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CHAOTIC DOUBLE PENDULUM CONSTRUCTION PROPOSALS

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Abstract. *Nonlinear dynamical systems are time-dependent systems that have a relation of cause and effect in accordance with its input and output variables. Among the behaviors existing at nonlinear dynamic systems, there is the chaotic behaviour. The chaotic systems are of great importance for the frequent occurrence in nature and other physical systems. The experimental verification of chaotic dynamical systems has provided an breakthrough in the study and development of new tools for the study of nonlinear dynamic systems. However, there is substantial difficulty to project and build experimental systems that exhibit such behaviour. Mechanical systems that exhibit chaotic dynamics deserve attention because of its didactic aspect. The literature reports several types of mechanical systems that exhibit chaotic behaviour. An example of such system is the double pendulum. The difficulty of building such a system is that there are few studies that show in detail its construction. The choice of sizes and kinds of materials to be used is another factor that hinders its construction. One way to evidence the chaotic behaviour of a system is to analyze its bifurcation diagram. This diagram shows the behaviour of this system for a given parameter this means, shows wich values of this parameters presents chaotic dynamics. This paper employs bifurcation diagrams of the double pendulum model to find the values of the constructive parameters parameters that allow experimental verification of chaos. Thus is given more options of materials and dimensions of easier access and cheaper montage to simplify and encourage the construction of such didactic platform for the study of chaos.*

Keywords: *Chaotic systems; Double pendulum; Bifurcation Diagram; Material selection.*

1. INTRODUCTION

The mechanical systems nonlinear dynamics are affected by interaction of frictional forces and external driving forces. In such systems, generally the energy is not conserved and the driving force can lead the system to Chaos. In the Chaotic motion, pairs of trajectories starting at a very close point in the phase space will exponentially diverge from each other. The dependence of the chaotic motion on the initial conditions is so strong that the long-term predictions appear to be impossible. A chaos feature is that the motion has a continuous frequency spectrum and it will never become periodic, which may be significant in applications.

The dynamics of the system can be viewed by bifurcation diagrams over a range of parameters. Bifurcation means the sudden changes of dynamical behaviour with the parameters. The Bifurcation diagrams allow to analise the transition to chaotic regime.

Another way to investigate the nonlinear behaviour of a system is to study its motion in the phase space. This is a multidimensional space of the coordinates and the canonically conjugate momenta. The Poincaré Map associates the trajectories with points of first return to a cross-section to these trajectories. This is similar to a motion stroboscopic view in the phase space. Unfortunately, there is no general method to choose the Poincaré Map.

Chaotic dynamics occur in a wide variety of practical situations and control of chaos has attracted considerable interest in mechanical engineering. It was selected some particular works about Control Chaos and the Double Pendulum to ilustrate the applications. Chen(2002) showed chaos synchronization in two identical gyroscopeis with linear plus cubic damping. Later, Chen and Lee(2004) have studied the control of chaotic dynamics in rigid body motion under appropriate feedback gains. Urbakh *et al.*(2004) has proposed the control of chaotic frictional forces by mechanical normal vibrations of small amplitude and energy. Yang and Chen(2005) have investigated numerically bifurcation and chaos in axially accelerating viscoelastic beams. The use of impact dampers for the chaos control of system with limited power supply was studied numerically by de Souza *et al.*(2005). KovalŠchuk and Lobas(2004) investigated bifurcation and chaos in a double pendulum with springs and friction under the action of an asymmetric follower force. Fradkov(2005) showed pulse-modulated control for a coupled double pendulum system in the oscillatory regime. Spong and Bullo(2005) have considered the double pendulum for modeling the passive walking in bipedal locomotion and nonlinear control and Singh et al(2008) have studied a self-impacting double pendulum with varying coefficient of restitution of the knee joint impact model on a leg.

Other papers have showed the use of the pendulum models applications to solve problems related to different physical systems, such as: the behaviour of the ship constrained to forced oscillations caused by the movement (Henry, 2001). The helicopter carrying a load suspended by a cable modeling (Cicol, 2001), and particularly on double pendulum, it is cited studies that make the modeling of behaviour of different parts of the human body during walking using a model based on double pendulum (Gutnik, 2005). In the robotics area, Berk(1999) reported the use pendulum models to study double pendulum to investigate the behaviour of manipulators. Besides the relevance of pendulum modeling it is considered useful in engineering education.

Mathematical models of pendulum are used for studies of different physical systems. This makes its behaviour study important, once the advances made in their mathematical model analysis can be applied to other systems, whose mathematical models are isomorphic to the pendulum ones (Heng, 1992).

In this paper it is sugested how to determine, from simulations, a range of values of some double pendulum construtive parameters and indicate some materials to achieve chaotic behaviour.

2. METODOLOGY

2.1 Bifurcation Diagram

A bifurcation diagram is defined as a graphical representation of the qualitative behaviour of orbits for each one of parameter of systems values. The horizontal axis corresponds to parameter values and the vertical axis the values of one of the states of the system in the Poincare section. The first points of the orbit is discarded in order to eliminate the numerical transient.

Similarly, it is useful to classify the types of a system behaviour it is also useful to identify ways in which these behaviors change. The term bifurcation is associated with a qualitative change in the solution structure, as a consequence of a system parameters variation when they pass through a critical value. This term was used first by Poincaré to express the changes in the number or kind of solutions of equilibrium. The phenomenon of bifurcation is closely related to the existence of chaos if dynamic system does not present any kind of bifurcation it is does not present a chaotic response. However, the reciprocal is not true, *ie*, system that bifurcates not necessarily presents a chaotic response. The bifurcation diagram shows in a compact way the system dynamic subject behaviour, depending on the parameter value. For each value of the parameter within a range it is calculated several points of solution of the system, and remove are some starting points to eliminate any transitory state. The parameter value used is on the x-axis in a grafic in two dimensions, and the points of the solution in the Poincaré section, is on the y-axis.

When bifurcation diagram shows several points for a value of the bifurcation parameter, it means that behaviour is not periodic and probably chaotic. On the other hand, few points indicate periodic behaviour.

2.2 The double pendulum

The dynamics of the double pendulum is governed by the equations of Newton and Euler. The time rate of change of the total angular momentum of the double pendulum is equal to the resultant torque of the external forces. These forces are the gravitational force, the motor, a linear friction and the constant friction caused by internal friction of the rod and the motor. In this work, we used a model that considers the friction as a combination of a term proportional to the angular velocity signal (Coulomb friction) plus a small term proportional to angular velocity (viscous friction). It was also considered an external excitation by a DC motor of 100W to overcome damping caused by the friction.

The building model proposed follows the length and mass dimensions given in (Firmo,2007) and shown in Figure 1 where the author shows the existence of chaos for these dimensions. This work aims to find other dimensions that allow the chaotic behaviour emergence.

This work was used as a simplificated model which consists in considering each bar represented by the double pendulum center of mass, and these point masses connected by rods of negligible mass. Figure 2 shows the representation of the simplified model used for simulations.

It is also studied the sensitivity to initial conditions of a damped double pendulum subjected to external excitation. In order to investigate sensitivity to initial conditions, it was made numerically the bifurcation diagram. The considered bifurcation parameter was the bars masses . The analysis of this diagram allows to determine the parameter ranges in which the system manifests chaotic dynamics. It was possible to investigate the existence of regions in the phase space where the trajectories are confined. Simulations were made in Fortran language and the software Scilab, which are free and open source.

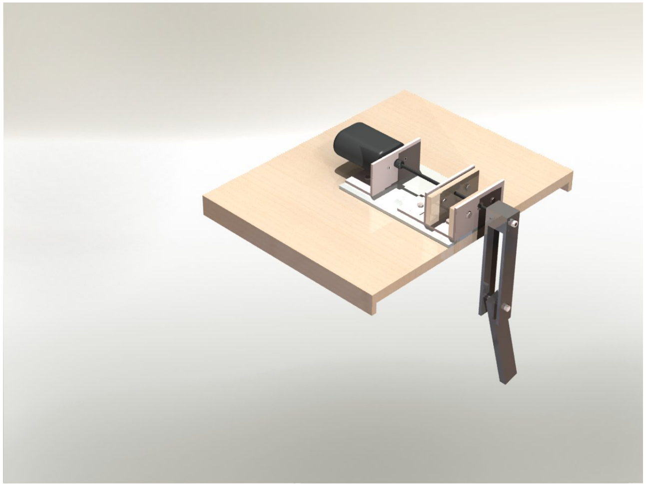

Figure 1. Design of the double pendulum.

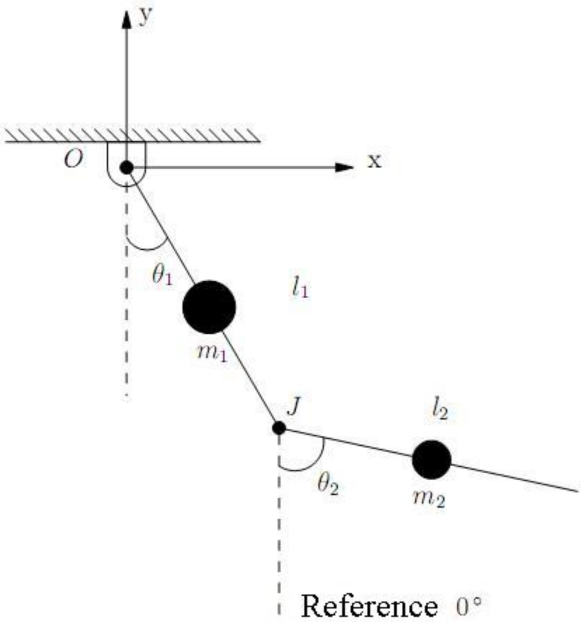

Figure 2. Model of the double pendulum.

2.3 Dimension and materials selection

Some materials are suggested to build the double pendulum. In the following is important to note that the density is the main parameter to project the double pendulum.

- Aluminum has a wide application in industrial products due to its mechanical properties. Aluminum has low density $2.7g/cm^3$, tensile strength up to $13Kg/mm^2$, high ductility close to 20 HB, electrical conductivity of 65% considering the Copper it has low elasticity modulus equal to $7000Kg/mm^2$ (ABAL, 2007). The elements of alloys help to improve the mechanical properties of the Aluminium as the iron which reduces its workability, and elements as Silicon and Copper which increase its tensile strength. The applications of Aluminium components are present in the several and modern products. According Malbrouki *et al.* (2008), nowadays, the automotive and aeronautic industries have a great demand of low-density materials, which can bear high load increase. Considering the wide range of metallic materials used in the moderns industries, aluminium alloys with a wide range of properties is one example of its use in engineering structures. For this reason Aluminium is used in many industries to make different products with the several details. Structural components made from aluminum and aluminum alloys are very important to the aerospace industry and have similar importance in other areas of transportation and building in which durability, strength and light weight are expected (Kurt *et al.*, 2008). For general production the 5000 and

6000 series alloys provide adequate strength combined with good corrosion resistance, high toughness and ease of welding. In addition to the application of Aluminium and its alloys include packaging, building and Architecture, High Pressure Gas Cylinders, Machined Components, Domestic and Office Furniture (Azom, 2008).

- MDF

  Medium-density fiberboard (MDF) is an engineered wood product formed by breaking down softwood into wood fibres, combining it with wax and a resin binder, and forming panels by applying high temperature and pressure. MDF has a typical density from 0.60 to 0.80g/cm3 and the thicknesses of the layers vary from 3mm to 60cm. The plates are made of MDF with different characteristics, which vary depending on the use, such as fire-resistant plates and plates resistant to water. There are also plates made with a larger amount of plastic, enabling them to applications requiring greater resistance to bending or impact. Its main advantage consists in strength and size, no tendency to divide in grains and low cost. Althought swells and breaks when waterlogged, it does not have a grain on the board surface, but into the board. Screwing into the edge of a board will generally cause it to split in a fashion similar to delaminating. And are subject to significant shrinkage in low humidity environments.

- Acrylic

  Acrylic (Poly - methyl methacrylat)) is thermoplastic and transparent plastic. It is often used as an alternative to glass, and in competition with polycarbonate (PC). It is often preferred because of its moderate properties, easy handling and processing, and low cost, but behaves in a brittle manner when loaded, especially under an impact force. Acrylic is often used as an alternative to glass, and in competition with polycarbonate (PC). It is often preferred because of its moderate properties, easy handling and processing, and low cost, but behaves in a brittle manner when loaded, especially under an impact force. Laser cutting may be used to form intricate designs from acrylic sheets. In this respect acrylic has an advantage over competing polymers such as polystyrene and polycarbonate, which require higher laser powers and give more messy and charred laser cuts. It has a density from 1.150 to 1.190g/cm3. This is similar to that of other plastics. Acrylic has a good impact strength higher and is softer and more easily scratched than glass. It has poor resistance to solvents, as it swells and dissolves easily. It also has poor resistance to many other chemicals on account of its easily hydrolyzed ester groups. Comonomers such as butyl acrylate are often added to improve impact strength.

2.4 O projeto

The double pendulum was built based on the dimensions used in Firmo(2007) and it is shown in Figure 3.

Figure 3. Picture of the double pendulum.

However, it was very difficult to build in exactly original proportions of the experiment. Thus arose the need to find a wide range of materials and dimensions to allow according to available resources, a simpler construction of the bench.

The bifurcation diagram was used to determine a range of values that allows the specification of the masses of the pendulum to emerge chaotic behaviour. The mathematical model used is a simplification of the mass parameter in considering them as values concentrated connected by rods of masses equal zero (Amaral, 2009). In the following, it was specified range of values and determined the extent possible for this parameter, considering the results we found constructive calculations and feasibility of the pendulum.

2.5 Determinação das dimensões

The dimensions of the bars must meet the following requirements. The mass of the pendulum rod can be defined by equation 1.

$$m = \rho \cdot v \tag{1}$$

where ρ is the density of the material, m is the mass and v is the total volume. v is given by:

$$v = L \cdot x \cdot y \tag{2}$$

then,

$$m = \rho \cdot L \cdot x \cdot y \tag{3}$$

The bar parameter is shown in the Figure 4.

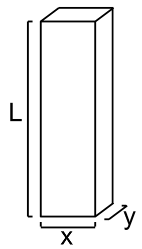

Figure 4. Dimension of the bar.

Therefore designing the material and dimensions of the bar, it should be noted that the parameters ρ, L, x, y are those that result in mass within the range given in the following section.

3. RESULTS

In order to investigate the global behaviour of the system was used the bifurcation diagram. Firstly, the bifurcation parameter was the pendulum excitation amplitude. The mass was choosed randomly in the range from 0.1 Kg to 0.5 Kg. The values of the mass was $m_1 = m_2$ =0.25 Kg. The others parameters was taken from firmo(2007). The bifurcation diagram achieved is presented in the Figure 5 (a). The diagram shows an interval of the chaotic behaviour to excitation amplitude approximately between 0.007 and 0.009. Then, it was choosed the value f=0.00837 to make the bifurcation diagrams therein. The Figure 5 (b) shows the chaotic attractor projection.

In the follow, it was investigated how the mass determinates the chaotic behaviour. So that, the bifurcation parameter was the mass. The values of each mass was fixed while the other mass was varied in the range from 0.1 Kg to 0.5 Kg. In the Figure 6 (a) is showed that if m_1=0.1 Kg the system presents chaotic behaviour for almost all values of the mass m_2. However, if the mass m_1 is equal to 0.5 Kg, the bifurcation diagram reveals chaotic behaviour for almost all values of the mass m_2.

This results is confirmed on the bifurcation diagrams in the Figures 7 (a) and 7 (b). In the diagram in the Figure 7 (a) almost all values of the mass m_1 are greater than the values of the mass m_2. In this case the chaotic behaviour is achieved in practicably all the range. The Figure 7 (b), when the mass m_1 is set closed to m_2 the chaotic behaviour is present.

4. CONCLUSION

In this paper was considered some proposals to build a chaotic double pendulum. The bifurcation diagrams were built using the excitation amplitude and the masses as bifurcation parameters. It was investigated some parameters range where chaotic behaviour come out.

The main result is that chaotic behavior is achieved for the mass values in intervals obtained by the diagrams. This occurs regardless of the material density and the width and thickness of bars. Besides, it is possible occur variations in the mass values due construction process without compromise the achievement of chaotic behavior, *ie*, the system presents robustness for parameters variations.

We conjecture that the choice of mass m_1 greater than the mass m_2 facilitates the achievement of chaotic behavior.

In future work we intend to investigate the influence of other constructive parameters and the effect of simultaneous variation of these parameters.

(a) (b)

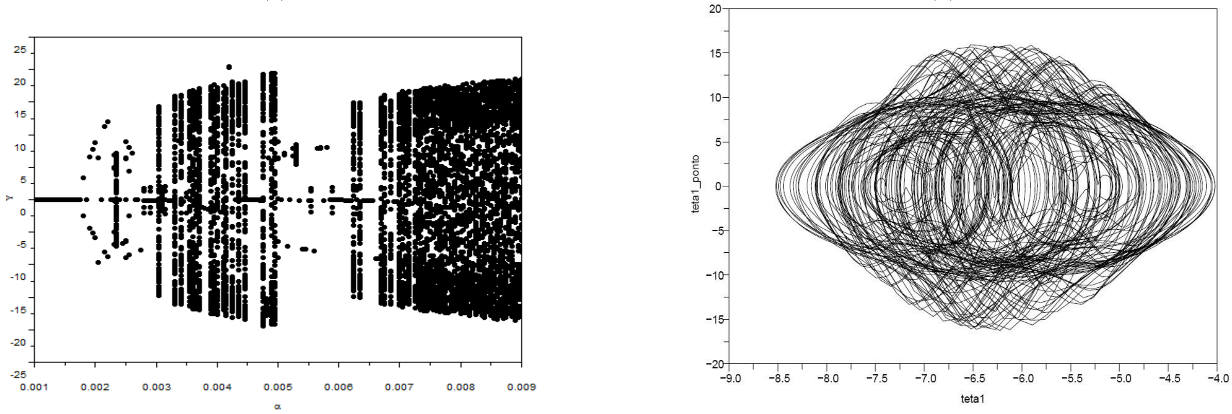

Figure 5. (a) Bifurcation diagram of the double pendulum. α, the excitation amplitude, is the bifurcation parameter (b) Projection in the $\theta \times \dot{\theta}$ plane of the chaotic attractor for α = 0.00837.

(a) (b)

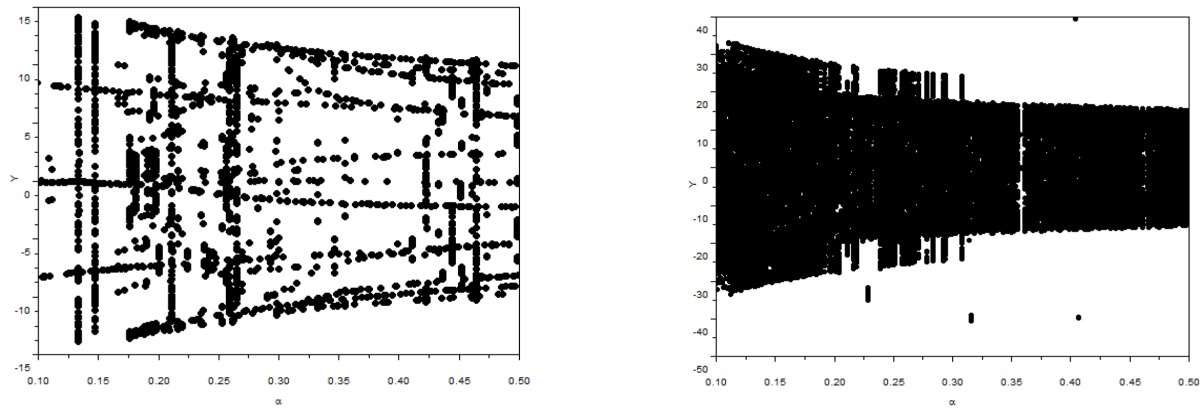

Figure 6. Bifurcation diagrams for the double pendulum for $\alpha = m_2$ (a) $m_1 = 0.1$ Kg (b) $m_1 = 0.5$ Kg. Depending on the m_1-value is possible to choose m_2 parameter in a large interval to produce chaotic behaviour.

(a) (b)

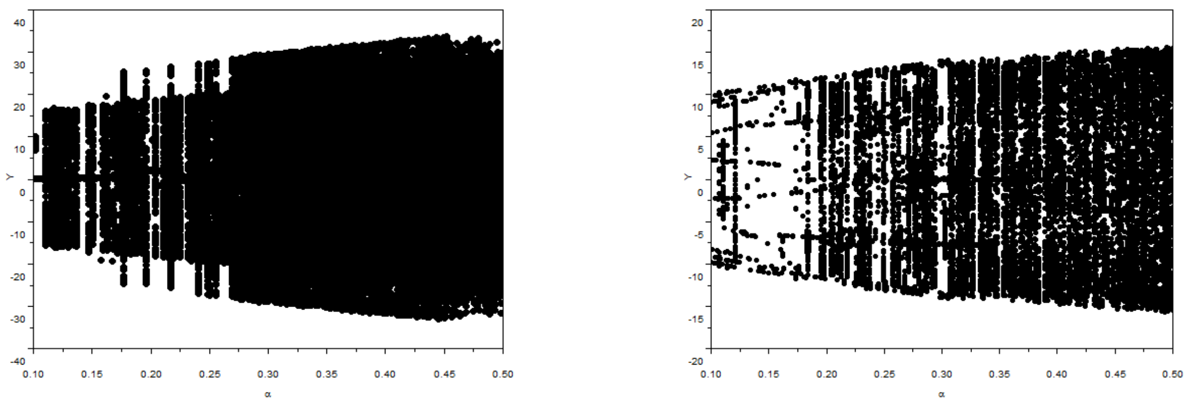

Figure 7. Bifurcation diagrams for the double pendulum for $\alpha = m_1$ (a) $m_2 = 0.1$ Kg (b) $m_2 = 0.5$ Kg.

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