APPLYING HYBRID VAN DER POL-RAYLEIGH OSCILLATORS FOR SIMULATING THE HUMAN CPG

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Abstract. *The biped gait is one of the most complex in Nature. It requires not only an adequate physical structure, but also an extremely accurate control system. Locomotion of humans and animals, performed by rhythmic and synchronized movements, involves a very large number of degrees of freedom, becoming essential a good coordination between them. The main part of this coordination is performed by spinal marrow, which generates signals according to the movement pattern of the desired gait. This process of signal generation can be modeled by a pattern generator, which can be designed as a network of nonlinear oscillators. Therefore, the objective of this work is the application of a coupled hybrid van der Pol-Rayleigh oscillators system for simulating the central pattern generator of human being. A 2D model was analyzed, with the three most important determinants of gait, that performs movements in the sagittal plane. Using oscillators with integer relation of frequency, the response as a function of time and the stable limit cycles of the network formed by three oscillators were determined, showing the behavior of hip and knees. From plotted graphics, the system provided excellent results when compared to experimental analysis, proving that the use of coupled hybrid van der Pol-Rayleigh oscillators can represent an excellent method to signal generation, allowing their application for feedback control of a walking machine, presenting optimized functioning in relation to the systems that use only van der Pol or Rayleigh oscillators.*

Keywords*: CPG, gait, locomotion, oscillators.*

1. INTRODUCTION

The first indications that the spinal marrow could contain the basic nervous system necessary to generate locomotion date to the beginning of 20th century. According to Mackay-Lyons (2002), nervous networks in the spinal marrow are capable to produce rhythmic movements, such as swimming, walking, and jumping, even when isolated of the brain and sensorial inputs. These specialized nervous systems are known as *nervous oscillators* or *central pattern generators* (CPGs).

The human locomotion is controlled by the central nervous system, in which the CPG supplies a series of pattern curves. This information is passed to the muscles by means of a network of motoneurons, and the muscular activity performing the locomotion. Sensorial information about the environment conditions or some disturbance are supplied as feedback of the system, providing a fast action of the CPG, which to adjust the gait to the new situation (Ivanenko et al., 2003). Other significant works about vertebrate locomotion controlled by CPG, including human locomotion, are presented by Grillner (1985), Pearson (1993), Collins and Richmond (1994), Calancie et al. (1994), and Dimitrijevic et al. (1998).

Nonlinear oscillators system can be used as a central pattern generator, providing necessary information for locomotion. Each pattern of movement requires a set of oscillator parameters and couplings. The gait can be modified by changing some parameters. A great number of studies about the application of this principle has been performed, specially the application in hexapod (Collins and Stewart, 1993), quadruped (Collins and Richmond, 1994) and biped models (Bay and Hemami, 1987, Zielinska, 1996, Dutra et al., 2003, Pina Filho et al., 2005, and Pina Filho, 2005). In this work, the CPG uses a set of hybrid van der Pol-Rayleigh oscillators, where each oscillator generates angular reference signals for the motion of a single link (knees and hip).

2. MECHANICAL MODEL

The modeling of natural biped locomotion is made more feasible by reducing the number of degrees of freedom taken into consideration. This is possible from the use of the determinants of gait, which represent the most important movements in the course of the locomotion cycle. According to Saunders et al. (1953), there are six determinants of gait: the compass gait, that is performed with stiff legs like an inverted pendulum; the pelvic rotation about a vertical axis; the pelvic tilt; the knee flexion of the stance leg, which effects combined with pelvic rotation and pelvic tilt achieve minimal vertical displacement of the center of gravity; the plantar flexion of the stance ankle; and the lateral displacement of the pelvis.

In order to further simplify the investigation, a 2D model that performs only movements in the sagittal plane will be considered. This model, presented in Fig. 1, characterizes the three most important determinants of gait: 1 (compass gait), 4 (knee flexion of the stance leg), and 5 (plantar flexion of stance ankle). The model does not take into account the motion of the joints necessary for the lateral displacement of the pelvis, the pelvic rotation, and the pelvic tilt.

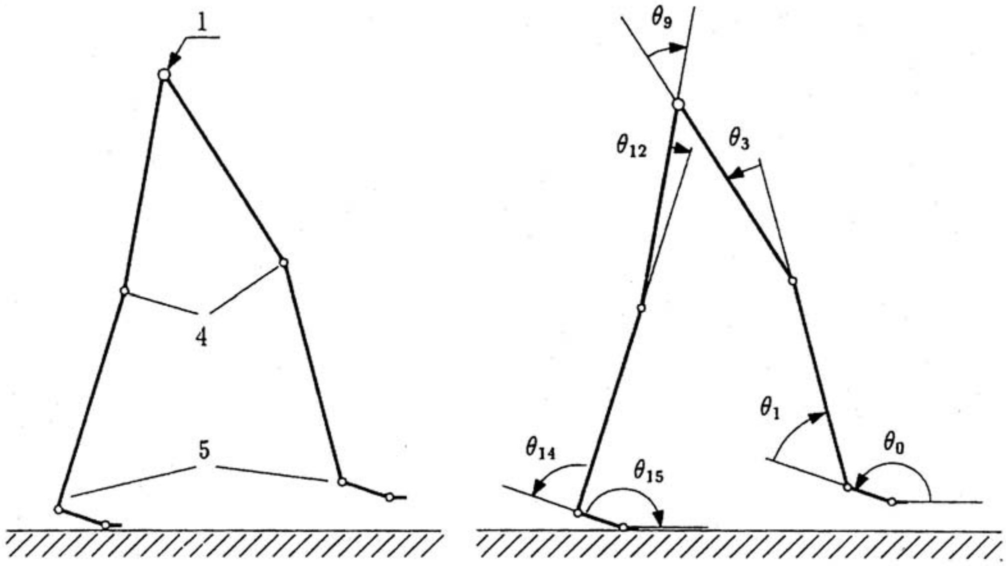

Figure 1. 2D model with the three most important determinants of gait and the relative angles.

The walking period can be divided in two intervals: single support phase (SSP), where one of the legs perform the movement of balance while the other is responsible for the support (the extremity of the support leg is assumed to be not sliding); and double support phase (DSP), where the transition of the legs occurs, i.e., the balance leg becomes the support leg and the other leg prepares to initiate the balance movement.

3. HYBRID VAN DER POL-RAYLEIGH OSCILLATOR

A hybrid oscillator have characteristics of two types of oscillators and whose equation presents a combination of terms of these oscillators. Considering the equations of van der Pol:

$$\ddot{x} - \varepsilon\left(1 - p(x - x_0)^2\right)\dot{x} + \Omega^2(x - x_0) = 0 \qquad \varepsilon, p \geq 0 \tag{1}$$

and Rayleigh:

$$\ddot{x} - \delta\left(1 - q\dot{x}^2\right)\dot{x} + \Omega^2(x - x_0) = 0 \qquad \delta, q \geq 0 \tag{2}$$

the equation of hybrid oscillator that will be used in the analysis is:

$$\ddot{x} - \varepsilon\left(1 - p(x - x_0)^2\right)\dot{x} - \delta\left(1 - q\dot{x}^2\right)\dot{x} + \Omega^2(x - x_0) = 0 \qquad \varepsilon, p, \delta, q \geq 0 \tag{3}$$

where ε, p, δ, q and Ω are the parameters of the oscillator. Using $y = \dot{x}$, we have the following autonomous system:

$$\begin{cases} \dot{x} = y \\ \dot{y} = \varepsilon\left(1 - p(x - x_0)^2\right)y + \delta\left(1 - qy^2\right)y - \Omega^2(x - x_0) \end{cases} \tag{4}$$

Choosing values for ε, p, δ, q, Ω and x_0, using a program that integrates ordinary differential equations (ODE), it is possible to plot the graphic representation of x and $\dot{x}$ as functions of time and the trajectory in the phase space.

Setting $\varepsilon = p = \delta = q = \Omega = 1$ and $x_0 = 0$, and using the Matlab, the graphic representation illustrating the hybrid van der Pol-Rayleigh oscillator behavior was generated. Figure 2 shows the periodic movement of the oscillator and the limit cycle.

An interesting characteristic of the hybrid oscillator is that depending on the relation between the values of ε and δ, it assumes a similar behavior to the van der Pol oscillator or Rayleigh oscillator. This is important because these oscillators have distinct behaviors, which can be assumed by the hybrid oscillator. In electric point of view, the oscillators answer to an increase of voltage of different form. In the case of the van der Pol oscillator, an increase of the voltage implies in increase of the frequency, while in the Rayleigh oscillator it implies in an increase of the amplitude.

More details about the characteristics and behavior of these oscillators are described in Hebisch (1992).

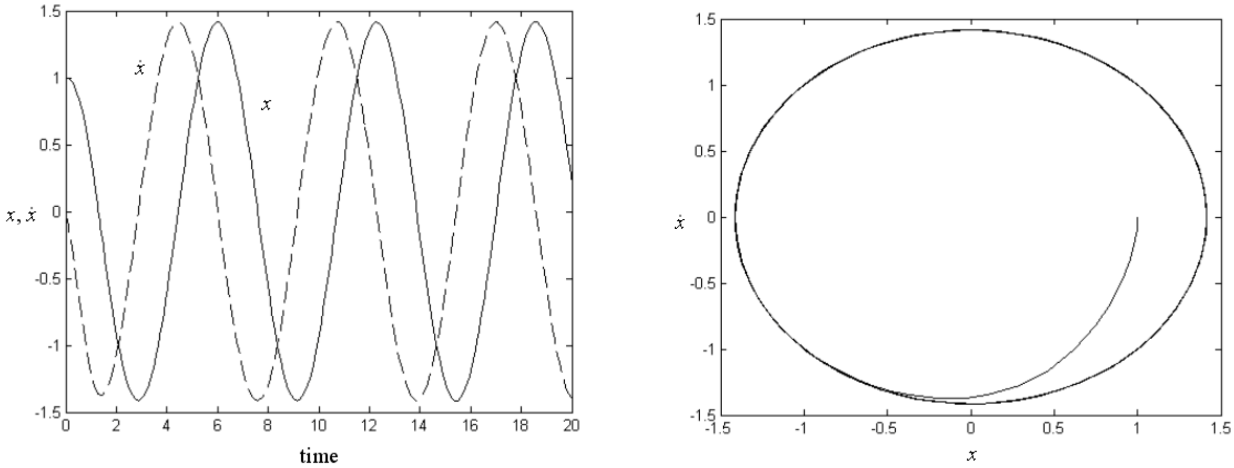

Figure 2. Graphical representation of x and $\dot{x}$ as a function of time, and the limit cycle.

4. OSCILLATORS SYSTEM

From the model shown in Fig. 1, the hip angle θ_9 and the knee angles θ_3 and θ_{12} will be determined by a nonlinear oscillators system. Experimental studies of human locomotion (Braune and Fischer, 1987) and Fourier analysis of these data (Dutra, 1995) show that the motion of knees and hip angles can be described very precisely by their fundamental harmonic, whether the biped is in DSP or SSP. Figure 3 shows the experimental results where θ_{3m}, θ_{9m} and θ_{12m} represent the curves built by means of experimental studies and θ_{3h}, θ_{9h} and θ_{12h} represent the approximation of these curves using the Fourier analysis. A set of three coupled oscillators was used to generate the angles θ_3, θ_9 and θ_{12}. These oscillators are mutually coupled by terms that determine the influence of one oscillator on the others (Fig. 3).

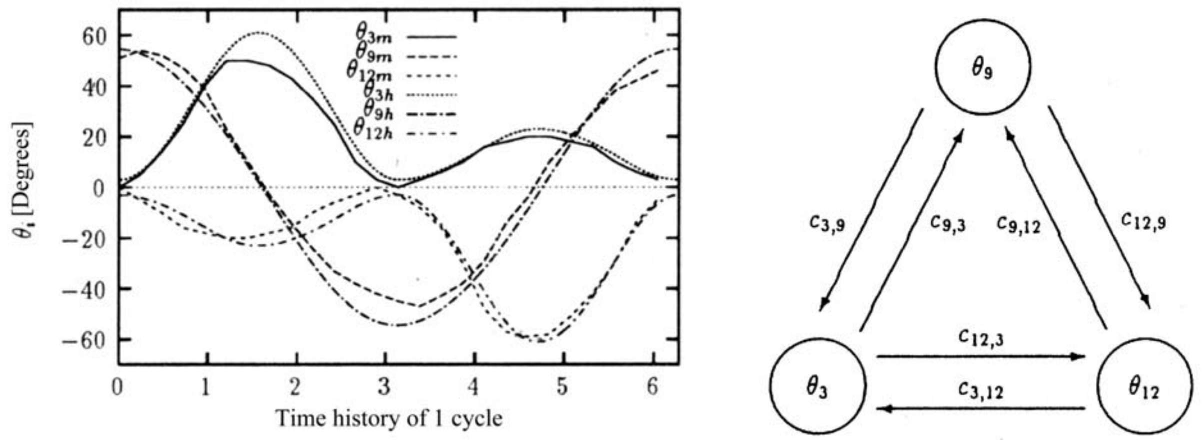

Figure 3. Experimental results and approximation using Fourier analysis, and coupled oscillators system.

Considering oscillators with the same frequency, a network of n-coupled hybrid van der Pol-Rayleigh oscillators can be considered. From Eq. (3) and adding a coupling term that relates the velocities of the oscillators, we have:

$$\ddot{\theta}_i - \varepsilon_i \left[1 - p_i\left(\theta_i - \theta_{io}\right)^2\right] - \delta_i\left(1 - q_i\dot{\theta}_i^2\right)\dot{\theta}_i + \Omega_i^2\left(\theta_i - \theta_{io}\right) - \sum_{j=1}^{n} c_{i,j}\left(\dot{\theta}_i - \dot{\theta}_j\right) = 0 \qquad i = 1, 2, ..., n \tag{5}$$

where ε_i, p_i, δ_i, q_i, Ω_i and $c_{i,j}$ are the parameters of this system.

For small values of parameters determining the model nonlinearity, we will assume that the response is approximated by low frequency components from full range of harmonic response. Therefore, periodic solutions can be expected, which can be approximated by:

$$\theta_i = \theta_{io} + A_i \cos(\omega t + \alpha_i) \tag{6}$$

We desired to determine the values of the parameters q_i, p_i e Ω_i. In this case all oscillators have the same frequency ω. Deriving Eq. (6) and inserting the solutions in Eq. (5), by the method of harmonic balance (Nayfeh and Mook, 1979), the following nonlinear equation system is obtained:

$$\begin{cases} A_i\left(\Omega_i^2 - \omega^2\right)\cos\alpha_i + A_i\omega\left[\varepsilon_i\left(1 - \dfrac{A_i^2 p_i}{4}\right) + \delta_i\left(1 - \dfrac{3\omega^2 A_i^2 q_i}{4}\right)\right]\operatorname{sen}\alpha_i + \omega\sum_{j=1}^{n} c_{i,j}\left(A_i \operatorname{sen}\alpha_i - A_j \operatorname{sen}\alpha_j\right) = 0 \\ A_i\left(\omega^2 - \Omega_i^2\right)\operatorname{sen}\alpha_i + A_i\omega\left[\varepsilon_i\left(1 - \dfrac{A_i^2 p_i}{4}\right) + \delta_i\left(1 - \dfrac{3\omega^2 A_i^2 q_i}{4}\right)\right]\cos\alpha_i + \omega\sum_{j=1}^{n} c_{i,j}\left(A_i \cos\alpha_i - A_j \cos\alpha_j\right) = 0 \end{cases} \tag{7}$$

Moreover, it is possible to verify the following relation proposal by Hebisch (1992):

$$q_i = \frac{\varepsilon_i p_i}{\delta_i \omega^2} \tag{8}$$

Then, substituting the value of q_i in Eq. (7) and solving, the parameters p_i and Ω_i are:

$$p_i = \frac{1}{A_i^2} + \frac{\delta_i}{A_i^2 \varepsilon_i} + \frac{1}{A_i^3 \varepsilon_i}\sum_{j=1}^{n} c_{i,j}\left[A_i - A_j \cos(\alpha_i - \alpha_j)\right] \qquad i = 1, 2, ..., n \tag{9}$$

$$\Omega_i = \sqrt{\omega^2 - \frac{\omega}{A_i}\sum_{j=1}^{n} A_j c_{i,j} \operatorname{sen}(\alpha_i - \alpha_j)} \qquad i = 1, 2, ..., n \tag{10}$$

Given the amplitude A_i and A_j, phase α_i and α_j, the frequency ω and the chosen values of ε_i, δ_i and $c_{i,j}$, the value of the parameters q_i, p_i and Ω_i can be calculated.

In the case of oscillators with different frequencies, the oscillators with frequency ω can be synchronized with other oscillators with frequency $n\omega$, where n is an integer. In analyzing human gait, we can observe that some degrees of freedom have twice the frequency of the others (n = 2), that can be seen in Fig. 3.

Therefore, a network of coupled hybrid van der Pol-Rayleigh oscillators can be described as:

$$\ddot{\theta}_h - \varepsilon_h[1 - p_h(\theta_h - \theta_{ho})^2]\,\dot{\theta}_h - \delta_h\left(1 - q_h\dot{\theta}_h^2\right)\dot{\theta}_h + \Omega_h^2(\theta_h - \theta_{ho}) - \sum_{i=1}^{m} c_{h,i}\left[\dot{\theta}_i(\theta_i - \theta_{io})\right] - \sum_{k=1}^{n} c_{h,k}\left(\dot{\theta}_h - \dot{\theta}_k\right) = 0 \tag{11}$$

where $c_{h,i}\left[\dot{\theta}_i(\theta_i - \theta_{io})\right]$ is responsible for the coupling between two oscillators with different frequencies, while $c_{h,k}\left(\dot{\theta}_h - \dot{\theta}_k\right)$ effects the coupling between two oscillators with the same frequencies.

Using a similar assumption as previously applied, we have:

$$\theta_h = \theta_{ho} + A_h \cos(2\omega t + \alpha_h) \tag{12}$$

$$\theta_i = \theta_{io} + A_i \cos(\omega t + \alpha_i) \tag{13}$$

$$\theta_k = \theta_{ko} + A_k \cos(2\omega t + \alpha_k) \tag{14}$$

Deriving Eq. (12)-(14) and inserting the solutions in Eq. (11), a nonlinear equation system is obtained, and solving this system, the parameters p_h and Ω_h are:

$$p_h = \frac{1}{A_h^2} + \frac{\delta_h}{A_h^2 \varepsilon_h} + \frac{1}{4A_h^3 \varepsilon_h} \sum_{i=1}^{m} A_i^2 c_{h,i} \cos(\alpha_h - 2\alpha_i) + \frac{1}{A_h^3 \varepsilon_h} \sum_{k=1}^{n} c_{h,k} \left[A_h - A_k \cos(\alpha_h - \alpha_k) \right] \tag{15}$$

$$\Omega_h = \sqrt{4\omega^2 + \frac{\omega}{2A_h} \sum_{i=1}^{m} A_i^2 c_{h,i} \operatorname{sen}(\alpha_h - 2\alpha_i) - \frac{2\omega}{A_h} \sum_{k=1}^{n} A_k c_{h,k} \operatorname{sen}(\alpha_h - \alpha_k)} \tag{16}$$

Given the amplitude A_h, A_i and A_k, phase α_h, α_i and α_k, the frequency ω and the chosen values of ε_h, δ_h, $c_{h,i}$ and $c_{h,k}$, the value of these parameters q_h, p_h and Ω_h can be calculated.

5. ANALYSIS OF THE SYSTEM AND RESULTS

Considering the oscillators system presented in Fig. 3, from Eq. (11) the system can be described as:

$$\ddot{\theta}_3 - \varepsilon_3 [1 - p_3 (\theta_3 - \theta_{3o})^2] \dot{\theta}_3 - \delta_3 \left(1 - q_3 \dot{\theta}_3^2\right) \dot{\theta}_3 + \Omega_3^2 (\theta_3 - \theta_{3o}) - c_{3,9} \left[\dot{\theta}_9 (\theta_9 - \theta_{9o})\right] - c_{3,12} \left(\dot{\theta}_3 - \dot{\theta}_{12}\right) = 0 \tag{17}$$

$$\ddot{\theta}_9 - \varepsilon_9 [1 - p_9 (\theta_9 - \theta_{9o})^2] \dot{\theta}_9 - \delta_9 \left(1 - q_9 \dot{\theta}_9^2\right) \dot{\theta}_9 + \Omega_9^2 (\theta_9 - \theta_{9o}) - c_{9,3} \left[\dot{\theta}_3 (\theta_3 - \theta_{3o})\right] - c_{9,12} \left[\dot{\theta}_{12} (\theta_{12} - \theta_{12o})\right] = 0 \tag{18}$$

$$\ddot{\theta}_{12} - \varepsilon_{12} [1 - p_{12} (\theta_{12} - \theta_{12o})^2] \dot{\theta}_{12} - \delta_{12} \left(1 - q_{12} \dot{\theta}_{12}^2\right) \dot{\theta}_{12} + \Omega_{12}^2 (\theta_{12} - \theta_{12o}) - c_{12,9} \left[\dot{\theta}_9 (\theta_9 - \theta_{9o})\right] - c_{12,3} \left(\dot{\theta}_{12} - \dot{\theta}_3\right) = 0 \tag{19}$$

The synchronized harmonic functions, corresponding to the desired movements, can be written as:

$$\theta_3 = \theta_{3o} + A_3 \cos(2\omega t + \alpha_3) \tag{20}$$

$$\theta_9 = A_9 \cos(\omega t + \alpha_9) \tag{21}$$

$$\theta_{12} = \theta_{12o} + A_{12} \cos(2\omega t + \alpha_{12}) \tag{22}$$

Considering $\alpha_3 = \alpha_9 = \alpha_{12} = 0$ and deriving Eq. (20)-(22), inserting the solution in Eq. (17)-(19), the necessary parameters of the oscillators (q_i, p_i e Ω_i, $i \in \{3, 9, 12\}$) can be determined. Then:

$$p_3 = \frac{4c_{3,12}(A_3 - A_{12}) + 4A_3(\varepsilon_3 + \delta_3) + A_9^2 c_{3,9}}{4A_3^3 \varepsilon_3} \tag{23}$$

$$\Omega_3 = 2\omega \tag{24}$$

$$q_3 = \frac{\varepsilon_3 p_3}{4\delta_3 \omega^2} \tag{25}$$

$$p_9 = \frac{1}{A_9^2} + \frac{\delta_9}{A_9^2 \varepsilon_9} \tag{26}$$

$$\Omega_9 = \omega \tag{27}$$

$$q_9 = \frac{\varepsilon_9 p_9}{\delta_9 \omega^2} \tag{28}$$

$$p_{12} = \frac{4c_{12,3}(A_{12} - A_3) + 4A_{12}(\varepsilon_{12} + \delta_{12}) + A_9^2 c_{12,9}}{4A_{12}^3 \varepsilon_{12}} \tag{29}$$

$$\Omega_{12} = 2\omega \tag{30}$$

$$q_{12} = \frac{\varepsilon_{12} p_{12}}{4\delta_{12} \omega^2} \tag{31}$$

From Eq. (17)-(19) and Eq. (23)-(31), and using the Matlab, the graphs shown in Fig. 4 and 5 were generated; they present, respectively, the behavior of the angles as a function of time and the stable limit cycles of the oscillators.

These results were obtained using the parameters shown in Tab. 1, as well as the initial values provided by Tab. 2. All values were experimentally determined.

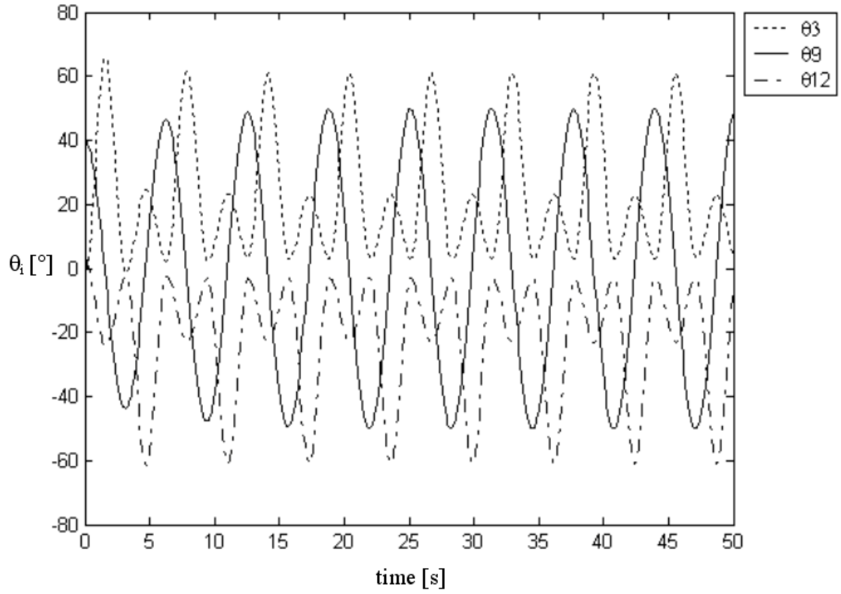

Figure 4. Behavior of θ_3, θ_9 and θ_{12} as a function of time.

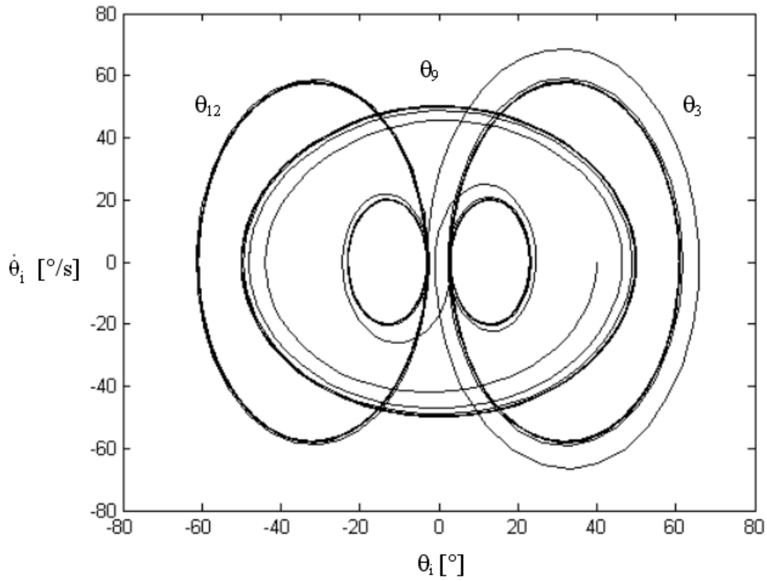

Figure 5. Trajectories in the phase space (stable limit cycles).

Table 1. Parameters of hybrid van der Pol-Rayleigh oscillators.

$c_{3,9}$	$c_{9,3}$	$c_{3,12}$	$c_{12,3}$	$c_{9,12}$	$c_{12,9}$	ε_3	ε_9	ε_{12}	δ_3	δ_9	δ_{12}
0.001	0.001	0.1	0.1	0.001	0.001	0.01	0.1	0.01	0.01	0.1	0.01

Table 2. Experimental initial values.

Cycle	A_3	A_9	A_{12}	θ_{3o}	θ_{9o}	θ_{12o}
$0 < \omega t \leq \pi$	–29	50	10	32	0	–13
$\pi < \omega t \leq 2\pi$	–10	50	29	13	0	–32

In Fig. 5 the great merit of this oscillator can be observed, i.e., if an impact occurs and the angle of one joint is not the correct or desired, it returns after a small number of periods to the desired trajectory. Considering, for example, a frequency equal to 1 s^{-1} and null initial velocities, with arbitrary initial values: $\theta_3 = -3°$, $\theta_9 = 40°$ e $\theta_{12} = 3°$, after some cycles we have: $\theta_3 = 3°$, $\theta_9 = 50°$ e $\theta_{12} = -3°$.

Comparing the results supplied for the coupling system using hybrid van der Pol-Rayleigh oscillators with the experimental results presented in recent work about human gait (Raptopoulos 2003), it is verified that the coupling system supplies similar results, which confirms the possibility of using coupled hybrid van der Pol-Rayleigh oscillators in the modeling of the CPG. Figures 6 and 7 present the results comparison.

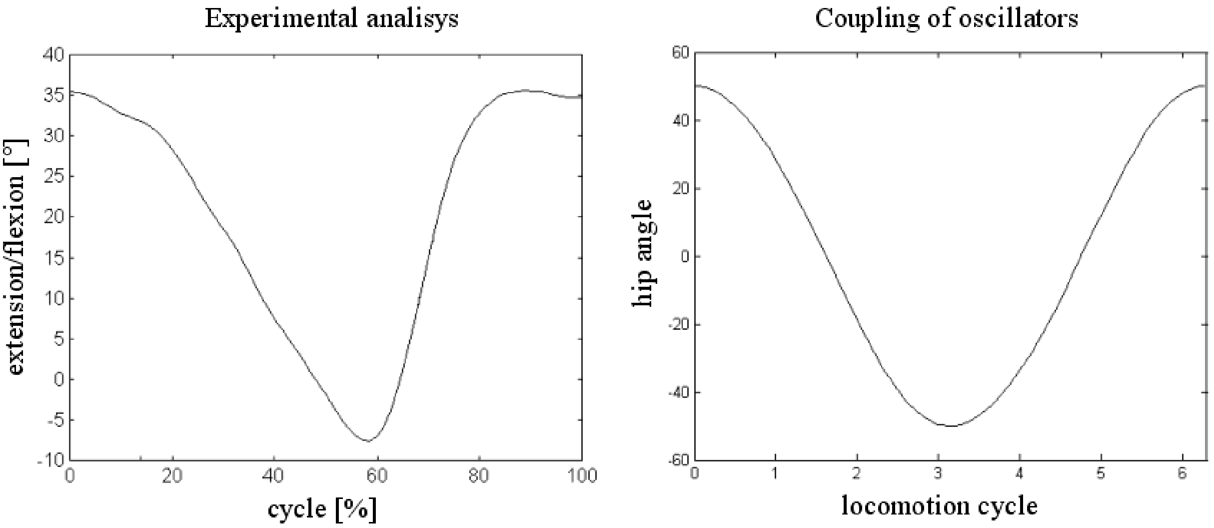

Figure 6. Behavior of the hip angle in the course of one locomotion cycle

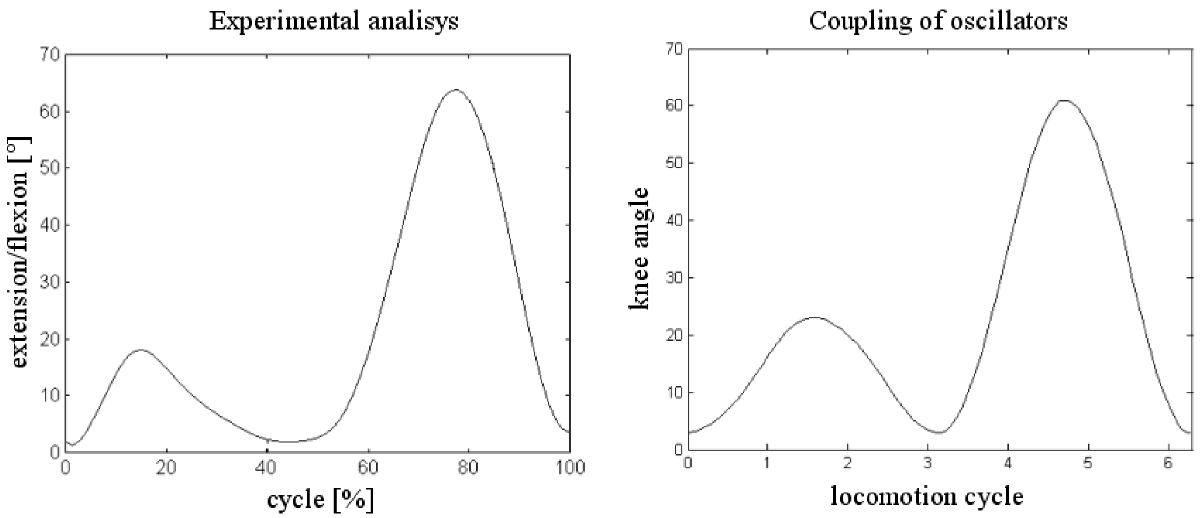

Figure 7. Behavior of the knee angle in the course of one locomotion cycle

An interesting characteristic of the hybrid oscillators, verified in the course of the tests, is the fact of it to recover the periodicity of the movement faster than the van der Pol oscillators, using similar parameters (Fig. 8). Therefore, if a small disturbance occurs and the angle of a joint is not the correct or desired, it is return more quickly to the desired trajectory. This difference in the recovery time is most significant in relation to the hip angle, but for the angles of the knees a small advantage for the hybrid oscillator still exists. The same advantage also exists in relation to Rayleigh oscillators, since the behavior of these is similar to van der Pol oscillators.

Finally, applying the system to the bipedal robot, we obtained the response shows in Fig. 9, which represents the gait with a step length of 0.5 m. The model have identical legs with 0.37 m for femur and tibia, and 0.11 for feet. Other step lengths and gaits can be simulated by changing some system parameters.

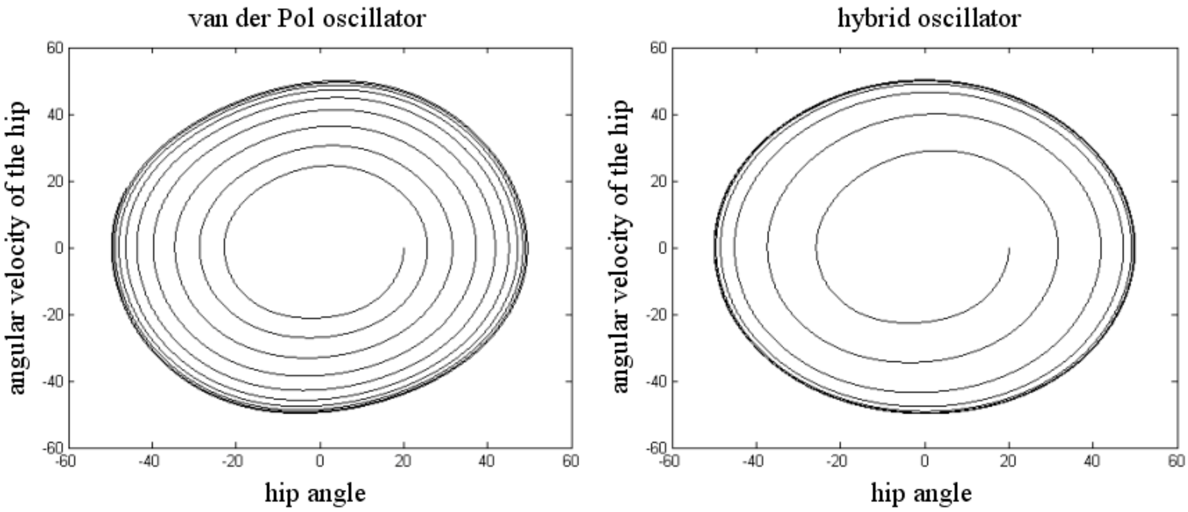

Figure 8. Comparison of the recovery of periodicity in the phase space

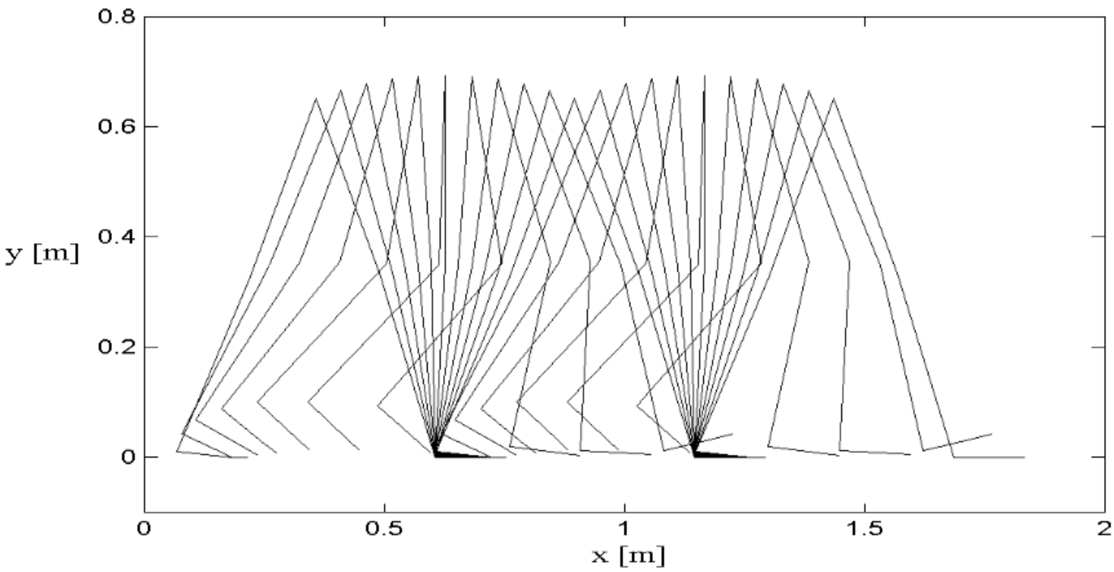

Figure 9. Stick diagram showing the gait with a step length of 0.5 m

6. CONCLUSION

The results presented in this work and their analysis and discussion lead us to the following conclusions about the application of hybrid van der Pol-Rayleigh oscillators for simulating the human CPG: (1) The use of these oscillators can represent an excellent way to signal generation, allowing their application for feedback control of a walking machine by synchronization and coordination of the lower extremities, presenting optimized functioning in relation to the systems that use only van der Pol or Rayleigh oscillators. (2) The model is able to characterize three of the six most important determinants of human gait. (3) By changing a few parameters in the oscillators, modification of the step length and gait frequency can be obtained. The gait frequency can be modified by means of Eq. (20)-(22) by choosing a new value for ω. The step length can be modified by changing the angles θ_9 and θ_{12}, with the parameters q_i, p_i e Ω_i, $i \in \{3, 9, 12\}$, being responsible for the gait transitions.

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8. RESPONSIBILITY NOTICE

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