AN ALTERNATIVE FORMULATION OF TRANSFER PATH ANALYSIS APPLIED TO THE FORCE IDENTIFICATION PROBLEM

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Abstract. *The transfer path analysis is one of the most popular methods for noise and vibration transmission investigation in vibro-acoustics. The method can be used to quantify and qualify the contributions of each instrumented sound path to the global emitted noise, and to identify inputted forces as well. This paper presents an alternative formulation of the method applied to the force identification problem. Basically, the difference is in obtaining the airborn transfer functions. The conventional scheme employs airborne and structural-borne transfer functions which need to be previously measured in laboratory. In practice, it is mandatory to maintain the same relative positions for the accelerometers and microphones so that the transfer functions could be used. The alternative scheme employs measured structural transfer functions, however the airborn transfer functions are calculated during system operation for an arbitrary microphone position, which represent an advantage in time and measurement flexibility.*

Keywords*: transfer path analysis, force identification, sound contribution.*

1. INTRODUCTION

Transfer Path Analysis (TPA) is a well known investigation technique used to quantify the contributions of the vibratory energy flow through the propagation paths of the system. The system is represented by air borne and structural borne transfer functions that connect the sources to a given receiver, that is, the inputs to the outputs of the system, respectively. The whole calculation consists in two steps,

$$x_i = A_{ij}\mathrm{f}_j \quad (1)$$
$$p_k = B_{kj}\mathrm{f}_j \quad (2)$$

where A_{ij} is the *mnth* element of the frequency response function matrix (FRF) which correlates the exciting forces, f_j, with its measured responses, x_i, and B_{kj} is the *mnth* element of the FRF matrix between the exciting forces and the receiver locations, p_k. Note that the system might have many different inputs and outputs. The FRF matrix is generally obtained by taking the ratio between known exciting forces and measured acceleration responses, during offline testing.

Janssen and Verheij (1999) published an application of the TPA method for experimental identification of structural-born sound transmission in ships. Eq. (1) and (2) were combined to evaluate the importance of each respective path for any specific receiver location.

$$p_k(\omega) = B_{kj}A_{ij}^{-1}x_i(\omega) \quad (3)$$

Hence, forces are required only as an intermediate parameter for the calculation of the path contributions. As a result, the sum of partial contributions, $\{p_k\}_{partial}$, was compared with the measured total response $\{p_k\}_{total}$, indicating good agreement. An extension of the method for multiple uncorrelated sources and measured acceleration responses was written by Noumura and Yoshida (2006).

$$p_k(\omega) = h_{ki}(\omega)x_i(\omega) \quad (4)$$

Equation (4) above establishes a correlation between the system outputs (vibrational responses) and the receiver (sound pressure at the microphone position). Thus, the overall emitted noise can be portioned with respect to the vibration transducers, which are supposed to represent each transmission path.

As it can be seen, the coefficients h_i combine the effects of structural and vibro-acoustic transfer functions. If vibration transducers are placed as close as possible to the exciting forces Eq. (4) returns a good approximation for the partial sound contributions produced by each exciting force.

The bottom line is that sometimes it is not possible to access the locations of the exciting forces, then the vibro-acoustic transfer functions B_{kj} are needed, according to Eq. (2).

In the traditional application of the TPA algorithm the vibro-acoustic functions or B-functions need to be previously measured, together with the structural transfer functions, or A-functions. This paper presents an alternative formulation to identify B-functions during system operation, using A-functions and measured outputs.

2. B-FUNCTION IDENTIFICATION BASED ON SYSTEM OUTPUTS

Let us consider, for example, a multiple-input-multiple-output (MIMO) system, instrumented with vibration transducers and a microphone as a receiver, as shown in Fig. 1.

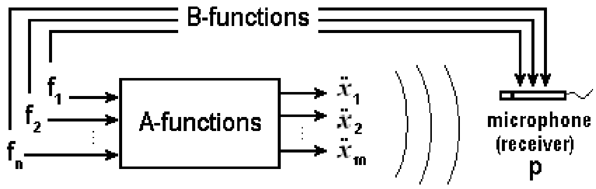


Figure 1. Schematic representation of the system with n exciting forces, m accelerometers and one microphone.

According to Eq. (1), the A-functions are the ratios between the system outputs, x_i, and system inputs, f_j, in frequency domain. Accelerations signals are conveniently used as system outputs. It writes $A_{ij}(\omega) = \ddot{x}_i(\omega)/\mathrm{f}_j(\omega)$. Similarly, the B-functions are the ratios between pressure and system inputs. It writes $B_{kj}(\omega) = p_k(\omega)/\mathrm{f}_j(\omega)$.

For a case of single excitation, f_1, and one receiver p_1, one may express Eq. (2) as a sum of m terms, where m is the number of accelerometers,

$$p_1 = \{p_1\}_1 + \{p_1\}_2 + \cdots + \{p_1\}_m = \{B_{11}\mathrm{f}_1\}_1 + \{B_{11}\mathrm{f}_1\}_2 + \cdots + \{B_{11}\mathrm{f}_1\}_m \tag{5}$$

Combining Eq. (3) and (5),

$$p_1 = \{B_{11}\}_1 A_{11}^{-1}\ddot{x}_1 + \{B_{11}\}_2 A_{21}^{-1}\ddot{x}_2 + \cdots + \{B_{11}\}_m A_{m1}^{-1}\ddot{x}_m \tag{6}$$

Equation (6) can be written for another single excitation, f_j, as follows.

$$p_k = \{B_{kj}\}_1 A_{1j}^{-1}\ddot{x}_1 + \{B_{kj}\}_2 A_{2j}^{-1}\ddot{x}_2 + \cdots + \{B_{kj}\}_m A_{mj}^{-1}\ddot{x}_m \tag{7}$$

For a time linear time invariant system it is possible to establish $\{p_k\}_m \propto \ddot{x}_m$. It means that $\{p_k\}_m$ is directly proportional to its respective transmission path, conveniently represented by $\ddot{x}_m$. Hence, the terms $\{B_{ij}\}_m A_{mj}^{-1}$ must be invariants for any fixed m, and of course for a fixed receiver location as well. It writes,

$$h_{ki} = \{B_{kj}\}_i A_{ij}^{-1} \tag{8}$$

Now, considering the case of full excitation, Eq. (2) returns,

$$\begin{aligned} p_k &= [B_{k1}\mathrm{f}_1] + \cdots + [B_{kn}\mathrm{f}_n] \\ p_k &= [\{B_{k1}\}_1 A_{11}^{-1}\ddot{x}_{11} + \cdots + \{B_{k1}\}_m A_{m1}^{-1}\ddot{x}_{m1}] + \cdots + [\{B_{kn}\}_1 A_{1n}^{-1}\ddot{x}_{1n} + \cdots + \{B_{kn}\}_m A_{mn}^{-1}\ddot{x}_{mn}] \\ p_k &= [h_{k1}\ddot{x}_{11} + \cdots + h_{km}\ddot{x}_{m1}] + \cdots + [h_{k1}\ddot{x}_{1n} + \cdots + h_{km}\ddot{x}_{mn}] \\ p_k &= h_{k1}(\ddot{x}_{11} + \cdots + \ddot{x}_{1n}) + \cdots + h_{km}(\ddot{x}_{m1} + \cdots + \ddot{x}_{mn}) \end{aligned} \tag{9}$$

The terms $\ddot{x}_{mn}$ represent the acceleration measured by each transducer 'm' due to each force 'n'. Thus, Eq. (9) can just be simplified to,

$$p_k = h_{k1}(\ddot{x}_1) + \ldots + h_{km}(\ddot{x}_m) = h_{ki}\ddot{x}_i \tag{10}$$

The B-functions can be reconstructed by solving the Eq. (11), defined below.

$$B_{kj} = \sum_i h_{ki}\, A_{ij} \tag{11}$$

Equation (11) needs to be measured many times for different excitation configurations in order to better approximate the B-functions. The pseudo-inverse algorithm was employed to solve the linear system of equations. Each term in the sum leads to a partial value of pressure, transferred by each propagation path due to each exciting force. The locations of accelerometers are often defined by trial and error.

It is shown that a pair of transfer functions A-B, A-h or an unusual B-h can be used to completely characterize the dynamic response of system, in terms of its vibrational and acoustic responses.

3. FORCE IDENTIFICATION

In practical applications, it is usual to have no physical or mathematical information available about the internal mechanisms of the system. For those situations the system behavior can be described by a certain relation between its inputs and outputs.

Two indirect methods are used to determine forces for TPA algorithms – the complex stiffness method and matrix inversion method (Padilha, 2006).

The complex stiffness method consists in determining the displacement response in each direction during systems operation, on both sides of the structural element. In this case, the structural element is treated as an ideal spring.

The element stiffness needs to be previously obtained by testing procedure. Therefore, for a given path 'j', and for each direction, the following equation can be written,

$$\mathrm{f}_j(\omega) = K_j(\omega)[x_{j,2}(\omega) - x_{j,1}(\omega)] \tag{12}$$

where $\mathrm{f}_j(\omega)$ is the operational force transmitted by path 'j', K_j is the complex stiffness, $x_{j,2}$ and $x_{j,1}$ are the measured displacements on both sides of the structural element. When data are obtained experimentally, displacements are so obtained by numerical integration of accelerometers signals.

The matrix inversion method consist in solving a system of equations, like those presented by Eq. (1) and (2), written as,

$$\{\mathrm{f}\} = [A]^{+}\{x\} \tag{13}$$
$$\{p\} = [B]\{\mathrm{f}\} \tag{14}$$

Each line of the system defined by Eq. (14) is obtained for one receiver, or microphone. The symbol '+' means the pseudo-inverse algorithm. For each receiver, a different set of B-functions needs to be determined by measurements or by using the procedure shown in item 2. Hence, forces can be also determined by solving,

$$\{\mathrm{f}\} = [B]^{+}\{p\} \tag{15}$$

It is similar to Eq. (14), but it is written in terms of sound pressures. Generally, forces are obtained by solving Eq. (13) because it needs a reduced set of measurements.

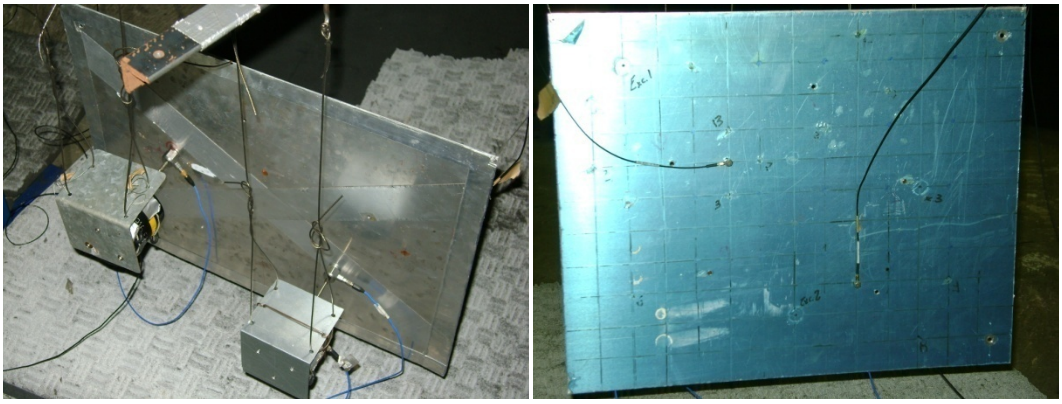
Conveniently, most of the force identification problems can be formulated in terms of a linear system of equations. When there is no noise in both input and output data the matrix inversion processes are computationally stables. However, if there is noise in the data these processes may become ill-conditioned, and the error can be largely amplified in the solution, so it becomes completely meaningless (Hansen, 1998).

In order to avoid matrices inversions, some algorithms have been developed to estimate forces by using numerical tools applied to systems modeling (Dobson and Rider, 1990), stochastic processes (Hans, 2007) based on modal parameters and distributed probabilities functions (Granger and Perotin, 1998), and mathematical algorithms based on the modal behavior of the structure (Silva, 2000).

4. EXPERIMETAL SETUP

A free aluminum plate of dimensions 520x520x2 [mm] was simultaneous excited by two electrodynamic shakers. White noise of the same order of magnitude, however, separated by two different noise generators were supplied to the shakers. On the other side of the plate, two accelerometers were placed at two arbitrary positions, as shown in Fig. 2.

The experiment was carried out in a hemi-anechoic chamber for better control of the reference pressure values, that is, the references values for B-functions and partial sound contributions.



(a) **(b)**

Figure 2. The free aluminum plate. (a) electrodynamic shakers and (b) accelerometers.

The microphone was placed 1.5 m distant from the plate, approximately at its center. The main calculation steps are itemized below.

1. A set of forty five (45) measurements was used to determine the h-functions by solving Eq. (10). A system of equation of size [45x2] was solved using a pseudo-inverse algorithm;
2. The B-functions are reconstructed by solving Eq. (8) and (11). In order to reduce the effect of random errors, the calculation was performed for a set of fifteen (15) measurements, and an averaging process was realized;
3. One block of measurements is needed to provide the references values for operational forces, B-functions and for the total sound pressure. Thus, the measured outputs are used firstly to obtain the operational forces by solving Eq. (1), and secondly to obtain the partial sound contributions by solving Eq. (2);

5. NUMERICAL RESULTS

The TPA algorithm allows determining firstly the amplitude of the exciting forces and secondly the partial sound contributions due to each exciting force. The partial sound contributions, together with the forces amplitudes, are essential for a complete understanding about the mechanisms of noise generation.

In order to test the algorithm sensibility to uncertainties, the Eq. (1) and (10) were contaminated with noise, as follows

$$\tilde{x}_i = \tilde{A}_{ij}\mathrm{f}_j \tag{16}$$

$$\tilde{p}_k = h_{ki}\tilde{x}_i \tag{17}$$

The symbol '~' denotes uncertainty in the data. Another reason for adding noise in the data is the fact that the experiment is seldom controlled in practice. A portion of the measured vibratory energy might come from the ground or from another external source outside the experiment, leading to incorrect estimations for operational forces and B-functions.

It was considered two different cases denoted by case A, without additional noise, and Case B, with an additional noise of 30% in the pressure values, 5% in the accelerations values and 15% in the A-functions.

Case A in Fig. 4, both the B1-functions and the B2-functions related to force #1 and force #2 respectively matched the reference quite well. Operational forces were also identified, but with few discrepancies around the low frequency bands.

In fact, the force identification process involves matrices inversions which use to amplify the effects of experimental errors. The sum of partial sound contributions matched the reference values in the whole frequency range.

For Case B, the B-functions presented a few discrepancies around the low frequency bands. The same errors appear in the operational forces, but in large scale. It was expected that the effect of additional errors would increase the discrepancies pointed by Case A.

Again, the sum of the partial sound contributions matched, except for low frequencies, with the reference values. It is clear that the process of obtaining partial sound contributions do not involves matrices inversions, therefore it is less sensitive to propagation errors.

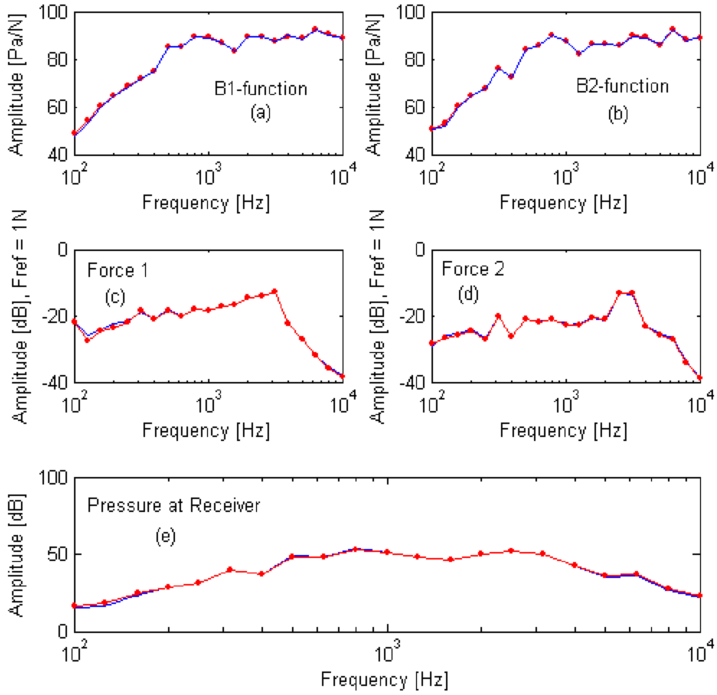


Figure 4 – Case A – For all plots the dash and the dotted lines represent the identified values and its references, respectively. (a) The B1-function for force #1; (b) the B2-function for force #2; (c) the identified operational force #1; (d) the identified operational force #2; (e) the sum of partial sound contributions at the microphone. The one-third octave band filter was used on the data. No additional noise was introduced in the data.

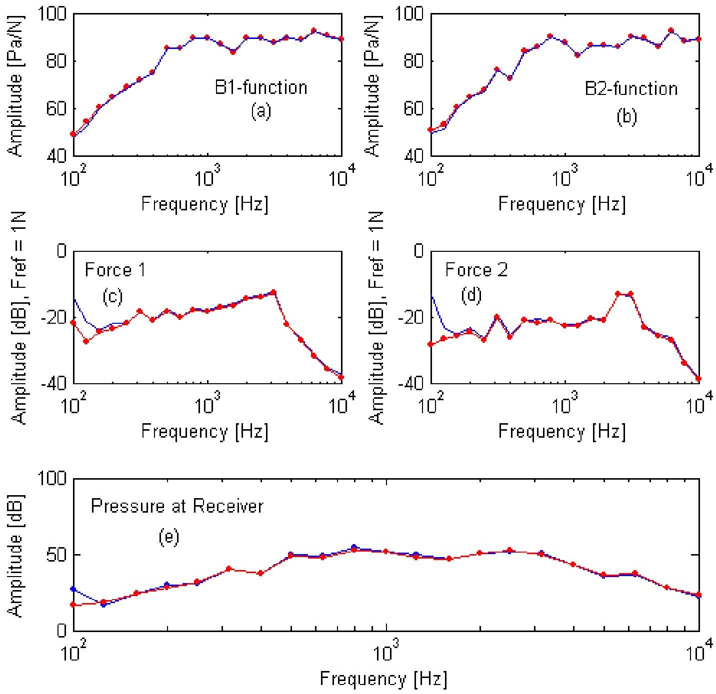


Figure 5 – Case B – For all plots the dash and the dotted lines represent the identified values and its references, respectively. (a) The B1-function for force #1; (b) the B2-function for force #2; (c) the identified force #1; (d) the identified operational force #2; (e) the sum of partial sound contributions at the microphone. The one-third octave band filter was used on the data. Additional noise was introduced in the data**.**

6. CONCLUSIONS

The presented alternative formulation of TPA has the attraction of measuring only A-functions before the operational test. The B-functions can be automatically determined for any receiver position, which leads to greater flexibility and faster testing.

The results obtained for B-functions and partial sound contributions, with additional noise in output data, fit well with its references. From the results it is clear that the B-functions determination process is not sensible to uncertainties, which was somehow expected because no inversion matrix process is needed, but least square algorithm.

It is worth saying that the matrix inversion method for force identification is generally ill-conditioned, and numerical instabilities may occur. For these cases, the use of regularization techniques and/or more precisely identification algorithms may be necessary.

7. ACKNOWLEDGEMENTS

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