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METAMODELING APPROACH USING RADIAL BASIS FUNCTIONS AND A CONTROLLED RANDOM SEARCH WITH APPLICATION TO INVERSE CASCADE DESIGN

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Abstract. *This paper discusses the viability of using a metamodeling technique in conjunction with a Controlled Random search Algorithm (CRSA) for treating the blade cascade inverse design. CRSA is a stochastic, population-set base algorithm, capable of performing global optimization tasks efficiently. The proposed metamodel technique is based on the iterative construction of response surfaces with radial basis functions (inverse multiquadrics and thin plate spline) and the application of heuristic criteria for updating the data bank during the optimization process. Ciclic search patterns for metamodel constrained optimizations are iteratively used for determining candidate points to be included into the data bank. These patterns induce distance constraints between the data bank points and the metamodel optima for balancing local and global searches and also for improving the metamodel construction. At each iteration the CRSA is applied several times in order to even out severe fluctuations due to the use of random numbers by CRSA. The best found point is included into the data bank. Some cases studies are presented for testing the efficiency and robustness of the proposed methodology. These cases include the Dixon-Szegö test functions and also an inverse cascade design example. Since the main objective of this paper is of a prospective nature, a low-fidelity flow solver is employed for the cascade design test. This solver employs a simplified viscous-inviscid interaction approach. The main objective is the minimization of a norm between the target and the iterative pressure distributions (inverse approach). The design parameters are the cascade stagger angle, the pitch-to-chord ratio and the weights of a Bezier parameterization of the blade contour.*

Keywords*: blade cascade design, inverse optimization, controlled random search, Bezier parameterization.*

1. INTRODUCTION

Blade cascade analysis still represents a fundamental tool in turbomachinery design context. Relying on 2-D flow models, cascade flow computations are much faster than 3-D models of similar physical complexity. Even so, however, full Navier-Stokes computations in 2-D dense grids may represent a practical bottleneck. This issue is evident when a great number of concurrent geometrical and flow parameters must be analyzed during the searching of a good solution for satisfying the design objectives. Normally, such task is better accomplished by means of a suitable optimization algorithm (OA). But taking into account real life constraints — such as the available computational environment and budget — the number of comparative evaluations required by an OA can become prohibitive in a specific design situation.

To overcome this drawback, several strategies have been conceived for accelerating the optimization task such as: (i) use of multiprocessing; (ii) use of better optimization algorithms; (iii) use of metamodels (surrogate models) for reducing the number of calls to the true solver model. From a strict engineering point of view, the 3rd strategy seems to be more inexpensive and universal, since it does not rely on costly hardware improvements neither on technical advances in optimization algorithms.

Metamodels (surrogate or response surface models) have been extensively used for representing expensive black box functions. A recent published monograph on the subject is already available Forrester and Keane (2008) focusing practical aspects. Praveen and Duvigneau (2007) classified surrogate models into four main categories: (i) data-fitting models, where an approximation of the expensive function is constructed using an available data bank; (ii) variable convergence models, where the expensive function depends of the numerical solution of a partial differential equation with a relaxed stopping criterion; (iii) variable resolution models, where a hierarchy of grids is used and the surrogate model is just the costly evaluation tool but run on a coarse grid; (iv) variable fidelity models, where an hierarchy of physical models is used. The first category is focused in this paper.

Often the constructed metamodel itself is of prime importance for the user when it is meant for utilization without further callings to the costly model. Sometimes, however, surrogate models are employed just for accelerating global optimization algorithms which otherwise would require a prohibitive number of costly function evaluations.

In this context, provisory metamodels are iteratively constructed from points already evaluated by the costly model (Jones, 2001). Here it is not so important to construct a representative metamodel over the whole search region but just in the vicinity of promising points. But the conscientious specification of newly evaluated points for entering the data bank is very important because they must provide a selective metamodel improvement during the iterative process.

As the prior knowledge of promising points is generally impossible, the use of global metamodels is advisable because they allow the inclusion of points in any part of the search region. Allied to suitable improvement criteria, global metamodels may accelerate dramatically certain optimization algorithms. Jones et al. (2001) developed a kriging-based response surface method called Efficient Global Optimization (EGO) where the next evaluation point is chosen to be the one that maximizes the expected improvement in the objective function value. However, it remains a conjecture whether such a method converges to the global minimum of any continuous function. Gutmann (2001) developed a radial basis function method which he proved converges to the global minimum of any continuous function. His method is based on a suitable measure of bumpiness for radial basis functions following a suggestion of Jones (1996) for general response surfaces.

Regis and Shoemaker (2005) developed a Constrained Optimization using Response Surfaces (CORS) method. Cyclic search patterns for metamodel constrained optimizations are iteratively used in order to specify newly evaluated points. These patterns induce distance constraints between the points already evaluated and the metamodel optima for balancing local and global searches and also for improving the metamodel construction. The method was shown to converge to the global minimizer of any continuous function on a compact set regardless of the response surface model and the basic optimization algorithm that are used. Regis and Shoemaker (2005) used radial basis functions (CORS-RBF) for the metamodel construction and the DIRECT method of Jones (2001) for global optimization.

This paper presents an implementation of the CORS-RBF method using inverse multiquadrics for metamodel construction and the Controlled Random Search Algorithm (CRSA). CRSA is a stochastic, population-set based algorithm, capable of performing global optimization tasks efficiently. CRSA was first proposed by Price (1977) and later improved by Ali et al. (1997 and 2004). Further improvements were introduced by Manzanares-Filho et al. (2005) and applied to inverse aerodynamic problems. The present metamodel strategy is also meant for this kind of application.

The paper is outlined as follows. Section 2 describes briefly the basics of the CORS strategy. Some implementation issues are discussed in Section 3. Section 4 presents and discusses the results obtained with the application of the proposed CORS implementation to the Dixon-Szegö test functions. Test results for an inverse cascade design example are presented and discussed in Section 5. Concluding remarks are considered in Section 6.

2. THE CORS STRATEGY FOR GLOBAL OPTIMIZATION USING METAMODELS (RESPONSE SURFACES)

The strategy is meant for finding a global minimum of a box-constrained continuous function f: $D \to \Re$, where $D = \{\boldsymbol{x} \in \Re^d : x_j^L \leq x_j \leq x_j^U,\ j = 1, ..., d\}$; x_j^L and x_j^U are lower and upper bounds for the d coordinates of $\boldsymbol{x}$ respectively. A point $\boldsymbol{x}^*$ is said to be a global minimum of f if $f(\boldsymbol{x}^*) \leq f(\boldsymbol{x})$, $\forall \boldsymbol{x} \in D$. The focus is on problems where f is a black box function that is expensive to evaluate. Thus it is important to find a point $\boldsymbol{x}' \in D$ such that $f(\boldsymbol{x}')$ is close to $f(\boldsymbol{x}^*)$ using only a relatively small number of function evaluations.

The following text is extracted from reference (Regis and Shoemaker, 2005): "The strategy is iterative and, in each iteration, the response surface model is updated and exactly one point is selected for costly function evaluation. The evaluation point is selected to be one that minimizes the current response surface model subject to the given constraints (as specified by D) and to some constraints on the distance from previously evaluated points. The guiding principle behind this method is that the selection of points for costly function evaluation has the dual goals of: (a) finding new points that have a low objective function value, and (b) improving the future response surface model by sampling regions of D for which little information exists. Hence, the selection of the next point for costly function evaluation is based on the minimization of current response surface model subject to constraints on how close the next point evaluated can be to previously evaluated points. Of course, there is a limit on how far a point can be from a previously evaluated point. If $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n$ are the previously evaluated points, then this limit is given by"

$$\Delta = \max_{\boldsymbol{x}' \in D} \min_{1 \leq j \leq n} \left\| \boldsymbol{x}' - \boldsymbol{x}_j \right\| \tag{1}$$

A point satisfying Eq. (1) is called a *maxmin point*. A scheme for calculating Δ is assumed. The next evaluation point is required to be at a distance no less than $\beta\Delta$ from all previously evaluated points, where $0 \leq \beta \leq 1$. The CORS general algorithm is given below (Regis and Shoemaker, 2005).

Step 1 (*Select initial points*). Set $i := 1$ and select a finite initial set of points $S_1 = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_k) \subseteq D$ for costly function evaluation.

Step 2 (*Do costly function evaluation*). Evaluate the function f at the points in S_1 and update the best function value encountered at every function evaluation.

Step 3 (*Iterate*). While termination condition is not satisfied do

Step 3.1 (*Fit or update response surface*). Fit or update a response surface model f_i' using the data points $D_i = \{(\boldsymbol{x}, f(x)\text{: } \boldsymbol{x} \in S_i\}$

Step 3.2 (*Select candidate point: auxiliary problem*). Select the candidate point $\boldsymbol{x}_{k+i}$ for function evaluation to be a point $\boldsymbol{x}$ that solves the following constrained optimization problem:

Minimize $f'(x)$
Subject to

$$\|\boldsymbol{x} - \boldsymbol{x}_j\| \geq \beta_i \Delta_i, \quad j = 1, \ldots, k+i-1$$
$$x \in D \tag{2}$$

where

$$\Delta = \max_{\boldsymbol{x}' \in D} \min_{1 \leq j \leq k+i-1} \|\boldsymbol{x}' - \boldsymbol{x}_j\| \tag{3}$$

and $0 \leq \beta_i \leq 1$ is a parameter to be set by the user.

Step 3.3 (*Do costly function evaluation*). Evaluate the function f at $\boldsymbol{x}_{k+i}$ and update the best function value encountered so far.

Step 4.4 (*Update information*). $S_{i+1} := S_i \cup \{\boldsymbol{x}_{k+i}\}$; $D_{i+1} := D_i \cup \{(\boldsymbol{x}_{k+i}, f(\boldsymbol{x}_{k+i}))\}$

Reset $i := i + 1$.
End.

Above, k is the number of initial evaluations points and i is the iteration counter. A search pattern $\langle \beta_1, \beta_2, \ldots, \beta_N \rangle$ is chosen with $1 \geq \beta_1 \geq \beta_2 \geq \ldots \geq \beta_N$ and applied in cycles of N iterations such that $\beta_i = \beta_{i+N}$. Regis and Schoemaker (2005) showed that for CORS strategy converges to a global minimum in D it is sufficient that the constraints in Eq. (2) be satisfied and the search pattern have at least one nonzero entry. The metamodel construction method and the optimization tools employed for solving the auxiliary problem in Step 2.3 are not fundamental for convergence. However, all these implementation aspects may be important for the convergence fastness of CORS strategy.

3. IMPLEMENTATION ISSUES

Regis and Schoemaker (2005) employed thin plate splines (radial basis functions) augmented by linear polynomials for the metamodel construction. For the auxiliary problem, they employed the DIRECT method of Jones (2001) for global optimization in conjunction with a gradient-based local solver (fmincon of Matlab Optimization Toolbox). Here we propose the use of other tools.

3.1 Metamodels Construction

The metamodels construction is made with inverse multiquadric radial basis functions and also whit thin plate splines, both augmented by a linear polynomial. Assume that we have n distinct points $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n \in \Re^d$ where the function values $f(\boldsymbol{x}_i)$ are known. The function interpolation in a point $\boldsymbol{x}$ is of the form

$$s(\boldsymbol{x}) = \sum_{i=1}^{n} \lambda_i \, \phi \|\boldsymbol{x} - \boldsymbol{x}_i\| + \mu_0 + \sum_{i=1}^{d} \mu_i x_i, \qquad \boldsymbol{x} \in \Re^d \tag{4}$$

where $r = \|\cdot\|$ is the Euclidean norm in $\Re^d$ and x_i is the i^{th} coordinate of the point $\boldsymbol{x}$; λ_i ($i = 1, \ldots, n$) and μ_i ($i = 0, \ldots, d$) are real coefficients to be determined.

The thin plate spline radial basis function is given by

$$\phi(r) = r^2 \log r \tag{5}$$

The inverse multiquadric radial basis function is given by

$$\phi(r) = \frac{1}{\sqrt{r^2 + c^2}} \tag{6}$$

where c is called *attenuation factor* having a strong influence on the interpolation. Increasing the c value makes the interpolation more predictive at expenses of ill-conditioning.

Consider the following matrices $\Phi \in \Re^{n\,x\,n}$ and $P \in \Re^{nx(d+1)}$ constructed by the radial basis functions and the linear polynomial respectively:

$$(\Phi)_{ij} = \phi\|\boldsymbol{x} - \boldsymbol{x}_i\|, \; i, j = 1, \ldots n, \tag{7}$$

$$P = \begin{pmatrix} 1 & x_{11} & \ldots & x_{1d} \\ \vdots & \vdots & x_{ij} & \vdots \\ 1 & x_n & \ldots & x_{nd} \end{pmatrix} \tag{8}$$

According to Powell (1992), the function interpolating the points (($\boldsymbol{x}_1$, $f(\boldsymbol{x}_1)$, …, ($\boldsymbol{x}_n$, $f(\boldsymbol{x}_n)$) is obtained by solving the system,

$$\begin{pmatrix} \Phi & P \\ P^T & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} F \\ 0_{d+1} \end{pmatrix}, \tag{9}$$

where $F = (f(x_1)), \ldots, f(x_n))^T$, $\lambda = (\lambda_1, \ldots, \lambda_n)^T \in R^n$ and $(\mu_0, \ldots, \mu_d)^T \in R^{d+1}$.

The coefficient matrix of order $n+d+1$ in Eq. (9) is symmetric and positive definite for both the inverse multiquadric function given in Eq. (6) and for the thin plate spline function given in Eq. (5). Thus the matrix is invertible and the system in Eq. (4) is solvable. Without the polynomial term the positiveness property is preserved only for the inverse multiquadric.

3.2 Approximation Example

Some preliminary tests were carried out for evaluating the approximation performance of the interpolations used in this paper. Two different Dixon-Szegö test functions were chosen for this purpose (see Section 4, Table 1). Two hundred points randomly distributed over the search space were employed.

Figures 1a and 1b show the approximation results for the Branin function using the thin plate spline with linear polynomial and the inverse multiquadric with linear polynomial, respectively. Both interpolations show good performance, but the multiquadric is clearly superior to the thin plate spline.

Figures 2b and 2a show the approximation results for the Goldstein-Price function using the inverse multiquadric with and without linear polynomial, respectively. Both interpolation show good performance and the polynomial addition has no substantial effect.

The results presented in Figs. 1 and 2 indicate that a metamodeling approximation using radial basis functions can be a good option for treating global optimization problems.

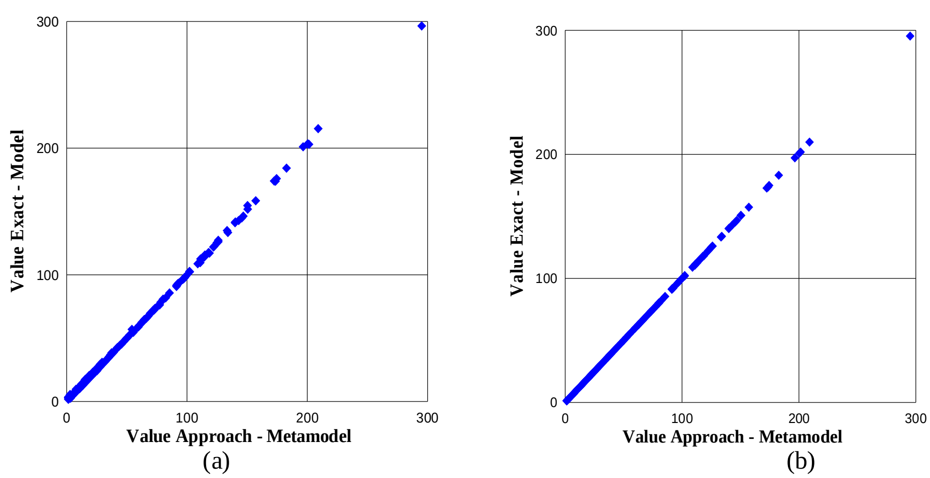
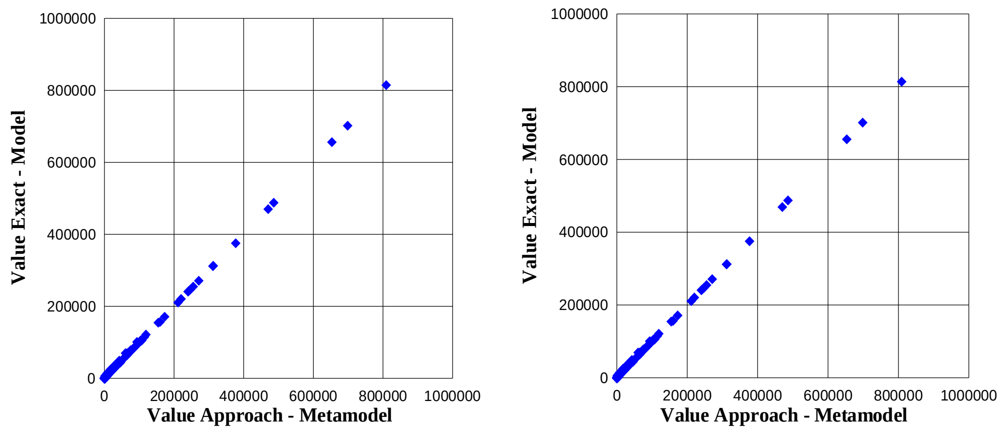


Figure 1 : Results for interpolation – function Branin, (a) Using thin plate spline radial basis function with a linear polynomial, (b) Using multiquadric inverse radial basis function with a linear polynomial (c = 0.5).



(a) (b)

Figure 2: Results for interpolation – function Goldstein Price, (a) Using multiquadric inverse radial basis function (c = 0.5), (b) Using multiquadric inverse radial basis function with a linear polynomial (c = 0.5).

3.3 Optimization Tool

For the solution of the auxiliary problem in Step 3.2 of CORS strategy we propose here the use of a Controlled Random Search Algorithm (CRSA). CRSA is an iterative, stochastic, population-set based algorithm, which promotes the substitution of the worst point of the population by a better one at each iteration (Price, 1977). The CRSA version used in this paper adopts improvements introduced by Ali et al. (1997 and 2004) and Manzanares-Filho et al. (2005). The extrema of coordinate-wise parabolic interpolations are used for selecting candidate points for entering the population. A penalty function scheme is used for treating the constraints in Eq. (3).

The CRSA may be applied several times in Step 3.2 in order to even out severe fluctuations due to the use of random numbers. When this is done, the better solution is adopted as the candidate point.

4. COMPUTATIONAL EXPERIMENTS

The proposed CORS implementation was tested on the Dixon-Szegö test functions (1991). Table 1 shows the characteristics of these functions. Their actual functional expressions can be found in reference (Dixon and Szegö, 1991).

Table 1: The Dixon-Szegö test functions (Dixon and Szegö, 1991)

Test function	Dimension	Domain	Nº of local minima	Nº of global minima	Global min value
Branin	2	[-5,10] x [0,15]	3	3	0.398
Goldstein-Price	2	[-2,2]	4	1	3
Hartman3	3	[0,1]	4	1	-3.86
Shekel5	4	[0,10]	5	1	-10.1532
Shekel7	4	[0,10]	7	1	-10.4029
Shekel10	4	[0,10]	10	1	-10.5364
Hartman6	6	[0,1]	4	1	-3.32

Each function was tested by using 10 independent runs. The CRSA was applied 10 times for solving the auxiliary problem in Step 3.2. A population equal to 10(d+1) is adopted, following a suggestion of Ali et al. (1997) (d is the problem dimension). Within the CORS framework, the CRSA stopping criterion was a population contraction tolerance below 10^{-4} (absolute difference between the function values of the worst and best points in the population) or the maximum number of functions evaluations equal to 10,000. The criterion of convergence of CORS was to get a relative error between calculated and known minimum values less than 1%. Besides the CORS strategy evaluation, tests were also made using the CRSA directly for comparison. In this case, the same criterion of convergence of CORS was used.

A run is considered unsuccessful when the 1% criterion is not satisfied after 10,000 function evaluations. The ratio between the successful runs and the total runs is called the ratio of success, RS.

During the tests it was observed that the attenuation factor c in the inverse multiquadrics exerted a strong effect on the convergence fastness. The Shekel functions require smaller values of c than the other functions. In the future it will be advisable to implement a scheme for optimizing the attenuation factor for decreasing this effect (Praveen and Duvgneau, 2007).

Table 2: Comparison of CORS implementations and direct CRSA on the Dixon-Szegö test functions

Test function	c	$CORS^{PP}$* (average)	$CORS^{PP}$ ** (average)	$CORS^{PP}$*** (average)	CORS [7]	CRSA (average)	CRSA RS
Branin	0.50	19.8	18.4	51.7	34 / 40	138	100 %
Goldstein-Price	0.50	46.5	47.1	59.7	49 / 64	257	100 %
Hartman3	0.50	36.3	33.1	46.7	25 / 61	113	100 %
Shekel5	0.05	57.6	103.4	142.8	41 / 52	621	55 %
Shekel7	0.05	57.3	64.5	94.1	46 / 64	588	55 %
Shekel10	0.05	46.0	62.8	76.6	51 / 64	553	80 %
Hartman6	0.50	102.0	62.6	82.6	104 / 108	427	80 %

The values in the table (except for c and RS) indicate the number of function evaluations to get a relative error less than 1%
* Results for inverse multiquadric radial basis function
** Results for inverse multiquadric radial basis function with a linear polynomial
*** Results for thin plate spline radial basis function with a linear polynomial (not is necessary a *attenuation factor*)

Table 2 shows the obtained results using three different radial basis function models with the search pattern ⟨0.95, 0.50, 0.25, 0.005, 0.0005, 0.00⟩, $N = 6$. The value of c used for each function is also included. $CORS^{PP}$ denotes the implementation of the present paper. The best and worst results obtained by Regis and Shoemaker (2005) using two different search patterns are also shown. The last two columns list the average values and the rate of success RS obtained with the direct application of CRSA.

Analyzing the results in Table 2, it can be noted that the choice of radial basis function or attenuation factor has a large influence on results. The inverse multiquadric, with or without polynomial, showed better results than the thin plate spline with polynomial. The polynomial addition is not always beneficial for the inverse multiquadric. On the other hand, the thin plate spline always requires the polynomial addition. Without it, the thin plate spline requires a very large number of expensive function evaluations (these results are not shown here). Maybe this behavior explains why Regis and Shoemaker (2005) presented results only for the thin plate spline with polynomial.

Using the average values of $CORS^{PP*}$ and CRSA, we see that the metamodeling strategy was able to reduce the number of function evaluations substantially. The minimum acceleration occurred for the Hartman3 function (~3 times) while the maximum acceleration occurred for the Shekel10 function (~12 times). Further, it is important to report that the rate of success of the CORS was 100% for all the tests what did not always occur with CRSA. Thus, the $CORS^{PP}$ was able not only to accelerate the optimization of costly functions but also to improve the reliability and robustness of the search process.

Table 3: Comparison of CORS implementations and direct CRSA on the Dixon-Szegö test functions with two different search pattern (thin plate spline with a linear polynomial)

Test function	$CORS^{PP}$ - SP1 - *** (average)	$CORS^{PP}$ (min /max)	$CORS^{PP}$ – SP2- *** (average)	$CORS^{PP}$ (min / max)
Branin	51.7	44 / 69	37.3	33 /40
Goldstein-Price	59.7	46 / 87	70.6	46 / 86
Hartman3	46.7	24 / 86	50.9	24 / 96
Shekel5	116.4	87 / 130	82.6	40 / 124
Shekel7	94.1	57 / 128	70.0	46 / 82
Shekel10	76.6	52 / 104	79.0	58 / 88
Hartman6	142.8	118 / 164	129.2	104 / 176

Table 3 shows results with two different search pattern — SP1 ⟨0.95, 0.50, 0.25, 0.005, 0.0005, 0.00⟩ and SP2 ⟨0.90, 0.75, 0.25, 0.05, 0.03, 0.00⟩. Note that the search pattern also influences the convergence of the method. SP2 was also used by Regis and Shoemaker (2005) for their better results. In general, the average results obtained with SP2 were better than those with SP1, except for Goldstein-Price and Shekel10 functions. The minimum and maximum numbers of functions evaluations along the 10 independent runs are also listed.

From the results of Tables 2 and 3, it is possible to conclude that the implementation of CORS presented in this paper is in general competitive with the original implementation of Regis and Shoemaker (2005).

5. INVERSE CASCADE DESIGN EXAMPLE

The shape aerodynamic problem treated in this example is that of an inverse airfoil cascade design. This kind of problem appears in dimensioning of the blade shape on cylindrical surface intersections of an axial flow turbomachine. The problem consists in determining the geometrical cascade parameters in order to satisfies a prescribed pressure or velocity distribution on the airfoil contour (the target distribution) and some other global characteristics like flow turning angle or lift coefficient. This problem will be treated as an optimization problem by seeking the minimum of a norm of the difference between the target distribution and the corresponding distribution resulting from the optimization process.

5.1. Airfoil Cascade Description

The parameters of a rectilinear airfoil cascade are illustrated in Fig. 3: the airfoil stager angle β, the chord to-spacing ratio (solidity) $\sigma = c/s$, the geometrical parameters defining the airfoil shape, the flow velocities and angles (W_1, β_1) and (W_2, β_2) far upstream and downstream of the cascade, respectively. According to the flow model adopted, parameters like Reynolds and Mach numbers can also appear.

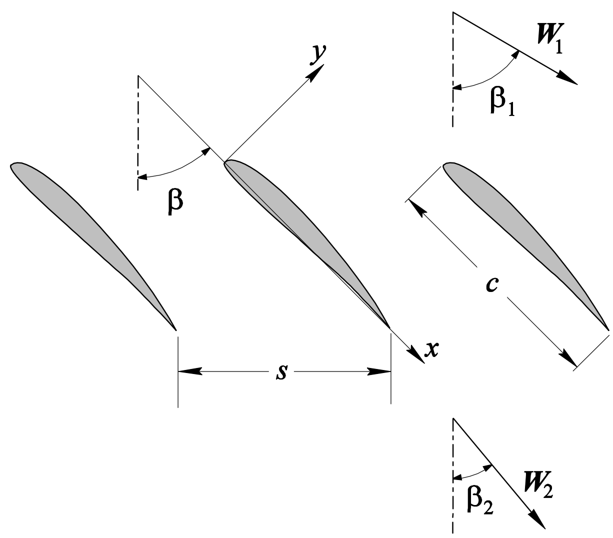


Figure 3: Scheme of a rectilinear cascade of airfoils

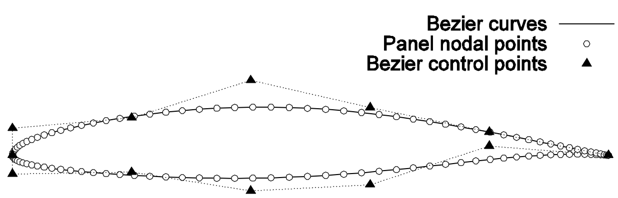


Figure 4: Approximation of airfoil NACA 65_1-412 by Bezier curves

5.2 Airfoil Shape Parameterization

The lower and upper surfaces of the airfoil are represented by Bezier curves of high degree. A Bezier curve of degree n_B in the (x, y) plane is defined by a set of $n_B + 1$ control points $\mathbf{P}_i = (Px_i, \; Py_i)$. A point $\mathbf{r}(x, y)$ of the curve is given by:

$$\mathbf{r}(t) = \sum_{i=0}^{n_B} \frac{n_B!}{i!(n_B - i)!} t^i (1-t)^{n_B - i} \, \mathbf{P}_i \tag{10}$$

where t represents the curve parameter varying between 0 and 1. The first control point coincides the initial curve point (t = 0); the last control point coincides the final curve point (t = 1).

The first and last control points of both curves are fixed at the airfoil leading and trailing edges respectively. The abscissas of the remaining control points are also fixed and only their ordinates are treated as design variables. Being n_{BE} and n_{BI} the degrees of upper (extrados) and lower (intrados) curves, respectively, it results a maximum number of design variables $n = n_{BE} + n_{BI} - 2$. Figure 3 shows an approximation of airfoil NACA 65_1-412 by the described Bezier curves with $n_{BE} = 6$ and $n_{BI} = 6$ and equally spaced abscissas. It is also shown a discretization of the airfoil contour using 120 panels for application of the flow solver described below.

5.3 The Flow Solver

Since the main objective of this paper is of a prospective nature, a low-fidelity flow solver is employed for the cascade design test. This solver employs a simplified viscous-inviscid interaction approach but only the inviscid potential formulation is applied in this example. The solver consists of a boundary element implementation of constant vortex panels with local curvature corrections for improving the accuracy. The Dirichlet boundary condition of null velocity in the interior of the airfoils is applied on a number of collocation points equal to the number of panels. For details on the formulation and numerical implementation of vortex element methods, including the consideration of cascade effects, the reader is referred to Lewis (1991). Details on panel methods can be found in Katz and Plotkin (1999).

5.4 Test Problem Description

For the inverse cascade test one considers a target pressure distribution used by Rogalsky et al. (1999) in one of their tests. The pressure coefficient $Cp = 1 - (w/W_1)^2$ is given as a function of the natural airfoil coordinate, $Cp(s)$. Thus the objective function $f(\boldsymbol{x})$ is simply equated to a norm of the difference between the target (Cp_{T}) and calculated (Cp) pressure coefficients, also used by Rogalsky *et al*. (1999):

$$f(\boldsymbol{x}) = \sum_{i=1}^{n_p} (C_{p\mathrm{T}} - C_p)_i^{\,2} \tag{11}$$

where n_p is the number of panels used in the airfoil discretization and i is the collocation point index. Besides the cascade stager angle β and solidity σ the only shape parameters considered as design variables are the ordinates of the three upper control points close to the airfoil leading edge, Py_1, Py_2, Py_3. The remaining Bezier parameters are fixed for test purposes. Thus, the dimension of the search space is 5 and the vector of design variables is $\boldsymbol{x} = (Py_1, Py_2, Py_3, \beta, \sigma)$. This reduced dimension allows the use of a full factorial sampling plan for the CORS initialization in Step 1. Future tests with higher dimensions may require other types of sampling plans (Latin hypercubes, for instance).

5.5 Results

At first the CRSA was applied directly for solving the cascade test problem. The flow inlet angle was fixed, $\beta = 30^{\mathrm{o}}$. The same parameters used for testing the Dixon-Szegö functions were set. Ten independent runs were carried out in order to identify a possible global minimum in the box constrained by $\boldsymbol{x}^L$ = (0.03, 0.025, -0.02, 0.5, 20) and $\boldsymbol{x}^U$ = (0.10, 0.70, 0.40, 1.5, 30). All tests converged to a solution vector $\boldsymbol{x}' \approx$ (0.0460, 0.288, 0.0666, 28.1, 0.906) with the final function value $f(\boldsymbol{x}') \approx 2.01553$. For these initial tests, a population contraction tolerance of 10^{-7} was assumed and the average number of function evaluations for convergence was 7254.

The final value of the objective function is very high for the norm defined in Eq. (8), indicating that the solution of the inverse problem would not be acceptable for practical purposes, i.e., the target and final pressure distributions were not sufficiently close. Probably it would not be possible to improve the situation using the parameterization shown in Fig. 4 in addition to the variation of only three Bezier parameters. But the main goal of the test is just to evaluate the CORS performance as applied to aerodynamic inverse problems and not to solve a real cascade design problem. The use of better parameterization schemes and more shape parameters will overcome these drawbacks in future tests.

Figure 5 shows the target and final pressure distributions confirming their bad concordance. Figure 6 shows the final airfoil shape. It is very like a Liebeck airfoil reported by Rogalsky *et al*. (1999).

Aiming comparison with the $\mathrm{CORS}^{\mathrm{PP}}$ strategy, additional tests of CRSA were made. A smaller box including $\boldsymbol{x}'$ was considered: $\boldsymbol{x}^L$ = (0.030, 0.025, 0.00, 0.7, 25) and $\boldsymbol{x}^U$ = (0.050, 0.300, 0.080, 1.0, 30). Ten new independent direct runs of CRSA were then carried out. For just attaining a relative error below 1% from the final value $f(\boldsymbol{x}') \approx 2.01553$, an average value around 421 function evaluations was shown to be sufficient in these new CRSA tests. The minimum and maximum numbers of function evaluations were 177 and 1500, respectively.

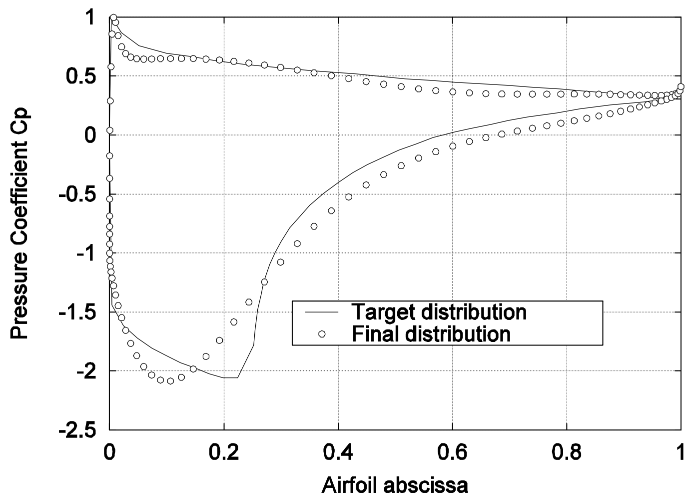


Figure 5: Target and final pressure distributions

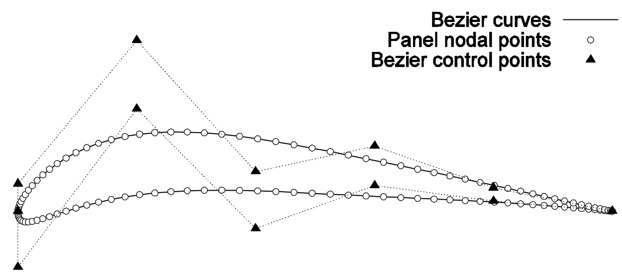


Figure 6: Final airfoil shape

Finally, the CORS^{PP} strategy was evaluated using the second box defined in the last paragraph. Only the inverse mutiquadric without polynomial was used. The same parameters used for testing the Dixon-Szegö functions were assumed, with the exception of the multiquadric attenuation factor. Now a value of c = 1 was shown to be quite adequate. Ten independent runs of the CORS^{PP} strategy were carried out. The average, minimum and maximum numbers of function evaluations for just attaining a relative error below 1% from the final value $f(\mathbf{x}') \approx 2.01553$ were 99, 83 and 120, respectively. An average acceleration factor around 4.6 was observed.

6. CONCLUDING REMARKS

This paper discussed an alternative implementation of the Constrained Optimization using Response Surfaces (CORS) method originally proposed by Regis and Shoemaker (2005). The response surfaces (metamodels) are constructed using inverse multiquadrics and thin plate spline radial basis functions with a linear polynomial. The optimization process is guided by a Controlled Random Search Algorithm (CRSA).

The CORS strategy presented in this paper was applied on the Dixon-Szegö test functions an also on an example of inverse cascade design. The results have shown that the strategy is promising in accelerating and improving the reliability of CRSA. Nevertheless some issues deserve more attention in the future, mostly the adequate choice of the attenuation factor used in the inverse multiquadrics. This value is problem dependent and strongly influences the CORS convergence fastness. Thus it is advisable to implement an automatic scheme for optimizing this factor in order to construct the provisory metamodels with more confidence and robustness.

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