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A Damage Assessment Strategy based on a Sequential Algebraic Algorithm with a Sensitivity Analysis

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Abstract. *The scattering of acoustic waves by non uniform one-dimensional and non dissipative media have been considered in many fields of practical interest. Relevant applications are reported in geophysics, medical ultrasound, and engineering non-destructive testing. In the present work, a sequential algebraic algorithm is considered for describing the wave propagation along a slender beam and used for damage identification purposes. In the formulation of the damage identification problem, the generalized impedance field, that minimizes the functional defined as the distance between the calculated echo and the experimental one, at the sensor position, is sought. The main advantage of the proposed method, differently of other methods in the literature, is that only one unknown impedance parameter is estimated in each iteration of the identification procedure. In order to assess the proposed method, different damage scenarios are considered, along with two levels of signal-to-noise ratio in the required data. It will be shown that the position and shape of the considered damage scenarios were successfully identified using Genetic Algorithms to solve the inverse problem derived from the sequential algebraic formulation. The sensitivity coefficients of the corresponding echoes to the considered damages are also presented.*
Keywords: *Wave Propagation, Inverse Problems, Sensitivity analysis, Damage Assessment, Genetic Algorithms*

1. Introduction

Damage identification is an essential issue for determining safety reliability and remaining lifetime of aerospace, civil and mechanical structures. The technological and scientific challenges posed by damage identification problems yielded a great research activity, within the engineering community, on this subject (Doebling et al.,1996, Santos et al., 2008, Zhongqing et al., 2006).

Different nondestructive damage identification approaches are proposed in the literature. These ones, encompassing deterministic or statistical perspectives, consider different types of data (modal, time series, frequency responses), several forms of excitations and experimental setups, distinct mathematical formulations and numerical algorithms for solving the corresponding inverse problem. Most of proposed approaches are built on the vibrational behavior of the structure, more specifically on the traditional modal analysis. Although these methods were proven to succeed in many damage identification problems, the high frequency effects of small defects, such as cracks, may be slightly or even not reflected in the required modal properties of the structure, making the damage identification a very difficult task. This great difficulty is not present in damage identification approaches built on wave propagation, since these approaches are highly sensitive to changes in local dynamic impedance (Zhongqing et al., 2006, Gangadharan et al., 2009, Grabowska et al., 2008, Fernandes et al., 2008a), such as that caused by small defects. Some successful applications are reported in the fields of geophysics (Mendell and Ashrafi, 1980, Robinson and Treitel, 1980), medical ultrasonics (Lefebre, 1985), and non-destructive tests (Tenenbaum and Zindeluk, 1986).

The basic idea of the wave propagation approach is as follows. In a wave propagation test, an excitation (an incident longitudinal stress, in the present case) is applied at the boundary of the structure and, consequently, a progressive wave propagates along it. This progressive wave is reflected whenever it encounters a local change of impedance, for instance, a boundary condition or a damage, generating a regressive wave (*echo*). Finally, the generated echo is measured at the sensor location and used to infer about the location and shape of the damage.

In the present work, the wave propagation along the beam is modeled by a sequential algebraic algorithm (Tenenbaum and Zindeluk, 1992a, Tenenbaum and Zindeluk, 1992b). The corresponding inverse problem of damage identification is formulated in such a way that the generalized impedance field, that minimizes the functional defined as the distance between the calculated echo and the experimental one, at the sensor position, is sought. Finally, a strategy based on Genetic Algorithms is used to solve the inverse problem (Goldberg, 1989). The sequential algebraic formulation of the wave propagation renders to the identification procedure the advantage of identifying only one generalized impedance parameter at each iteration. Different damage scenarios, along with two levels of signal-to-noise ratios, are considered in order to assess the capability of the proposed method. The numerical results show that, in all considered damages, the identification procedure based on Genetic Algorithms is robust, with respect to noise, and succeeded in determining the location and shape of the damage.

2. Wave Propagation Modeling

In the present work, a sequential algebraic algorithm is considered for describing longitudinal wave propagations along a slender beam (Tenenbaum and Zindeluk, 1992a, Tenenbaum and Zindeluk, 1992b). Here, an incident longitudinal stress $f(t)$ is considered to be applied at the free end of a clamped-free beam yielding the regressive wave $g(t)$.

According to the proposed model, the generalized acoustic impedance of a semi-infinite medium, defined as

$$Z(\tau) = \rho c A(\tau), \tag{1}$$

where ρ is the density, c is the sound wave speed and A is the cross section area, may be approximated by a sectionally constant function $Z_j(\tau)$, within constant intervals of traveling time $\Delta\tau$, as

$$Z_j(\tau) = Z\left(\left(j - \frac{1}{2}\right)\Delta\tau\right), \quad j = 1, 2, ... \tag{2}$$

Assuming the beam as a sectionally homogeneous medium with equal travel time layers $\Delta\tau = \Delta t$, and the data sampled at a frequency $\nu_a = 2\Delta\tau$, the amplitude G_j of the discretized outgoing echo, for clamped-free boundary conditions, is given by (Tenenbaum and Zindeluk, 1992a)

$$G_j = \sum_{n=1}^{j}\left(R_n + \sum_{p=1}^{n-2} Q_n^p\right) F_{j-n+1}\,, \qquad j = 1, 2, \ldots, N, \tag{3}$$

where F_j is the amplitude of discretized incoming pulse (excitation function) and the polynomials Q_n^p have the general recursive formula

$$Q_n^p = R_{n-p}\left[\frac{Q_{n-1}^p}{R_{n-p-1}} - R_{n-p-1}\left(\sum_{l=1}^{p-1} Q_{n-1}^l + R_{n-1}\right)\right], \quad n \geq 3, \quad p = 1, 2, ..., n-2 \tag{4}$$

and the *reflection coefficient* R_i at the $i\text{-}th$ layer of the homogeneous medium is defined as

$$R_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i}. \tag{5}$$

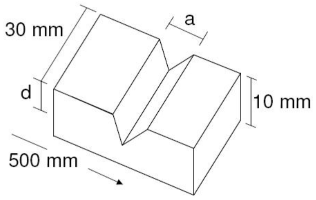
Therefore, the mathematical procedure, in the wave propagation approach, consists in the following steps. The medium, with cross-section area A and generalized impedance Z, is discretized into N elements. Then, the reflection coefficients R_i are computed from Eq. (5). In the sequence, the polynomials Q_n^p are calculated from Eq. (4). Finally, the amplitude of the echo G_j is computed from Eq. (3).

2.1 Description of Damage

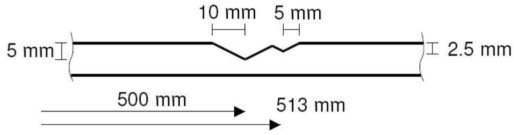
In the numerical analysis that follows, the nominal geometric properties of the beam are considered as: Length $l_0 =$ 1 m; width $w_0 = 30$ mm; and height $h_0 = 10$ mm, yielding the area moment of inertia $I_0 = 2.5 \times 10^{-9}$ m^4. Besides, the assumed nominal material properties are: Young modulus $E_0 = 7.1 \times 10^{10}$ N/m^2; and density $\rho_0 = 2.7 \times 10^3$ kg/m^3, yielding the sound speed $c_0 \approx 5128$ m/s, since $c = \sqrt{E/\rho}$.

In order to assess the proposed damage identification approach, three different damage scenarios were considered (Case 1 to 3), as depicted in Fig. 1. In Case 1, the geometric parameters adopted for the damage represented by a prismatic triangular cut were: $a = 30$ mm and $d = 5$ mm. Different values for this damage parameters were considered for assessing their influence in the wave propagation issues (Fernandes et al., 2008a) and for comparing the results of damage identification using three kinds of optimization methods (Fernandes et al., 2008b). In Case 2, two superposed triangular cuts were considered, as shown. In Case 3, the damage is modeled by a circular hole with radius $h_0/2$ at the middle of the beam and crossing its entire width.

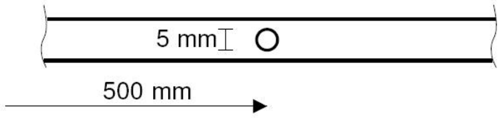
For each damage scenario in Fig. 1, a pulse-echo experiment is performed and the impulse response is taken in the damage identification procedure. In this kind of test, the excitation is given by a longitudinal impact at one end of the beam, which generates progressive (pulse) and regressive (echo) waves propagating along it due to the inhomogeneity. All pulses considered in this work are of the kind of the Dirac's delta, $\delta(t)$, obtained by a deconvolution pre-processing. Other kinds of pulses, like rectangular shaped ones, were tested with no noticeable differences in the identification results. Since the impulse response (IR) for the pulse-echo test can always be obtained by a well controlled deconvolution technique, this will be the excitation considered here.



(a) Case 1



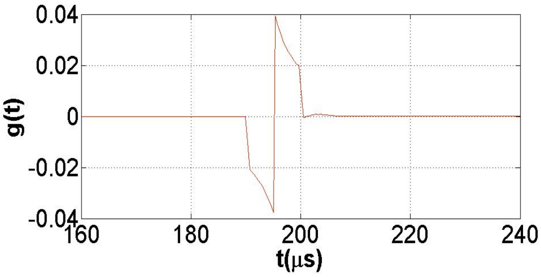
(b) Case 2



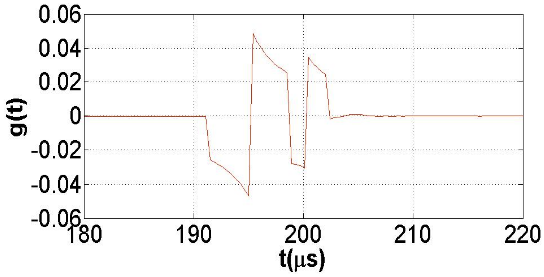
(c) Case 3

Figure 1. Three different damages imposed to the slender beam: (a) Case 1; (b) Case 2; and (c) Case 3.

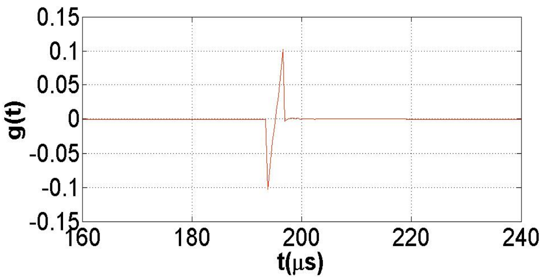
The beam impulse responses for the three damage cases are presented in Fig. 2. These are the functions that, when discretized in time, will furnish the data set for the identification process. It is worthwhile emphasizing here that the echoes considered in the damage identification process are truncated so that they do not contain the influence of the boundary conditions at $x = l_0$. Therefore, the echoes depicted in Fig. 2 indicate the presence of a change of impedance (damage) along the beam.



(a) Case 1



(b) Case 2



(c) Case 3

Figure 2. IR's, $g(t)$, of the pulse-echo tests for the three damage scenarios: (a) Case 1; (b) Case 2; and (c) Case 3.

From Fig. 2, one may observe that the first part of the obtained echo for the damage in Case 1 is negative, which corresponds to a decrease in the acoustic impedance of the beam (cross-section area, in the present case). The second part, on the other hand, is positive due to an increase in the acoustic impedance. The same behavior is observed in the echoes for Cases 2 and 3. It is worthwhile emphasizing that the echoes depicted in Fig. 2 clearly indicate the damaged regions along the beam, since the position may be directly obtained from these plots in time considering constant the wave speed c.

3. Damage Identification

Inverse problems are usually ill-conditioned what may lead to poor estimates for the unknowns with an excessive amplification of the measurement errors.

The sensitivity analysis is an important tool in the formulation and solution stages of inverse problems because the use of experimental data with higher quality, i.e. higher sensitivity to the parameters to be estimated, potentially provides better estimates for such unknowns. It is crucial that the sensitivity to the unknowns to the determined be large enough in order that the calculated values of the observable quantities, which will be measured experimentally, reflect even small changes in the unknowns. Besides, when more than one unknown are estimated simultaneously, the sensitivity coefficients must be linearly independent indicating that the unknowns are uncorrelated.

In the present work we use an implicit formulation, with the inverse problem written as an optimization problem. Therefore, much of the effort is devoted to the minimization of the objective function given by the summation of the squared residues between the calculated and the measured values of the observable quantity, the echo of a wave propagation as a function of time.

4. Sensitivity analysis

The sensitivity coefficients are

$$X_A = \frac{\partial g(t)}{\partial A(x)} \Rightarrow X_i^j = \frac{\partial G_j}{\partial A_i}, \quad i = 1, 2, ..., N, \qquad j = 1, 2, ..., N, \tag{6}$$

where $A(x)$ represents the beam cross section to be determined, which in its discretized form is represented by A_i, $i = 1, 2, ..., N$, $g(t)$ is the echo which in its discretized form is written as G_j, and N is the total number of unknowns to be determined.

The sensitivity coefficients were calculated using a centered finite difference approximation,

$$X_i^j = \frac{\partial G_j}{\partial A_i} \cong \frac{G_j(A_i + \epsilon) - G_j(A_i - \epsilon)}{(A_i + \epsilon) - (A_i - \epsilon)} = \frac{G_j(A_i + \epsilon) - G_j(A_i - \epsilon)}{2\epsilon}, \tag{7}$$

with $i = 1, 2, ..., N$ and $j = 1, 2, ..., N$, with a second order truncation error $O(\epsilon^2)$.

The derivative related to each discretized damage is calculated. Discretizing the in N points, the matrix of the derivatives has a dimension $N \times N$ and a total of N^2 derivatives must be performed. The direct problem formulation with the algebraic sequential algorithm, is solved for the computation $(G_j(A_i + \epsilon))$ and $(G_j(A_i - \epsilon))$, leading to a total of $2N^2$ derivatives. When an excitation with $l \neq 0$ mm is considered, the length of the excitation pulse must be added to the number of points used in the damage discretization.

In order to have a general implementation for beams of any size, we normalized the cross section areas. For beams without damage we consider $A_i = 1.0$, and with damage $0 \leq A_i < 1$.

In Figure 3 are represented the sensitivity coefficients X_i^j for a total of 58 parameters A_i, for the time period of observation discretized in j time instants related to Case 1, shown in Fig. 1, with $f(t) = \delta(t)$. Considering the first point, with $A_1 = 1.0$ (undamaged), it is observed that $X_1^1 \neq 0$ and $X_1^2 \neq 0$ in the time instants t_1 and t_2, respectively. For the later times, $X_1^j = 0$, with $j = 3, ...N$. In this figure can be observed a higher sensitivity in the damage region, and a smaller sensitivity to the parameters A_i in the undamaged region.

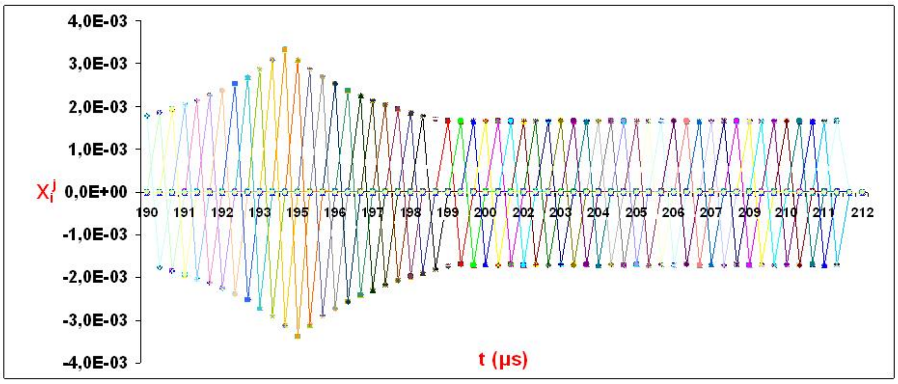


Figure 3. Sensitivity coefficients to the Case 1

In Fig. 4 are shown the sensitivity coefficients for Case 2 (Fig. 1). The two regions with higher amplitude area related to the damage with two overlapping triangles. In this Case also the sensitivities X_i^j are different from t_j an $t_j + 1$.

In Fig. 5 are shown the sensitivity coefficients for Case 3 (Fig. 1) with the circular damage. In this graph there are 38 parameters A_i, and the time interval adopted is similar to the ones considered for the two previous cases, beginning with $A_1 = 1.0$ (undamaged) and with $A_2 \neq 1.0$ (damage).

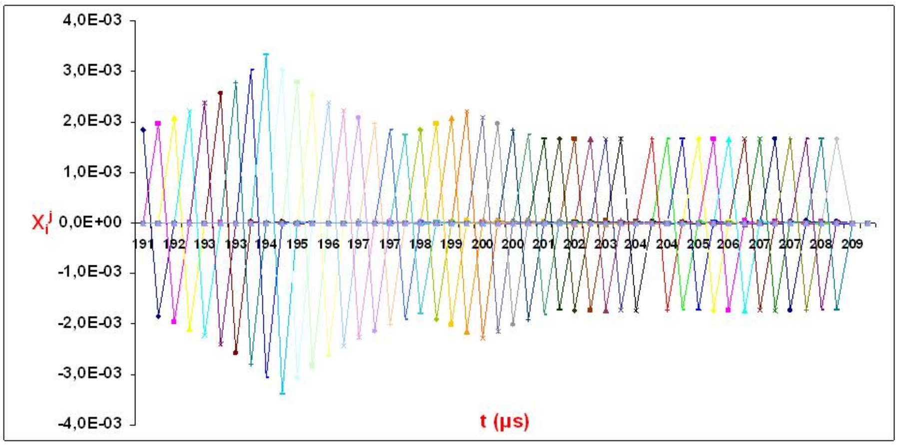


Figure 4. Sensitivity coefficients to the Case 2.

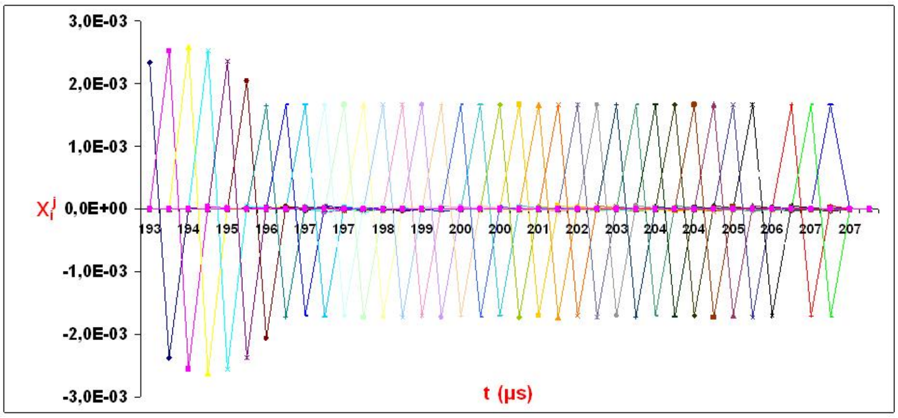


Figure 5. Sensitivity coefficients to the Case 3.

The main difficulty with the inverse problem at hand is not to determine where the damage begins, but where it finishes, because while $G_j = 0$ there is an undamaged region. When the damage finishes, i.e. when we go had to the situation of $A_i = 1.0$, the echoes G_j slowly diminishes. Therefore, we have established that there is still damage if $G_j > 10^{-5}$.

For the damage assessment inverse problem we estimate the discretized parameters A_i using an optimization method. For the solution of this problem we need to solve the direct problem many times. For that purpose we use the classic finite difference or finite element methods for the solution of the partial differential equations that model the direct problem. In such methods the echoes G_j are calculated in a global form from a linear algebraic system.

Using the algebraic sequential algorithm considered this work, the echoes G_j are not calculated globally, and the echo G_j depends only on $G_{j-1}, G_{j-2}, ..., G_1$. To calculate the value G_1 we need only the parameter A_1. To calculate the value G_2 we need A_1, A_2 and G_1. To calculate G_3 we need A_1, A_2, G_1 and G_2 and so on. This procedures is continued until $G_j \leq 10^{-5}$.

From the sensitivity analysis we conclude that the response signals G_j are independent. The sensitivity to the time instant t_j, i.e. $X_i^j \neq 0$, and for the estimation of A_i is only required the echo G_j.

The major difficulty in the identification process is the determination of the damage profile and size. The number of parameters to be estimated is linked to the number of points used in the discretization of the beam. In the problems we are dealing with we have considered a total of 20, 50 and up to 200 parameters for each damage. A literature review indicates that the traditional methods are not adequate to handle such large number of unknowns, and in fact only very coarse grids are used in the discretization of the beam yielding a very small number of unknowns.

From the sensitivity analysis performed and the inherent structure of the algebraic sequential algorithm use in the the direct problem formulation and solution (Tenenbaum and Zindeluk, 1992a), we can identify one parameter at a time, A_i, even when a large number of unknowns N is to be estimated.

4.1 Identification procedures

Consider the non homogenous beam, with damage, shown in Fig. 1. The damage region is represented by a vector with unknowns values for cross section areas,

$$\mathbf{A} = \{A_1, A_2, \ldots, A_N\}^T, \tag{8}$$

where A_i is the *i-ith* point of the dicretization of the damage region, and N is the total number of points used in its dicretization.

The damage assessment problem is based on a minimization problem,

$$\mathbf{A}^* = \ ? \qquad \text{such that} \qquad \mathrm{E}(\mathbf{A}^*) = \min_{\mathbf{A}} \mathrm{E} \tag{9}$$

where E is a measure associated with the vector of residues $\mathbf{r}(\mathbf{A})$ which is defined is

$$\mathbf{r}(\mathbf{A}) = \left\{ \begin{array}{c} G_1(\mathbf{A}) - \overline{G}_1 \\ G_2(\mathbf{A}) - \overline{G}_2 \\ \vdots \\ G_N(\mathbf{A}) - \overline{G}_N \end{array} \right\}, \tag{10}$$

where N is the total number of experimental data considered in the identification process, $G_j(\mathbf{A})$ is the calculated echo obtained with the computational model, and $\overline{G}_j$ is the experimental echo measured at the time instant t_j.

In the present work we consider the L^2 norm, and therefore,

$$E(\mathbf{A}) = r^T(\mathbf{A})r = \sum_{j=1}^{N} \left[G_j(\mathbf{A}) - \overline{G}_j\right]^2 \tag{11}$$

In Fig. 6 is presented the pseudo-code of the identification process based on the sensitivity analysis and the direct problem algebraic sequential algorithm. The N experimental data are used on at a time at each step of the iterative procedure. The values A_1 and $G_1(A_1)$ are obtained from $\overline{G}_1$. The parameters A_1 and $G_1(A_1)$ are updated and then A_2 and $G_2(A_1, A_2)$ are obtained from A_1, $G_1(A_1)$, $\overline{G}_2$ and $\overline{G}_1$. This procedure is continued until $G_j \leq 10^{-5}$.

```
i = 1
do while ( i ≤ N )
      Solver ( A_i , G_i - with: G_i-1 , G_i-2 , ..., G_1 ; A_i-1 , A_i-2 , ..., A_1 ; Ḡ_i , Ḡ_i-1 , Ḡ_i-2 , ..., Ḡ_1 )
      Update A_i
      i = i+1
end do
```

Figure 6. Pseudo-Code of the identification procedures

As actual experimental data was not available we generated synthetic experimental data with

$$\overline{G}_j(\mathbf{A}) + n_r\varepsilon \tag{12}$$

where ε is standard deviation of measurement errors and n_r is a computer generated pseudo-random number in the range [-1,1].

4.2 Genetic Algorithms (GA)

In order to minimize the Euclidian norm given by Eq. (9) along with the identification procedure represented in Fig. 6, we have use Genetic Algorithms (GA). This method is very well known and will not be described here. The interested reader will find a detailed description of the method in the works by (Silva Neto and Becceneri, 2009).

For our purposes it is enough to mention that the method is based on crossover and mutation operations, with the corresponding probabilities p_c and p_m, in order to evolve a initial population of n individuals (candidates solutions $\mathbf{A}$ for the minimization of the Euclidian norm E) to better solutions of the inverse problem along a number of n_g generations.

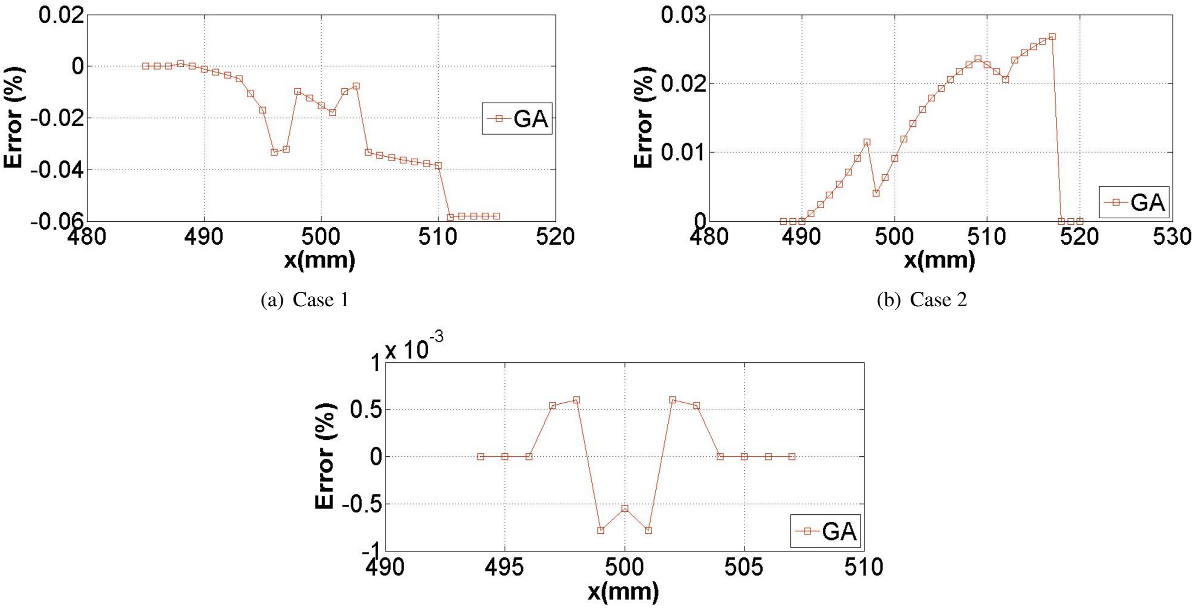
5. Main Identification Results

The general procedure for the inverse problem solution is, briefly, as follows. An impulsive wave, $f(t)$, is given as the input for the direct problem, ie, as a progressive plane wave propagating along the beam, for each one of the studied damages, as described in (Fernandes et al., 2008a, Fernandes et al., 2008b). Then, the corresponding synthetized echoes are assumed as experimental impulse responses for each damaged structure. In the sequence, the optimization GA method is applied, with the aim at updating the generalized impedance profile of the model in order to fit its predicted echo to the synthetized one.

5.1 Noiseless Results

In order to verify the accuracy of the optimization method a noiseless data situation is considered. For this idealized condition the recovery of the generalized impedance profile may be considered as almost complete.

With the GA optimization method applied to the problem, the errors (%) obtained in the profiles recovery for Cases 1, 2 and 3 are plotted in Fig. 7. It is worth noting that, actually, the GA method presents some identification error even in the absence of additive noise, but the discrepancy is restricted to no more than 0.03% at maximum for these three cases. The conclusion is that the algorithm is quite reliable for damage assessment purposes.



(a) Case 1

(b) Case 2

(c) Case 3

Figure 7. Relative (%) error in the identification for Cases 1, 2 and 3

5.2 Noisy Data Results

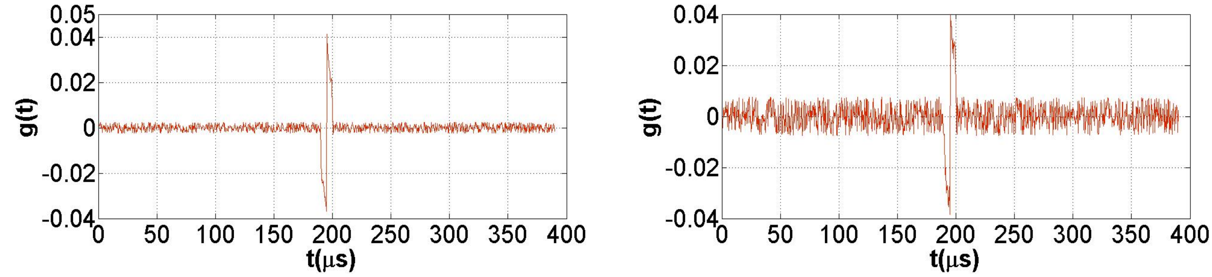
In the identification of actual damages with pulse-echo essays, of course, there will be some level of additive noise present in the experimental data. To verify the influence of distinct levels of signal to noise ratio, SNR, two different levels of random null average (white) noise were added to the output signal, corresponding to SNR $= 10$ dB and SNR $= 0$ dB. Note that in this last and worst case the noise average power is equivalent to the signal average power, a condition almost never found in a well controlled experimental setup.

In Fig. 8, it is shown the impulse response for Case 1, with $a = 25$ mm and $d = 5$ mm (see Fig. 1), added with a pseudo-random null average noise, with SNR of 10 dB (a), and 0 dB (b).

Figure 9 presents the generalized impedance identification results, compared with the actual exact profile, with the two considered signal to noise ratios, for the triangular damage of Case 1, with length $a = 25$ mm and depth $d = 5$ mm (see Fig. 1a), and the identification being performed with the GA method. The recovery of the generalized impedance profile seems to be quite good for both noise levels, showing a small deviation at the end of the curve for SNR$= 0$ dB.

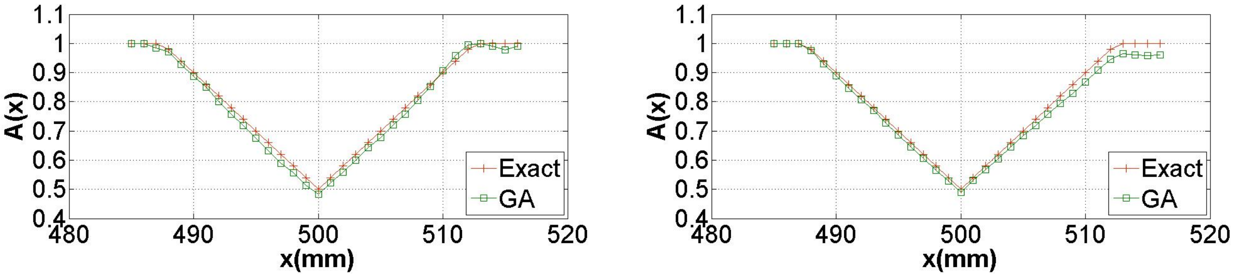
In Fig. 10 are shown the damage identification results for Case 2, again with the two SNR's. For both situations some error in the identification is found. However, it is worth noting that the triangle superposition is clearly identified and the general damage shape is preserved.

Finally, Fig. 11 presents the identification results for Case 3. Note that, since plane wave propagation is considered, the damage shown in the plots represent the cross section area profile, corresponding to the imposed circular hole shown in Fig. 1. The actual shape (and position in the vertical axis) of the hole, however, is not recognizable with the plane wave propagation model. It is worth noting that for Case 3 and high signal to noise ratio (0 dB) it was found the greater



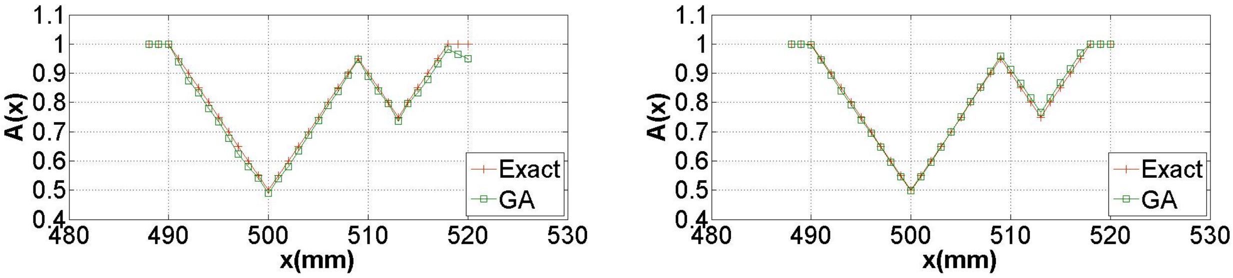
(a) SNR = 10 dB (b) SNR = 0 dB

Figure 8. Impulse responses for Case 1, with SNR of 10, and 0 dB



(a) SNR = 0 dB (b) SNR = 10 dB

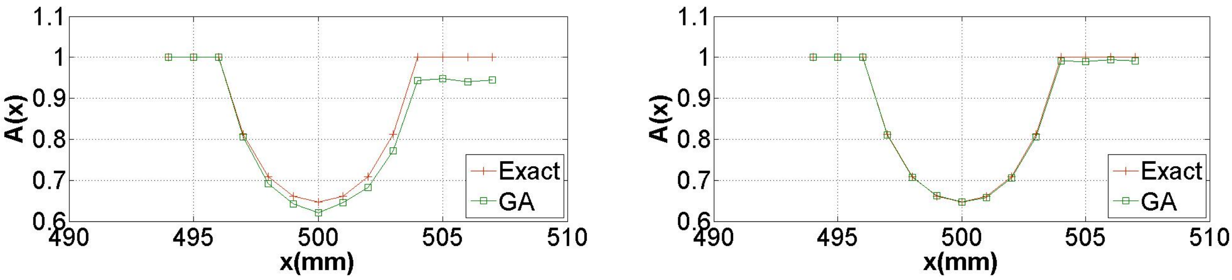
Figure 9. Damage identification with two levels of signal to noise ratio (SNR) levels, Case 1



(a) SNR = 0 dB (b) SNR = 10 dB

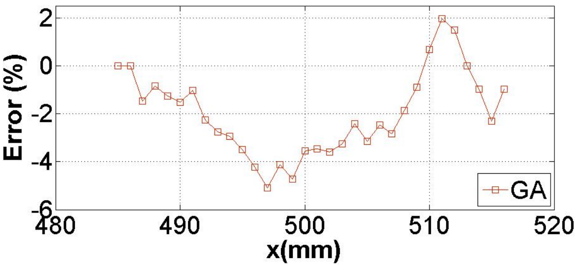
Figure 10. Damage identification with two levels of signal to noise ratio (SNR) levels, Case 2.

discrepancy between the exact impedance profile and the recovered one. The relative (%) errors in the identification for the three cases, for SNR = 0 dB are presented in Fig. reffig:error0db.

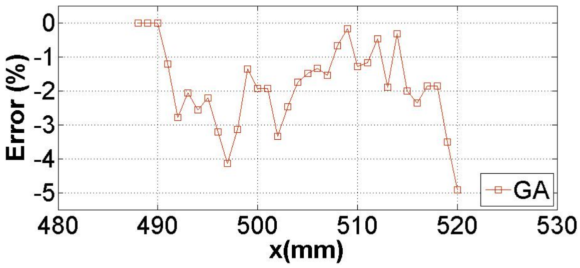


(a) SNR = 0 dB (b) SNR = 10 dB

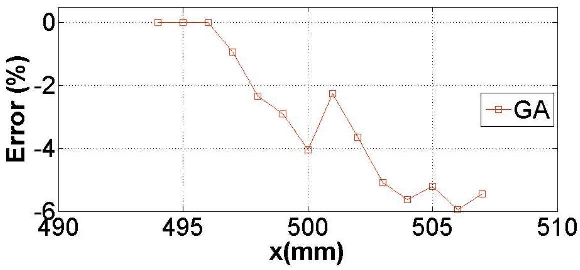
Figure 11. Damage identification with two levels of signal to noise ratio (SNR) levels, Case 3.



(a) Case 1



(b) Case 2



(c) Case 3

Figure 12. Relative (%) error in the identification for Cases 1, 2 and 3 for $\mathrm{SNR} = 0$ dB.

6. Conclusions

The damage influence on the wave propagation issues of a slender beam impulse response was studied in this work. For this, an aluminium beam with an imposed damage scenario was considered. In the wave propagation framework the presence of a damage is clearly recognized by measuring the generated echo. Both the damage location and its intensity provide information to the signal output.

The sensitivity analysis showed that the wave propagation model, described by Eqs. (3–5), is a very good description for the phenomenon. All computed coefficients presented good sensitivity in the time interval of interest.

Since the direct model is recursive and sequential, the identification procedure needs to estimate only one unknown for each iteration, furnishing a huge computational gain. This would not be possible, for instance, using finite differences to solve the non-homogeneous hyperbolic wave equation (Tenenbaum and Zindeluk, 1992a).

Analyzing the inverse problem solution using noiseless data, it was showed that the stochastic GA method is suitable to deal with this problem, since the method leads to excellent identification results in the absence of additive noise.

In order to verify the suitability of the adopted technique, distinct damage shapes and positions were projected as well as different damage intensities were tested. Furthermore, two different SNR levels were provided in order to examine the actual capability of the methods to deal with noisy corrupted data.

For the tests run with SNR of 10 and 0 dB, i.e., with moderate to high noise levels, the performance showed to be still good. Tests were also performed with SNR of 20 and 30 dB, but the results were so close to the noiseless identification profiles that they were not included in this text.

As a final conclusion, the number of parameters identified was, for each case, of the order of 30. This high number of parameters leaded to a detailed recovery of the impedances profiles — in this case, a cross section profile. So, it is expected that a not so regular profile, as in the cases analyzed here, could be identified in a somewhat detail. This will confirmed, in a near future, by some experimental setup being prepared.

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7. REFERENCES

Doebling, S.W., Farrar, C. R., Prime, M.B. and Sheritz, D.W., 1996, "Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in their Vibration Characteristics: A Literature Review". Los Alamos National Lab., Rept. LA-13070-MS, Los Alamos.

Fernandes, K.M., Stutz, L.T., Tenenbaum, R.A. and Silva Neto, A.J., 2008a, "Vibration and wave propagation approaches applied to assess damage influence on the behavior of Euler-Bernoulli beams — Part I: Direct problem", Proceedings of the Ninth International Conference on Computational Structures Technology, Athens.

Fernandes, K.M., Stutz, L.T., Tenenbaum, R.A. and Silva Neto, A.J., 2008b, "Vibration and wave propagation approaches applied to assess damage influence on the behavior of Euler-Bernoulli beams — Part II: Inverse problem", Proceedings of the Ninth International Conference on Computational Structures Technology, Athens.

Gangadharan, R., Roy Mahapatra, D., Gopalakrishnan, S., Murthy, C.R.L. and Bhat, M.R., 2009, "On the sensitivity of elastic waves due to structural damages: Time frequency based indexing method", Journal of Sound and Vibration, 320, 915–941.

Grabowska, J., Palacz, M. and Krawczuka, M., 2008, "Damage identification by wavelet analysis", Mechanical Systems and Signal Processing, 22, 1623–1635.

Goldberg, D.E., 1989, "Genetic Algorithms in Search, Optimization and Machine Learning", Addison- Wesley.

Lefebre, J.P., 1985, "La tomographie d'impédance acoustique", Trait. Signal, V.2, n.2, 103–110.

Mendell, J.M. and Ashrafi, F.H., 1980, "A survey of approaches to solving inverse problems for lossless layered media systems", IEEE Trans. Geosci. Rem. Sensing, GE-18, n.4, 320–330.

Robinson, E.A. and Treitel, S., 1980, "Geophysical Signal Analysis", Englewood Cliffs, Prentice Hall Edition.

Santos, J.V.A., Maia, N.M.M. and Soares, C.M.M., 2008, "Structural Damage Identification: A Survey". [Book Auth.] Topping, B.H.V., and Papadrakakis, M., Trends In Computational Structures Technology. Stirlingshire, Scotland: Saxe-Coburg Publications.

Silva Neto, A.J. and Becceneri, J.C. (Eds.), 2009, "Técnicas de Inteligência Computacional Inspiradas na Natureza – Aplicação em Problemas Inversos em Transferência Radiativa", SBMAC.

Tenenbaum, R.A. and Zindeluk, M., 1986, "Classical signal processing techniques applied to variable impedance identification by impulse testing", Proceedings of Internoise 86, V.II, 1241–1246.

Tenenbaum, R. A. and Zindeluk, M., 1992a, "An exact solution for the one-dimensional elastic wave equation in layered media", J. Acoust. Soc. Am., v.92, n.6, 3364–3370.

Tenenbaum, R.A. and Zindeluk, M., 1992b, "A fast algorithm to solve the inverse scattering problem in layered media with arbitrary input", J. Acoust. Soc. Am., V.92, n.6, 3371–3378.

Zhongqing, S., Lin, Y. and Ye, L., 2006, "Lamb waves for identification of damage in composite structures: A review", Guided, Journal of Sound and Vibration, 295, 753–780.