of-freedom (d.o.f's). $\boldsymbol{M} \in R^{N\times N}$ is the mass (symmetric, positive-definite) matrix, and $\boldsymbol{K}_e \in R^{N\times N}$ and $\overline{\boldsymbol{K}}_v \in R^{N\times N}$ are the stiffnesses matrices (symmetric, nonnegative-definite) corresponding to the elastic and viscoelastic substructures, respectively. $\boldsymbol{Q}(\omega,T) \in R^N$ and $\boldsymbol{F}(\omega) \in R^N$ are, respectively, the vectors of displacements and external loads. The receptance or FRF matrix is expressed as:

$$\boldsymbol{H}(\omega,T) = \left[\boldsymbol{K}(\omega,T) - \omega^2 \boldsymbol{M}\right]^{-1} \tag{12}$$

Based on Eq. (12), the interest is to evaluate the influence of the viscoelastic damping on the amplitudes resonance peaks of the FRFs.

4. SENSITIVITY ANALYSIS OF STRUCTURAL RESPONSES

The global finite element matrices appearing in Eqs. (6) and (8), establish the dependence of the response of the system with respect to a set of design parameters. Such functional dependence can be expressed as follows:

$$\boldsymbol{r} = \boldsymbol{r}[\boldsymbol{M}(\boldsymbol{p}), \boldsymbol{K}(\boldsymbol{p})] \tag{13}$$

where $\boldsymbol{r}$ and $\boldsymbol{p}$ designate vectors of structural responses and design parameters, respectively.

The sensitivity of the responses with respect to a given parameter p_i, evaluated for a given set of values of the design parameter $\boldsymbol{p}^0$ can be estimated by finite differences by computing successively the responses corresponding to $p_i = p_i^0$ and $p_i = p_i^0 + \Delta p_i$, and then computing defined as the following partial derivative:

$$\left.\frac{\partial \boldsymbol{r}}{\partial p_i}\right|_{p^0} \approx \left\{ \frac{\boldsymbol{r}\left[\boldsymbol{M}\left(p_i^0 + \Delta p_i\right), \boldsymbol{K}\left(p_i^0 + \Delta p_i\right)\right]}{\Delta p_i} - \frac{\boldsymbol{r}\left[\boldsymbol{M}\left(p_i^0\right), \boldsymbol{K}\left(p_i^0\right)\right]}{\Delta p_i} \right\} \tag{14}$$

Such approach is in general inefficient from the computation point of view. In fact, for finite element models of composite structures composed by many thousands of degrees-of-freedom (d.o.f's), the time to compute the finite differences by the successively computation of the dynamic responses can become prohibitive. Moreover, the results depend upon the choice of the value of the parameter increment Δp_i, which has to be small compared to the corresponding parameters p_i. Another strategy consists in computing the analytical derivatives of the structural responses with respect to the parameters of interest. This approach is considered in the following section.

4.1. Sensitivity of the FRFs with respect to structural parameters.

Consider the complex FRF matrix of a composite sandwich plate incorporating viscoelastic material as given by Eq. (12). Sensitivity with respect to a given structural parameter p_i can be computed by deriving the relation $\boldsymbol{H}(\omega,T,p)\boldsymbol{H}^{-1}(\omega,T,p) = \boldsymbol{I}$:

$$\left.\frac{\partial \boldsymbol{H}(\omega,T,p)}{\partial p_i}\right|_{\left(\omega,T^0,p^0\right)} = -\boldsymbol{H}\left(\omega,T^0,p^0\right)\left[\frac{\partial \boldsymbol{K}\left(\omega,T^0,p^0\right)}{\partial p_i} - \omega^2 \frac{\partial \boldsymbol{M}\left(p^0\right)}{\partial p_i}\right]\boldsymbol{H}\left(\omega,T^0,p^0\right) \tag{15}$$

Regarding the equation above, it should be noted that when the parameter p_i appears explicitly in matrices $\boldsymbol{M}$ and/or $\boldsymbol{K}(\omega,T)$, the computation of the derivatives of these matrices with respect to such parameter is straightforward, generally resulting in sparse matrices. It is important to mention that in the present work the computations of the partial derivatives are performed by considering that the design parameters retained herein are not correlated. Nevertheless, if some are related to each other, the proposed method will not be able to describe accurately the sensitivity of the global response to the different parameters and the correlations between the corresponding variables must be taking into account.

The principal advantages of the analytical approach when compared with the numerical method (finite difference), is the fact that it can be used to approximate the behavior of the modified composite structures without the re-actualization of the nominal system during the iterative optimization and/or model updating processes, leading to a drastically reduction of the time required for computing the FRFs. Moreover, this approach can be advantageously adapted to several others structural domains based on iterative processes: stochastic structural dynamics, nonlinear mechanics, and reliability-optimization-based design. Nevertheless, the computation of the first-order derivatives of the responses with respect to parameters of interest requires that these parameters appear explicitly in the finite element matrices by performing the parameterization process when possible (Lima *et al.*, 2006; 2009), constituting a great disadvantage of the analytical method.

4.2. Sensitivity of the FRFs with respect to temperature.

The computation of the derivatives of FRFs with respect to temperature requires that such parameter appear explicitly in the viscoelastic stiffnesses matrices. This is possible through the use of the so-called FTSP principle, also known as *Williams, Landell and Ferry (WLF) Principle* (Nashif *et al.*, 1985), which establishes a relation between the effects of the excitation frequency and temperature on the properties of thermorheologically-simple viscoelastic materials. This implies that the viscoelastic characteristics at different temperatures can be related to each other by changes (or shifts) in the actual values of the excitation frequency. This leads to the concepts of *shift factor* and *reduced frequency*, symbolically expressed as:

$$G(\omega,T) = G(\omega_r,T_0) = G(\alpha_T\omega,T_0) \tag{16.a}$$

$$\eta(\omega,T) = \eta(\omega_r,T_0) = \eta(\alpha_T\omega,T_0) \tag{16.b}$$

where T is an arbitrary value of the temperature, T_0 is a reference value of temperature, $\omega_r = \alpha_T(T)\omega$ is the reduced frequency, ω is the actual frequency, and $\alpha_T(T)$ is the *shift function*. Functions $G(\omega_r)$ and $\alpha_T(T)$ can be obtained from experimental tests for specific viscoelastic materials (Nashif *et al.*, 1985). Drake and Soovere (1984) suggest analytical expressions for the complex modulus and shift factor for various commercially available viscoelastic materials. The following equations represent, respectively, the complex modulus and shift factor as functions of temperature and reduced frequency in the intervals $210 \le T \le 360K$ and $1.0 \le \omega \le 1.0x10^6\,Hz$, for the 3M™ ISD112 viscoelastic material (3M) (which is considered in the numerical applications that follow), as provided by those authors:

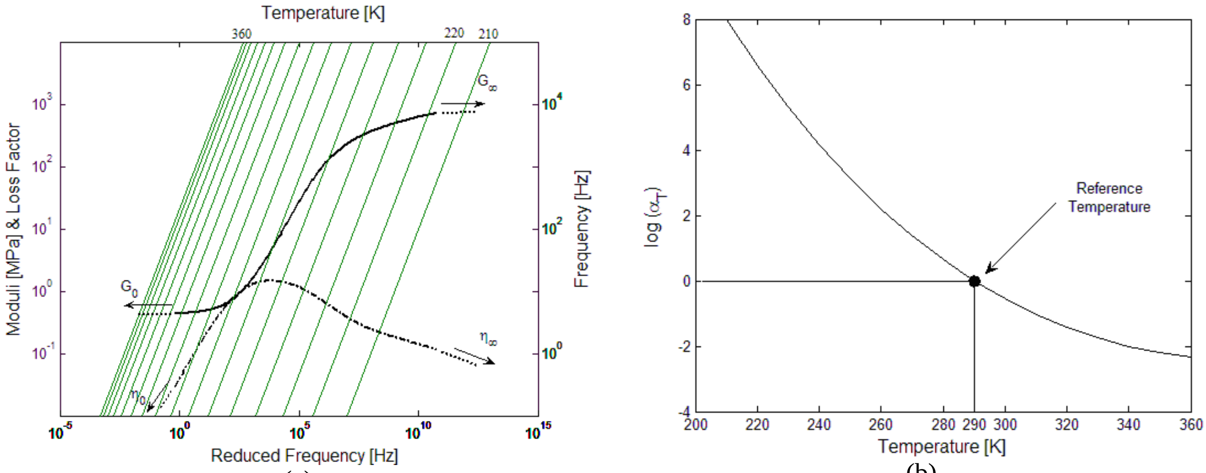
$$G(\omega_r) = B_1 + B_2 \Big/ \left(1 + B_5 (i\omega_r/B_3)^{-B_6} + (i\omega_r/B_3)^{-B_4}\right) \tag{17.a}$$

$$log(\alpha_T) = a\left(\frac{1}{T} - \frac{1}{T_0}\right) + 2.303\left(\frac{2a}{T_0} - b\right)log\left(\frac{T}{T_0}\right) + \left(\frac{b}{T_0} - \frac{a}{T_0^2} - S_{AZ}\right)(T - T_0) \tag{17.b}$$

where:

$$B_1 = 0.4307MPa;\ B_2 = 1200MPa;\ B_3 = 1543000;\ B_4 = 0.6847;\ B_5 = 3.241;\ B_6 = 0.18$$
$$T_0 = 290K;\ T_L = 210K;\ T_H = 360K;\ S_{AZ} = 0.05956K^{-1};\ S_{AL} = 0.1474K^{-1};\ S_{AH} = 0.009725K^{-1}$$
$$C_A = (1/T_L - 1/T_0)^2;\ C_B = (1/T_L - 1/T_0);\ C_C = (S_{AL} - S_{AZ});\ D_A = (1/T_H - 1/T_0)^2;\ D_B = (1/T_H - 1/T_0)$$
$$D_C = (S_{AH} - S_{AZ});\ D_E = (D_B C_A - D_A C_B);\ a = (D_B C_C - C_B D_C)/D_E;\ b = (D_C C_A - C_C D_A)/D_E$$

The final result of a damping material analysis is a reduced temperature nomogram, which expands the limited number of test results to a graph from which the designer can obtain the damping material's properties (modulus and loss factor) at any given combination of temperature and frequency. Fig. 2 depicts the standardized curves representing the variations of the storage and loss moduli and the loss factor as functions of the reduced frequency, as obtained from Eq. (17.a), and a plot of the shift factor as a function of the temperature, as given by Eq. (17.b).



(a) (b)

Figure 2. (a) Master curves (—— G' ; – · – η) for the 3M™ ISD112 material; (b) Shift factor curve for the 3M™ ISD112 material (adapted from Lima *et al.*, 2006).

The procedure for reading the nomogram is as follows: select a combination of temperature and frequency, for example, 280 K and 200 Hz and find the point for 200 Hz on the right-hand axis. Follow from that point horizontally to the line for 280 K temperature. At this intersection, draw a vertical line which defines the reduced frequency on the below axis, and the storage modulus and the loss factor on the left-hand axis. In this example $G'(200{,}280) = 116MPa$ and $\eta(200{,}280) = 0.854$.

By combining Eqs. (15) and (16) with Eq. (17), one writes:

$$\left.\frac{\partial \boldsymbol{H}(\omega, T, p)}{\partial T}\right|_{(\omega, T^0, p^0)} = -\boldsymbol{H}\left(\omega_r, T^0, p^0\right)\left[\frac{\partial G\left(\omega_r, T^0\right)}{\partial T}\overline{\boldsymbol{K}}_{\boldsymbol{v}}\left(p^0\right)\right]\boldsymbol{H}\left(\omega_r, T^0, p^0\right) \tag{18}$$

$$\frac{\partial G\left(\omega_r, T^0\right)}{\partial T} = \frac{\partial G}{\partial \omega_r}\frac{\partial \omega_r}{\partial T} = \frac{\partial G}{\partial \omega_r}\frac{\partial \alpha_T}{\partial T}\omega \tag{19}$$

5. NUMERICAL RESULTS

5.1. Composite plate with structural damping.

To illustrate the computation procedure of the FRFs, as the first example, numerical tests were performed using the FE model of a simple supported square (*a=b=0.16m*) composite plate as shown in Fig. 3(a). Fig. 3(b) illustrates the model composed by a total number of 64 finite elements and 225 nodes. The following simply supported boundary conditions are applied on the square composite plate [17]: $u_0 = w_0 = \psi_z = \zeta_x = \zeta_z = 0$ at $y = 0$ and $y = a$, and $u_0 = w_0 = \psi_z = \zeta_y = \zeta_z = 0$ at $x = 0$ and $x = b$. The composite plate consists of 5 layers of the same thickness *h/5* (*h=a/128m*), with the layups $\theta = \left(45^\circ/0^\circ/45^\circ/0^\circ/45^\circ\right)$. The real values of the material properties characteristics of each layer are [17]: $\overline{E}_1 = 172.4GPa$, $\overline{E}_2 = \overline{E}_3 = 6.89GPa$, $\overline{G}_{12} = \overline{G}_{13} = 3.45GPa$, $\overline{G}_{23} = 1.38GPa$, $\nu_{12} = \nu_{13} = 0.25$, $\nu_{23} = 0.30$ and $\rho = 1566Kg/m^3$ (adopted). The value of the structural damping factor considered herein is $\eta = 0.001$, so that $E = \overline{E} \times (1 + i \times \eta)$ and $G = \overline{G} \times (1 + i \times \eta)$, respectively. The computations consist in obtaining the sensitivities of the driving point FRF corresponding to the point *I*, denoted by $\boldsymbol{H}_{II}(\omega, p)$, as shown in Fig. 3(b).

In this example, the thicknesses and orientations of the layers were considered as the design variables in the computation of the sensitivities of the FRF $\boldsymbol{H}_{II}(\omega, p)$. The real and imaginary parts of the complex sensitivity functions obtained by using the first-order derivatives according to Eq. (15) are shown in Figs. 4 to 6, where they are compared to the approximate sensitivity functions calculated by finite differences having been adopted the variations from *5%* of the nominal values of $h_1 = h/5$, $h_3 = h/5$ and $\theta_3 = 45^\circ$, respectively. Also, in the same figures, the real and imaginary parts of the FRF $\boldsymbol{H}_{II}(\omega, p)$, multiplied by convenient scale factors, are shown.

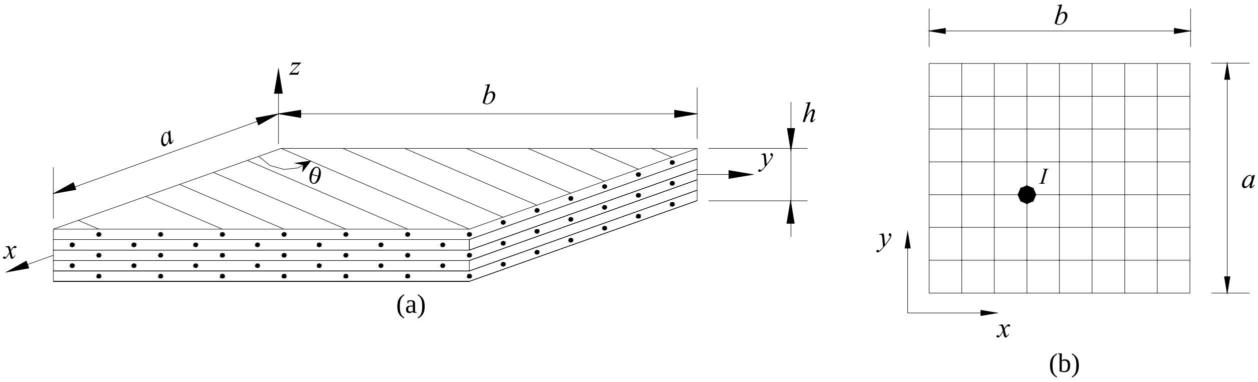


Figure 3. Illustration of the composite plate geometry (a) and the FE model discretization (b).

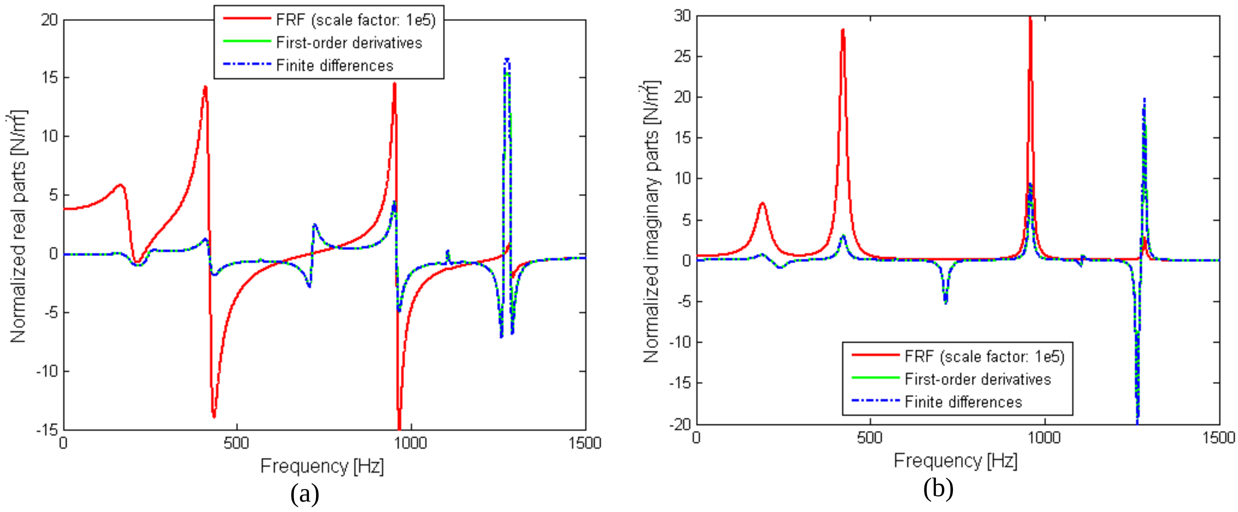


Figure 4. Sensitivities of the FRF $\boldsymbol{H}_{II}(\omega,T,p)$ with respect to h_1 for a variation from 5%.

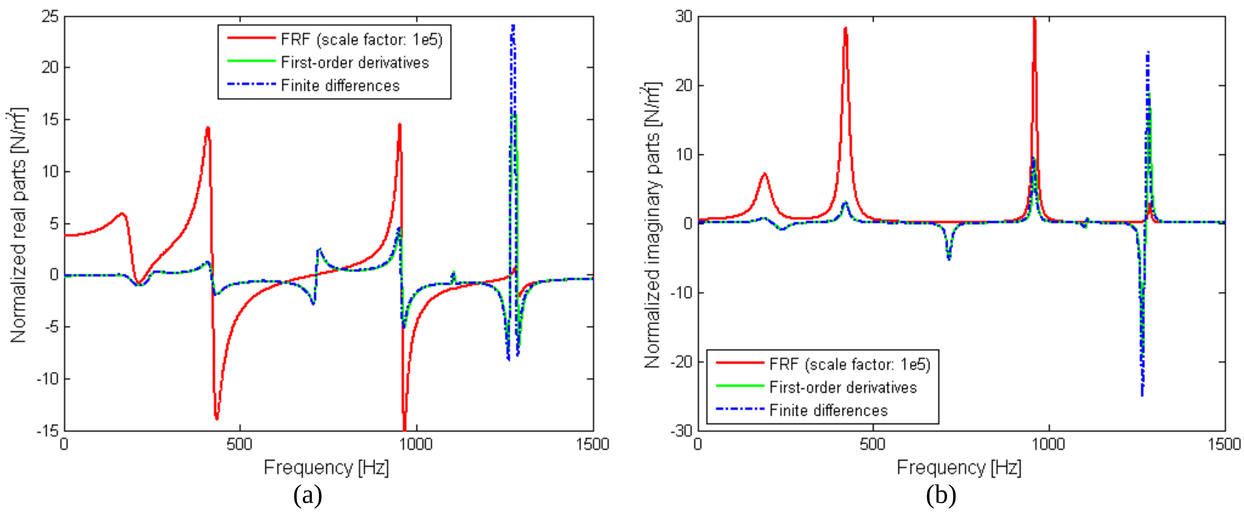
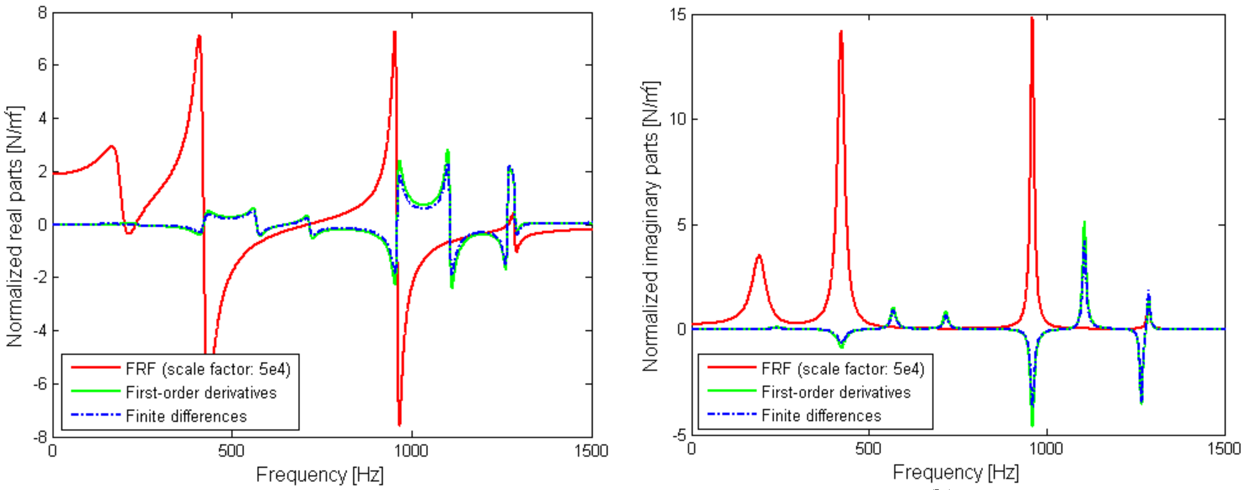


Figure 5. Sensitivities of the FRF $\boldsymbol{H}_{II}(\omega,T,p)$ with respect to h_3 for a variation from 5%.



(a) (b)

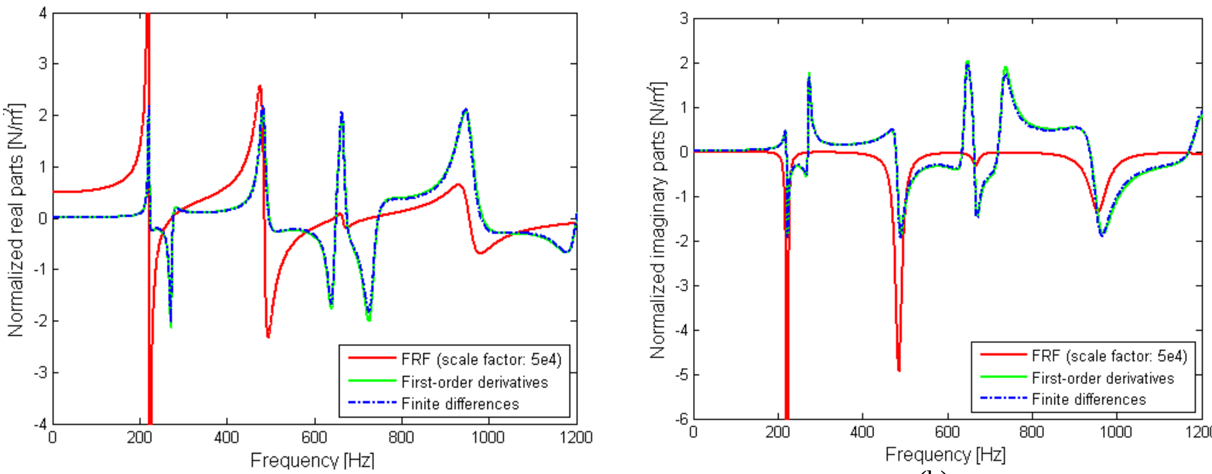
Figure 6. Sensitivities of the FRF $\boldsymbol{H}_{II}(\omega,T,p)$ with respect to θ_3 for a variation from 5%.

Figs. 4 to 6 enable to evaluate the accuracy of the computed first-order derivatives that compare fairly well with finite differences. In addition, based on the amplitudes and sign of the sensitivity functions one can evaluate the degree of influence of the design variables upon the amplitudes of the FRF in the frequency band considered, being also a very tool for the design, performance analysis and optimization of composite structures.

5.2. Composite sandwich plate with viscoelastic layer.

The finite element discretization and the geometrical characteristics of the composite sandwich structure considered here is the same as depicted on Section 5.1, with the exception that the middle layer consists of the 3M™ ISD112 viscoelastic material (see Section 4.2) with the thickness $h_v=8\times h/5$. The upper and the bottom layers have the same thickness and material properties characteristics of the laminated layers as presented in previously section.

Figure 7 shows the normalized real and imaginary parts of the sensitivity functions of the FRF $\boldsymbol{H}_{II}(\omega,T,p)$ with respect to nominal temperature value of $25°C$, as compared with the corresponding counterparts calculated by finite differences, using a variation from 5% of the nominal temperature value. Here again, the real and imaginary parts of the FRF, multiplied by a convenient scaling factor are also presented. Moreover, Fig. 7 enables to evaluate the accuracy of the computed first-order derivatives, as demonstrated by their agreement with the results obtained by the first-order finite differences. In addition, it is possible to evaluate the degree of influence of temperature variations within the frequency band of interest, and the influence of the variation interval on the computation of the sensitivity functions.



(a) (b)

Figure 7. Sensitivities of the FRF $\boldsymbol{H}_{II}(\omega,T,p)$ with respect to temperature of the viscoelastic material for a nominal temperature of $25°C$ – for a variation from 5%.

6. CONCLUDING REMARKS

In this paper, the sensitivity analysis based on finite element models of composite sandwich plates containing viscoelastic materials is addressed. A parameterization-based formulation has been developed for the computation of first-order derivatives of FRFs with respect to two different kinds of parameters, namely: the physical and/or geometrical structural parameters, and temperature, which appear explicitly in the finite element matrices by performing the parameterization process. Applications have been made to rectangular composite sandwich plates though the method can potentially be applied to other types of structural components, which can be very convenient in a number of practical situations. As illustrated in the numerical applications presented, the sensitivities of complex FRFs convey valuable information about the influence of the design parameters on the dynamic behavior of the composite sandwich structures with viscoelastic layers, being also a very useful tool for the design, performance analysis and optimization and/or model updating.

The use of the complex modulus approach, combined with the concepts of shift factor and reduced frequency - justified by the principle of superposition frequency-temperature - has shown to be an adequate strategy to account for the typical dependency of the viscoelastic characteristics with respect to frequency and temperature in the finite element models of complex composite sandwich plates with viscoelastic layers. However, the limitations of the first-order approximations in the prediction of response variations should be remembered.

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