MAGNETO-HYDRODYNAMIC DOUBLE-DIFFUSIVE CONVECTION IN A SQUARE CAVITY WITH AIDING AND OPPOSING TEMPERATURE AND CONCENTRATION GRADIENTS

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Abstract. *The finite-volume method is used to predict numerically the characteristics of Magneto-hydrodynamic double-diffusive convective flow of a binary gas mixture in a square cavity with the upper and lower walls being insulated. Constant temperatures and mass concentrations are imposed along the left and right walls of the enclosure and a uniform magnetic field is applied in the x-direction. Numerical results are reported for the effect of the Hartmann number on the contours of streamline, temperature, mass concentration. In addition, results for the average Nusselt and Sherwood numbers are presented and discussed for various parametric conditions. In this study, the thermal and compositional buoyancy forces are assumed to be aiding and opposite.*

Keywords*: CFD, Double diffusive, Hydro magnetic, Finite volume method*

1. INTRODUCTION

Natural convection is of great importance in many industrial applications. Convection plays a dominant role in crystal growth in which it affects the fluid-phase composition and temperature at the phase interface; it is the foundation in modern electronics industry to produce pure and perfect crystals to make transistors, lasers rods, microwave devices, infrared detectors, memory devices, and integrated circuits. Natural convection adversely affects local growth conditions and enhances the overall transport rate. In addition, the applications of a magnetic field in various research areas has significantly increased in recent years. The development of super-conducting magnets has allowed the generation of magnetic fields up to 20 Tesla (or higher with hybrid magnets) as reported by [1 - 3].

Many researchers studied the simple rectangular and square enclosure with temperature gradients experimentally and numerically work in clean enclosure or filed by porous materials. A good review was reported by [4 - 6].

One of the effective means practiced in industry for thermally induced melt flow control is magnetic damping, which is derived from the interaction between an electrically conducting melt flow and an applied magnetic field to generate an opposing Lorentz force to the convective flows in the melt. The damping effects depend on the strength of the applied magnetic field and its orientation with respect to the convective flow direction.

Substantial theoretical and numerical work, thus far, has appeared on magnetic damping for natural convection was reported by [7 - 8] conducted a numerical analysis of the magnetic damping effect in a cubic cavity with two vertical walls at different temperatures. They found that strongest damping effect is achieved with magnetic field applied perpendicular to the hot wall. This is consistent with the work of [10], who use an asymptotic approach, and found that for a rectangular box, the damping effect is the weakest when the applied magnetic field is horizontal and parallel to the hot wall.

During the magnetic liquid encapsulated Czochralski (MLEC) growth of compound semiconductor crystal, a single – crystal seed in lowered through the encapsulate which initiates solidification and crystal growth begins in the presence of an externally applied magnetic field. [11] Presented a model of dopant transport during MLEC process. Previous investigators have studied the effect of a steady magnetic field on two-dimensional natural convection in rectangular enclosures. [12 - 13] conducted a model of dopand transport during Bridgman crystal growth with magnetically damped buoyant convection and followed it by a parametric study of segregation effects during vertical

Bridgman crystal growth with an axial magnetic field. Recently [14 - 18] conducted a several numbers of numerical researches in different methods of crystal growth with electric and magnetic fields in alloys manufacture.

[19] Studied numerically the hydromagnetic double-diffusive convection in a rectangular enclosure with opposing temperature and concentration gradients. They found that the effect of magnetic field reduced the heat transfer and fluid circulation within enclosure. Also, they concluded that the average Nusselt number increased owing to the presence of the heat sink while it decreases when heat source was present. And they reported that the periodic oscillatory behavior in the stream function inherent in the problem was decayed by the presence of the magnetic field. This decay in the by transient oscillatory behavior was speeded up by the presence of a heat source. [21] extended their previous work by changing the boundary conditions of verticals walls to be a constant heat and mass fluxes.

Finally, [**Error! Reference source not found.**] did a parametric study and extension for [19 - 21] works. A wide range for thermal Rayleigh number was studied from 10^3 to 10^6. This range covers most of engineering and industrial applications.

This study intends to do a complementation of [1] and applied the numerical solution in a square cavity submitted with cross gradients of temperature and concentration in the presences of magnetic fields. In addition, a strong magnetic field as required by electronics devices [1], for this reason the Hartmann number is increased to 25 .

2. MATHEMATICAL FORMULATION

Consider steady, laminar, hydromagnetic, double-diffusive natural convection inside a square cavity. The schematic of the problem is shown in Fig. 1. Constant values of temperature and mass concentration are imposed along the left and right walls while the top and bottoms walls are keep adiabatic and impermeable to mass transfer. The enclosure is filled with an electrically conducting and in this work is not admitting the internal heat generating or absorbing binary gas mixture. A magnetic field with uniform strength B_0 is applied in the $x-$direction normal to left and right walls. The fluid is assumed to be incompressible, viscous and Newtonian. Both the viscous dissipation and Joule heating are assumed to be negligible. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. No electric field is assumed to exist and the MHD Hall effect is negligible.

The governing equation for this work are based on two-dimensional balance of mass, linear momentum, thermal energy and mass concentration of chemical species modified to include the magnetic field. Taking account the previous assumptions along with Boussinesq approximations with opposite thermal and compositional buoyancy forces, these equations can be written in dimensional form as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2}$$

$$\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-g\beta_T\left(T-T_{ref}\right)+g\beta_c\left(c-c_{ref}\right)-\frac{\sigma B_0{}^2}{\rho^2}\mu \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right) \tag{5}$$

Where x and y are the horizontal and vertical distances, respectively. u,v,p,T and c are the velocity components in $x-$ and $y-$ direction , pressure, temperature and concentration of chemical species, respectively. β_T and β_c are the thermal and compositional expansion coefficients, respectively. ρ,μ,α,c_p And D are the fluid density, dynamic viscosity, thermal diffusivity, specific heat at constant pressure and the species diffusivity respectively. g,σ and B_0 are the gravitational acceleration, electrical conductivity, magnetic induction. T_{ref} and c_{ref} are the reference temperature and concentration.

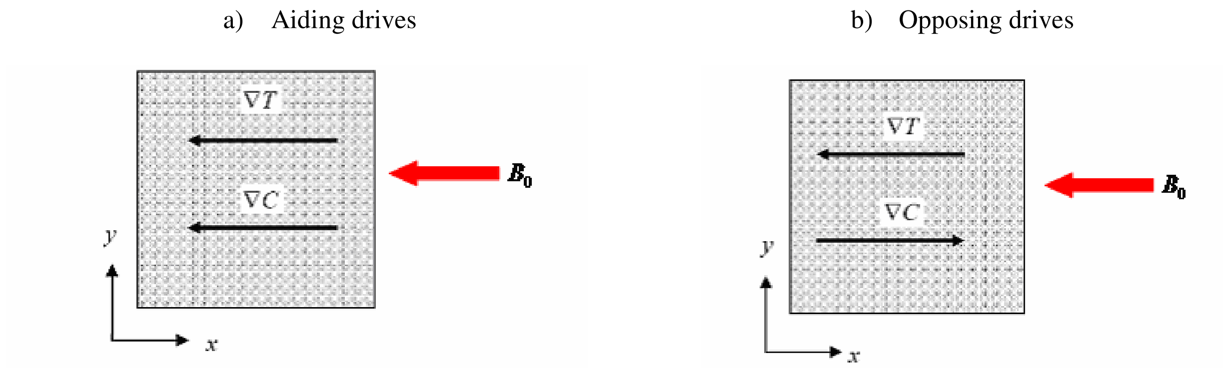


Figure 1 –Schematic diagram enclosure and imposed conditions: a) Aiding drives: $\Delta T = \Delta c = -1$ and b) Opposing drives: $\Delta T = -1, \Delta c = +1$.

The boundary conditions for the problem can be written as

$x = x, y = 0$:

$$u = 0,\ v = 0,\ \frac{\partial T}{\partial y} = 0 \text{ and } \frac{\partial c}{\partial y} = 0$$

$x = 0, y = L$:

$$u = 0,\ v = 0,\ \frac{\partial T}{\partial y} = 0 \text{ and } \frac{\partial c}{\partial y} = 0$$

$x = 0, y = y$ - aiding flow

$$u = 0,\ v = 0,\ \frac{\partial T}{\partial y} = 1 \text{ and } \frac{\partial c}{\partial y} = 1$$

$x = 0, y = y$ - opposing flow

$$u = 0,\ v = 0,\ \frac{\partial T}{\partial y} = 1 \text{ and } \frac{\partial c}{\partial y} = 0$$

$x = L, y = y$ - aiding flow

$$u = 0,\ v = 0,\ \frac{\partial T}{\partial y} = 0 \text{ and } \frac{\partial c}{\partial y} = 0$$

$x = 0, y = y$ - opposing flow

$$u = 0,\ v = 0,\ \frac{\partial T}{\partial y} = 0 \text{ and } \frac{\partial c}{\partial y} = 1 \quad (6)$$

Where L is the height (H) and width (W) of the square enclosure (aspect ratio - $A_r = H/W = 1$).
The majors dimensionless parameters using in this research are:

$$Ha = B_0 W\sqrt{\frac{\sigma}{\mu}},\ N = \frac{Gr_S}{Gr_T},\ Gr_S = \frac{\rho^2 \beta_c \Delta c H^3}{\mu^2},\ Gr_T = \frac{\rho^2 \beta_T \Delta T H^3}{\mu^2}, \Pr = \frac{\mu}{\rho\alpha},\ Ra_T = Gr_T \cdot \Pr \text{ and } Le = \frac{\alpha}{D} \quad (7)$$

Where are Hartmann number, buoyancy ratio, solutal Grashof number, thermal Grashof number, Prandtl number, Rayleigh number and Lewis number respectively.

3. NUMERICAL METHOD

The numerical method employed for discretizing the governing equations in the control-volume approach. The flux blended deferred correction which combines linearly the Upwind Differencing Scheme (UDS) and Central Differencing Scheme (CDS), was used for interpolating the convective fluxes. The well-established SIMPLE algorithm [20] is followed for handling the pressure-velocity coupling. Individual algebraic equation sets were solved by the SIP procedure of [22]. Details on the validation of the numerical tools, optimization and checking of numerical grids here employed can be found in previous publications, which include buoyant and forced flows in porous , hybrid and clean media for different geometries and boundary conditions, for more details see refers [21-33].

The Nusselt and Sherwood numbers are averaged and evaluated along the left boundary of the square cavity which may be expressed as

$$\overline{Nu} = -\int_0^L \left(\frac{\partial T}{\partial x} \right) dy \tag{8}$$

$$\overline{Sh} = -\int_0^L \left(\frac{\partial c}{\partial x} \right) dy \tag{9}$$

The numerical computations were performed for all cases using a 82 x 82 stretched grid with several points inside the boundary layers, as shown in Fig.2. The convergence criterion required that the difference between the current and the previous iterations for all of the variables at least be 10^{-5}.

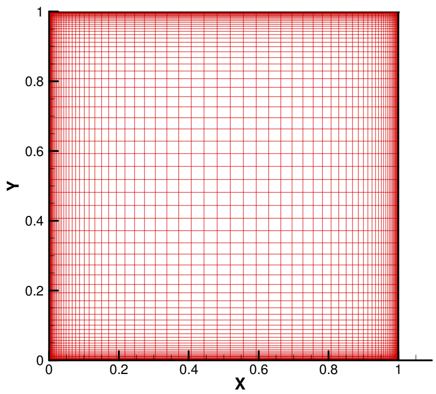


Figure 2 – Problem considered: grid used in all calculations

4. RESULTS and DISCUSSION

In the order the check on the accuracy of numerical technique employed for the problem considered in the present study, it was validate by performing, first, simulation for free convection flow in square cavity that is isothermally heated from the left, T_H , and cooled from opposing side, T_C . The other two walls are insulated. These boundary conditions are widely applied when solving buoyancy-driven cavity flows.

Figure 3 shows the Comparison between the present results and the data find in the literature for the laminar solutions with averaged Nusselt number at the hot wall. It is clear from this figure that good agreement the results exists.

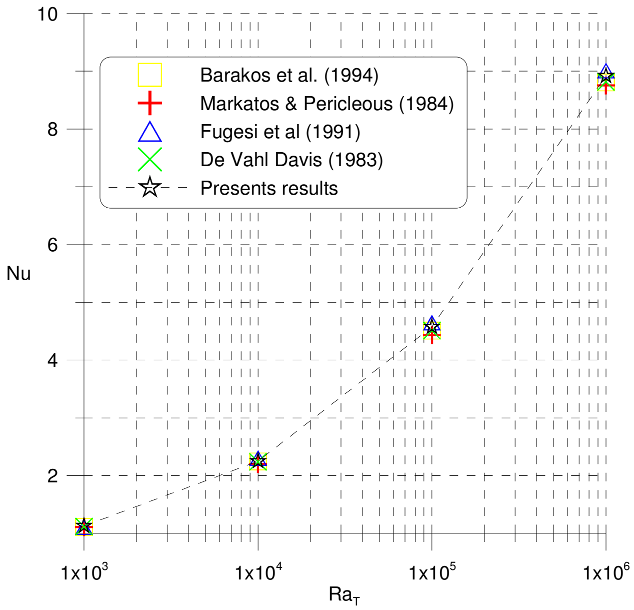
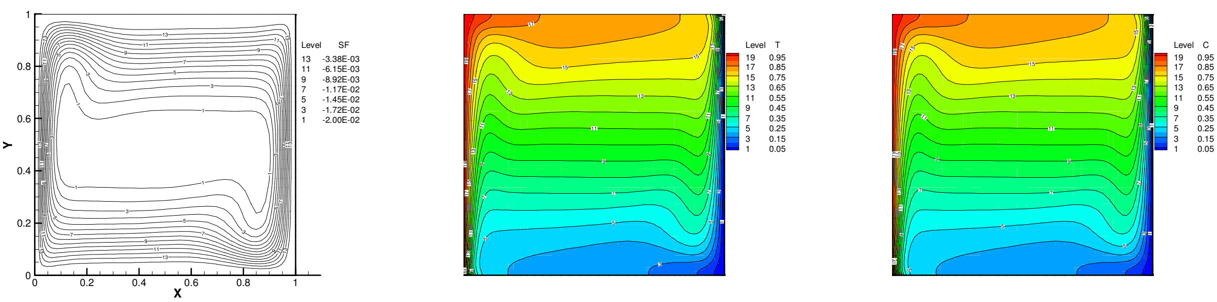


Figure 3 – Comparison between the present results and the data find in the literature for the laminar solutions with averaged Nusselt number at the hot wall

Figures 4 and 5 presents steady-state contour maps for the streamlines, temperature and mass concentrations for various values of the Hartmann number (Ha) for Le = 1, $|N|$ =1, Pr = 1.0, Ra = 1.E+06. The condition N =1.0 indicates that the flows is dominated by equal but opposing and aiding effects of both thermal and compositional buoyancies. When $N < 1.0$, the flow is primarily dominated by thermal buoyancies effects whereas for $N > 1$ the flow is mainly dominated by mass buoyancies effects.

In the absence of magnetic field (Ha = 0), Fig. 4 for the case aiding flow the streamlines is characterized by a primary recalculating eddy of relatively high velocities around the entire cavity. Because of boundary layer effects, both the temperature and mass concentration lines are characterized by sharp drops in their values near the vertical walls of the cavity. For opposing flow , in the absence of magnetic fields, the streamlines is characterized by two symmetric cell of recirculation that divided the cavity on mid – line, there are more two smalls recirculation areas on booth left and right side of cavity walls. The temperature and concentration is characterized by action of diffusion transport.

The application of transverse magnetic field has a tendency in to slow down the movement in the inner of cavity , as direct consequence of this is the recirculation cell to be stretched ,deforming and / or elongated in the vertical direction. This process continuing as the Hartmann number Ha as incremented until the flow separates forming recalculating eddies (aiding flow – two recalculating eddies, one around almost entire cavity and another , close to right wall; opposing flow – two symmetric cell regions , and on the top of cavity , in the junction of cell appears two small recalculating areas).



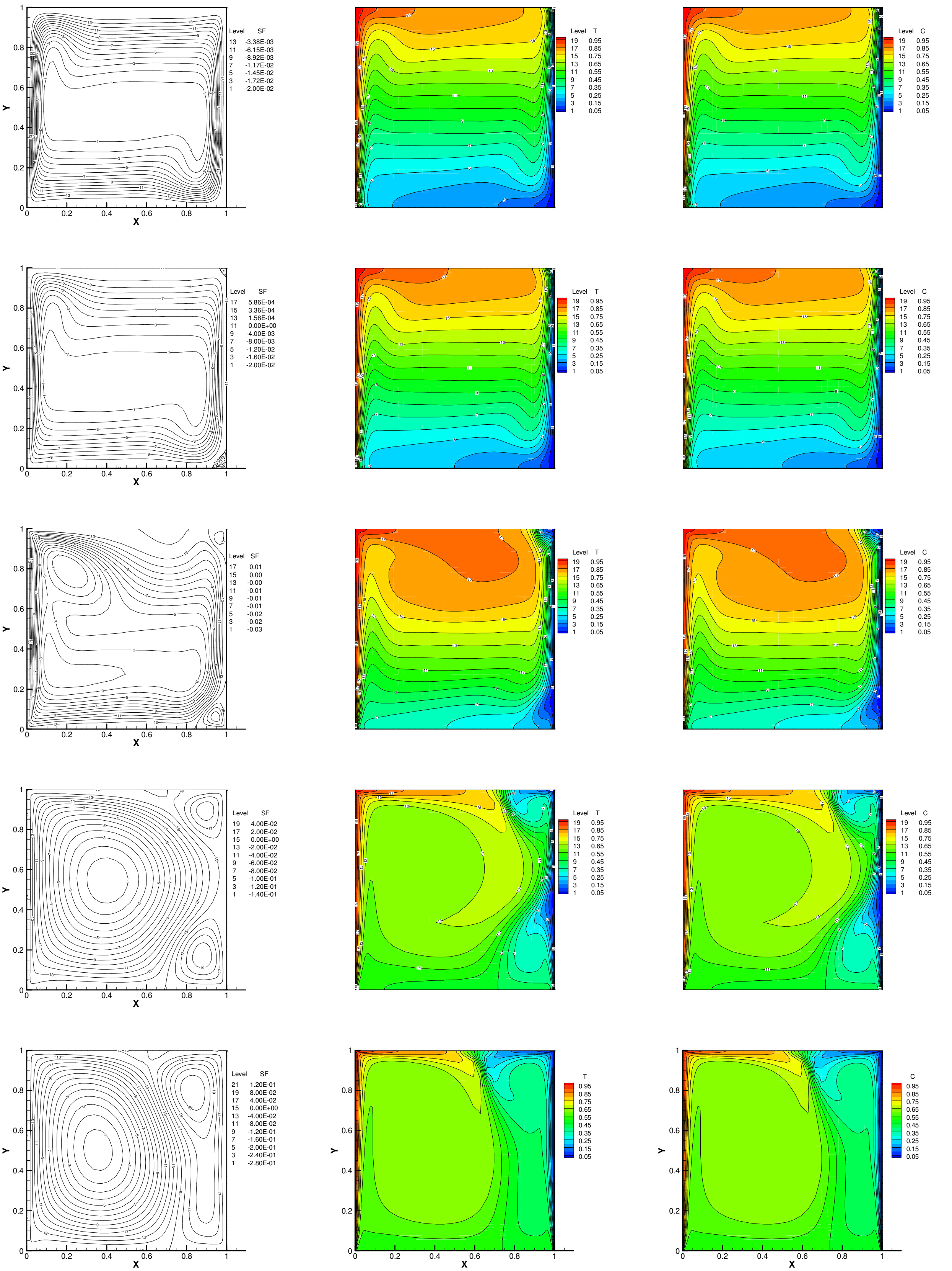
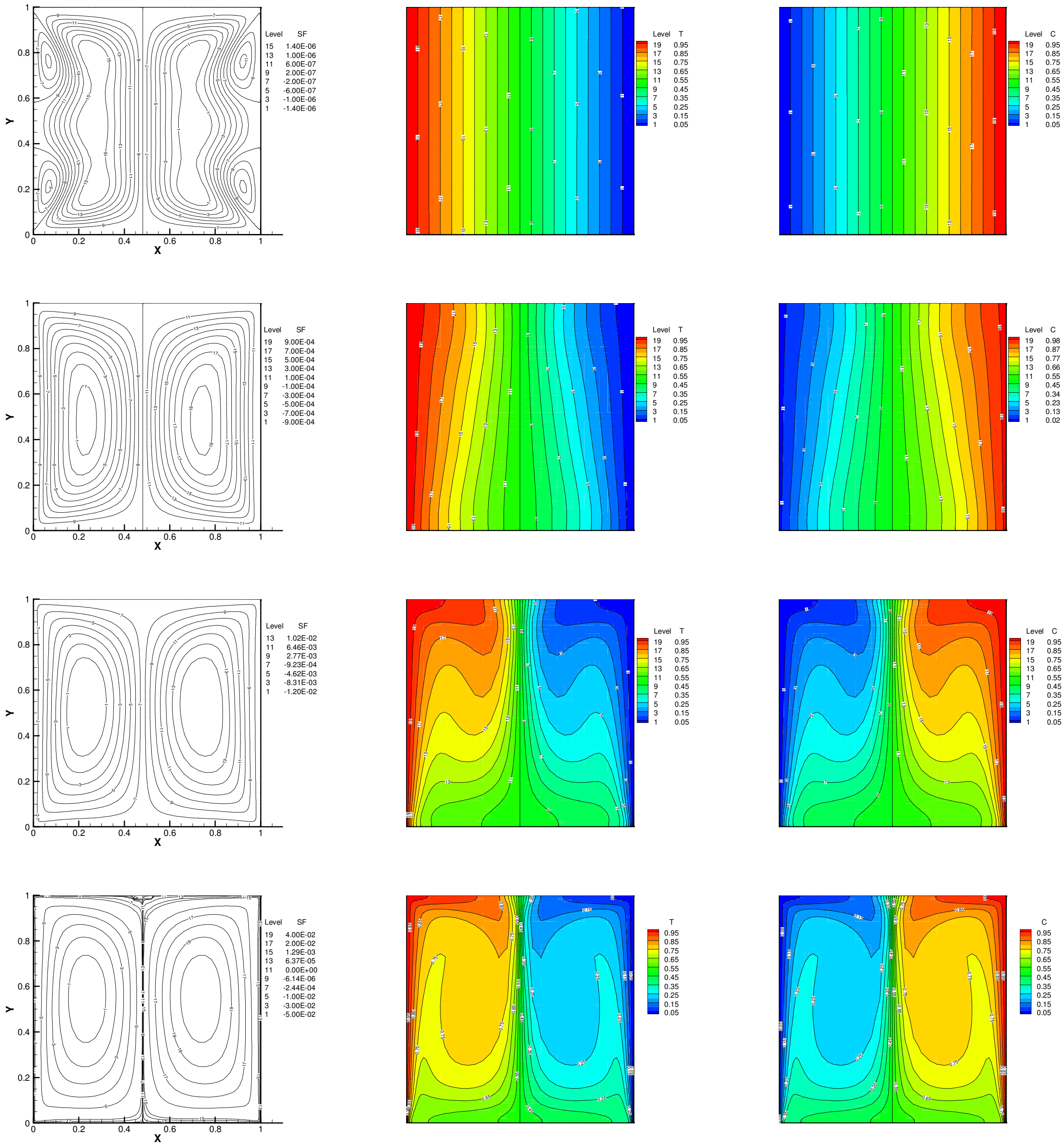


Figure 4 –Effect of Ha on the contours of streamlines, temperature and concentration for mass concentration for Le = 1, N=1 , Pr = 1.0 and Ra = 1.E+06.

Fig. 6 present typical profiles for the horizontal and vertical velocities, temperature and mass concentration at mid-section of the square cavity for several values of the Hartmann number, respectively. These profiles show an antisymmetric comportment about the mid horizontal (x =0.5) of the square cavity. In the region close of the left wall of enclosure, the component u of the velocity decreases while all of temperature and concentration increases due to increases in the Hartmann number. The opposite effect is observed in the region close of the right vertical wall of cavity. The vertical component of velocity (v) has an ambiguous comportment because in booth sides of cavity tend the assuming values equal a zero.



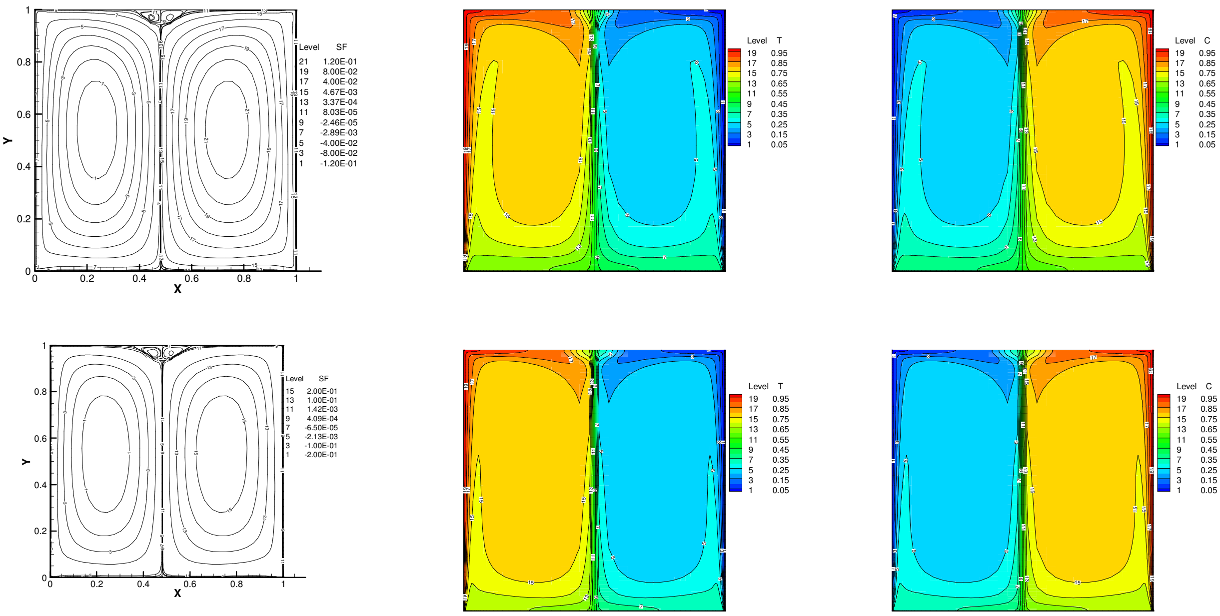
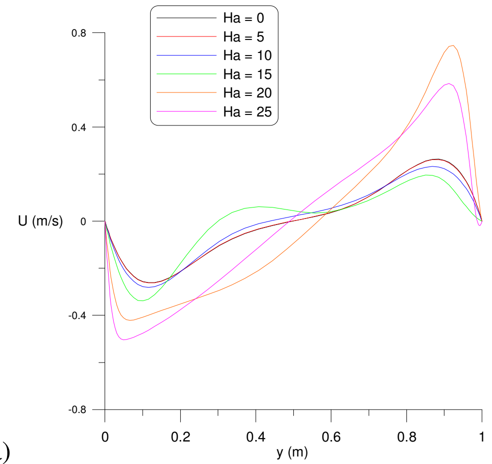
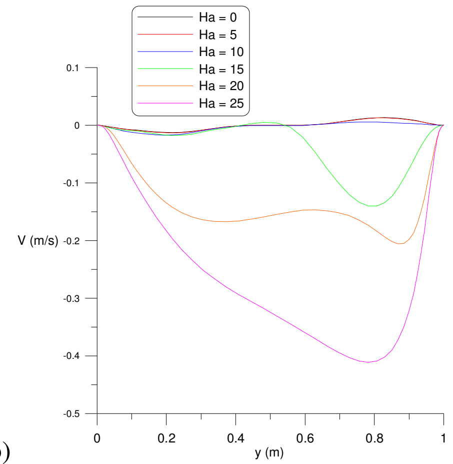


Figure 5 - Effect of Ha on the contours of streamlines, temperature and mass concentration for Le = 1, N=-1 , Pr = 1.0 and Ra = 1.E+06.

The effects of Hartmann number (Ha) on the average Nusselt number (Nu_{av}) and average Sherwood number (Sh_{av}) for the buoyancy ratio of N = 1, $Ra = 1.E+06$ and $Le = 1$ are presented in Fig. 7 a) and b) respectively. It is observed that the booth of Nu_{av} and Sh_{av} have an incrementing trend with increasing values of Ha (this comportment is more sensitive for the opposing flow cases).



a)



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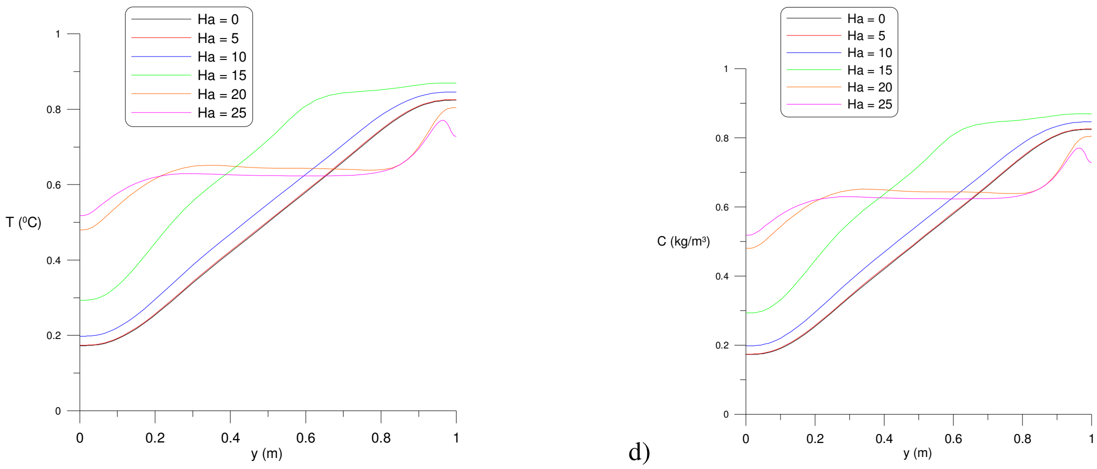

c) d)

Figure 6 – Effect of Ha number at enclosure mid-section – aiding flow: a) on x-component velocity profile b) on y-component velocity profile c)on temperatures profile and d) on concentration profile.

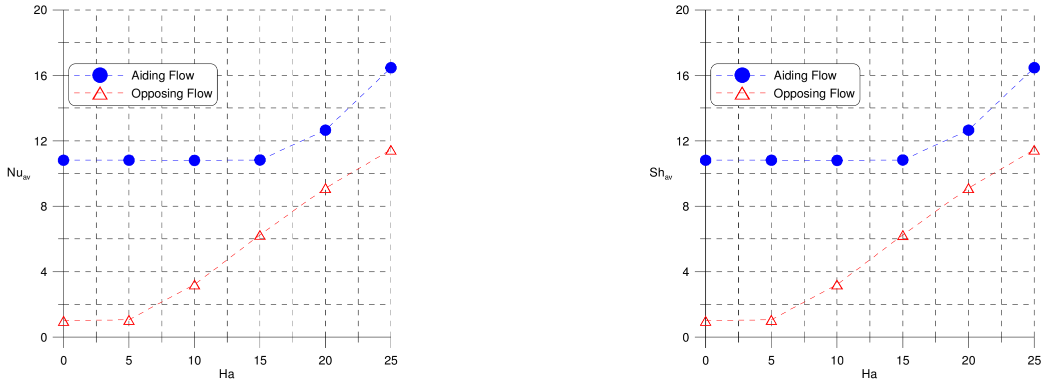

Figure 7 –Effects of Ha on the average Nusselt and Sherwood numbers for Le = 1.0, Pr = 1.0 and Ra = 1.E+06.

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