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METAMORPHIC ROBOTS: ENUMERATION OF CONFIGURATIONS AND MOTION PLANNING

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Abstract. *This paper considers how to enumerate the basic set of all the non-isomorphic configurations of a planar metamorphic robotic system. Metamorphic robotic systems are being widely studied because their shape changing abilities make them potentially useful for a larger set of tasks that conventional robotic systems are unable to develop, for example reconnaissance, exploration, satellite recovery, or operation in constrained environments inaccessible to humans, (e.g., nuclear reactors, space or deep water). A metamorphic robotic system is a collection of mechatronic modules that can dynamically self-reconfigure in a variety of configurations, kinematic chains, to meet different or changing task requirements. However, due to typical symmetries in module design, different assemblies may generate isomorphic robotic structures. To solve this problem, we use group theory tools for the identification of symmetries of metamorphic robotic systems. In particular, we define the concept of binary orbits of the automorphism group of the graphs associated with the metamorphic robot configurations. Another issue considered in this paper is the motion planning of a metamorphic robot system, i.e., how to determine a sequence of module movements required to go from a given initial configurations to a desired final configuration. The paper solved the fundamental problem which is to determine the set of all possible configurations. Knowing all the possible configurations, the motion planning is solved with algorithms proposed in the literature.*

Keywords: *metamorphic robot configurations, group theory, symmetry, isomorphisms, automorphisms, orbits, motion planning.*

1. INTRODUCTION

A metamorphic robotic system is a collection of mechatronic modules that can dynamically self-reconfigure (Chirikjian, 1994). A change in the macroscopic morphology results from the locomotion of each module over its neighbors. Chirikjian (1994, 1996) present some applications of metamorphic robotic systems. One application in particular, civil structures in times of emergency, evince the importance of previously knowing all the possible configurations that a predetermined finite number of modules can assume.

There are some intriguing questions in the literature of modular and metamorphic robots which are sometimes implicit in the context:

1. How to enumerate all possible configurations that a metamorphic robotic system can assume (Chen and Burdick, 1998);
2. How to find the optimal configuration for a predetermined task (Chen and Burdick, 1995; Bi et al., 2003);
3. How to plan the movement of a metamorphic robot system, *i.e.* how to determine a sequence of module movements required to go from a given initial position to a desired goal configuration (Pamecha et al., 1997; Chiang and Chirikjian, 2001).

Questions 2 and 3 are relatively frequent in the metamorphic robot literature. Chen and Burdick (1995) consider the problem of finding an optimal module assembly configuration for a specific task. Their solution was formulated as a discrete optimization procedure. Bi et al. (2003) define the configuration space as the set of all feasible configuration variations of the robotic system and evaluate system adaptability for reconfigurable robotic systems with large variations in configurations. They also described as to achieve task-oriented configuration design of reconfigurable robotic systems.

Chirikjian and Pamecha (1996) proposed lower and upper bounds to the number of moves needed to change such systems from any initial to any final specified configuration. Pamecha et al. (1997) introduced the concept of distance between metamorphic robot configurations and demonstrate that this distance satisfies the formal properties of a metric. These metrics are applied to the automatic self-reconfiguration of metamorphic systems for computing the optimal sequence of movements required to reconfiguration. Dumitrescu et al. (2004) present a number of fast formations for both rectangular and hexagonal systems, and presented lower and upper bounds on the speed of locomotion. Kamimura et al. (2003) propose an offline method to generate a locomotion pattern automatically for a modular robot in an arbitrary module configuration.

Question 1, the problem of enumerating the set of kinematically distinct modular robot assembly configurations from a given set of modules, was addressed by Chen and Burdick (1998) . They introduced a representation of a modular robot assembly configuration as an assembly incidence matrix and defined equivalence relations based on symmetries in module geometry and graph isomorphisms on the assembly incidence matrix. They also presented an algorithm to identify the kinematically equivalent robots. Chitta and Ostrowski (2006) also focused on enumeration of distinct configurations of a modular robot.

This paper focuses firstly on question 1 above, *i.e.* how to enumerate the basic set of all the non-isomorphic configurations of a metamorphic robotic system or, in other words, how many different kinematic chains may be spanned by a given finite set of modules. By different kinematic chains, it is meant all the non-isomorphic configurations of a metamorphic robotic system. Other configurations may be obtained later by basic operations of group, reflection symmetry and mirror symmetry.

The remaining questions 2 and 3 are straightforwardly solved when all the non-isomorphic configurations of a metamorphic robotic are obtained. Using algorithms proposed in literature (Walter et al., 2005; Walter et al., 2004; Chiang and Chirikjian, 2001; Casal and Yim, 1999; LaValle, 2006) basic reconfiguration operations between the configurations can be precomputed, optimized and stored. The sequence of module movements required to go from a given initial position to a desired goal configuration, already known, consists of an ordered series of simple, precomputed sub-reconfigurations.

Common planar module designs are square (Pamecha et al., 1996; Dumitrescu et al., 2002; Chiang and Chirikjian, 2001), hexagonal (Pamecha et al., 1996; Abrams and Ghrist, 2004; Walter et al., 2004; Dumitrescu et al., 2002; Walter et al., 2002), For spatial metamorphic systems there are cubic (Rus and Vona, 2001; Yoshida et al., 1998) and dodecahedral (Yim et al., 1997; Yim et al., 2001) modules. Due to the inherent symmetries of these module designs, different assemblies of these modules may lead to several kinematically isomorphic robotic structures. To identify these symmetries, hence eliminating isomorphisms, in metamorphic robotic systems we use group theory, in particular the concept of orbits of automorphisms groups. This concept was previously applied to identify all inversions of a kinematic chains by Simoni et al. (2008, 2009). This tool helps avoiding isomorphisms in enumeration of planar metamorphic robots configurations; therefore, all non-isomorphic configurations are enumerated.

The remainder of this paper is structured as follows. Section 2 introduces basic definitions, tools and examples of group theory that are used in method of enumeration of configurations of metamorphic robots. Section 3 identifies the symmetry of classical modules by group theory and present a concept of binary orbits, the basic concept of the proposed technique for enumeration of configurations of metamorphic robot systems is described in Section 4. Section 5 discuses the question on how to plan the movement of a metamorphic robot system. Section 6 discusses some implementation details of the technique. The conclusions and final remarks are presented in section 7.

2. GROUP THEORY TOOLS

Groups are abstract structures used in mathematics and science in general to capture the internal symmetry of a structure in the form of group of automorphisms. To enumerate all possible configurations of a metamorphic robotic system is fundamental to know which is the symmetries of the structure to prevent isomorphic structures. Below we present the essential definitions of group theory found in the literature (Alperin and Bell, 1995; Burrow, 1993; Rotman, 1995; Scott, 1964).

Definition 1 (Group) *A group is a set G with a binary operation* $*: G \times G \to G$ *that satisfies the following 3 axioms:*
(i) Associativity: For all a, b *and* c *in* G, $(a * b) * c = a * (b * c)$.
(ii) Identity element: There is an element e *in* G *such that for all* a *in* G, $e * a = a * e = a$.
(iii) Inverse element: For each a *in* G, *there is an element* b *in* G *such that* $a * b = b * a = e$, *where* e *is an identity element.*

Definition 2 (Subgroup) *A set G' is a subgroup of a group G if it is a subset of G and is a group using the operation defined on G.*

Definition 3 ((Left) group action) *A left group action of a group* G *on a set* X *is a binary function*

$$G \times X \to X$$

$$(g, x) \mapsto g \cdot x$$

which satisfies the following two axioms:
(i) $(gh) \cdot x = g \cdot (h \cdot x)$ *for all elements* g, h *in the group* G *and* x *in the set* X.
(ii) $e \cdot x = x$ *for every element* x *in the set* X *(where* e *denotes the identity element of the group* G*).*

Analogously, the right group action is defined. From now on, we use the term action for left action, unless otherwise stated.

Definition 4 (Symmetric group) *The symmetric group on a set* X, *denoted by* S_X, *is the group whose underlying set is the set of all bijective functions from* X *to* X, *in which the group operation is that of composition of functions.*

The symmetric group on the finite set $X = \{1, 2, \ldots, n\}$ is denoted as S_n and all $\sigma \in S_n$ will be denoted by

$$\sigma = \begin{pmatrix} 1 & 2 & \cdots & n \\ \sigma(1) & \sigma(2) & \cdots & \sigma(n) \end{pmatrix}.$$

Subgroups of S_n are called permutation groups. Permutations can also be represented by a binary matrix operation. For instance,

$$\sigma = \begin{pmatrix} a & b & c \\ b & a & c \end{pmatrix}$$

can be represented as (left group action)

$$\begin{bmatrix} b \\ a \\ c \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

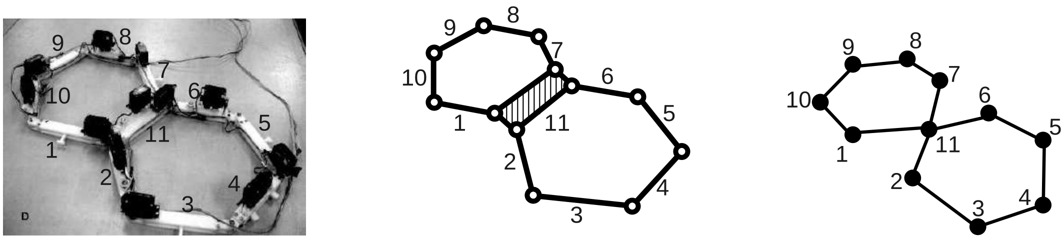
The set of graph vertices $V_n = \{1, 2, 3, \ldots, n\}$ form a permutation group and the definitions above can be applied.

Example 1 *Figure 1(a) shows the metamorphic robot with two hexagonal modules presented by Pamecha et al. (1996). Figure 1(b) shows the kinematic chain of this metamorphic robot configuration and Fig. 1(c) its graph representation (G). Figures 2(a) and 2(b) shows the action of σ_1 and σ_2 in G, respectively, on the labels of the metamorphic robot configuration, where*

$$\begin{aligned} \sigma_1 &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 2 & 1 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 11 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 3 \\ 10 \end{pmatrix}\begin{pmatrix} 4 \\ 9 \end{pmatrix}\begin{pmatrix} 5 \\ 8 \end{pmatrix}\begin{pmatrix} 6 \\ 7 \end{pmatrix}\begin{pmatrix} 11 \\ 11 \end{pmatrix} \\ &= (12)(310)(49)(58)(67)(11) \end{aligned}$$

and

$$\begin{aligned} \sigma_2 &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 10 & 9 & 8 & 11 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}\begin{pmatrix} 2 \\ 6 \end{pmatrix}\begin{pmatrix} 3 \\ 5 \end{pmatrix}\begin{pmatrix} 4 \\ 4 \end{pmatrix}\begin{pmatrix} 8 \\ 10 \end{pmatrix}\begin{pmatrix} 9 \\ 9 \end{pmatrix}\begin{pmatrix} 11 \\ 11 \end{pmatrix} \\ &= (17)(26)(35)(4)(810)(9)(11) \end{aligned}$$



(a) (b) (c)

Figure 1. (a) metamorphic robot with two hexagonal modules (Fig. 4 of (Pamecha et al., 1996)); (b) kinematic chain and (c) graph representation.

(a) σ_1 (b) σ_2

Figure 2. Action of σ_1 and σ_2 in G.

Definition 5 (Isomorphism) *Let G_1 and G_2 be two groups. A homomorphism of G_1 in G_2 is an application $\phi : G_1 \to G_2$ such that, for all x and y in G_1*

$$\phi(x \cdot y) = \phi(x) \cdot \phi(y).$$

If ϕ is bijective, the application is an isomorphism.

Therefore, two graphs H and H', with vertices $V_n = 1, 2, ..., n$, are said to be isomorphic if there is a permutation σ of V_n such that $\{x, y\}$ is in the set of graph edges $E(H)$ if and only if $\{\sigma(x), \sigma(y)\}$ is in the set of graph edges $E(H')$.

An isomorphism is called an automorphism if $G_1 = G_2$.

Definition 6 (Automorphism) *Let G be a group. A isomorphism of G in G is called an* automorphism.

An automorphism of a graph is a graph isomorphism with itself, *i.e.* a mapping of the vertices of a given graph H from the vertices of H such that the resulting graph is isomorphic with H. The sets of these permutations which map the graph into itself form a group called the group of automorphisms of the graph. This group of automorphisms is said to be a vertex-induced group. The group of automorphisms of the graph is a subgroup of the symmetric group and contains all possible permutations of the vertices that preserve the adjacency. The group of automorphisms of a graph characterizes its symmetries, and are, therefore, quite useful for determining some of its properties.

Definition 7 (Orbit) *Consider a group G acting on a set X. The orbit of the point $x \in X$ is denoted by*

$$\mathcal{O}_x = \{g \cdot x \mid g \in G\}.$$

The orbit of a point x in the set X is the set of elements of the set X to which the point x can be moved by the elements of the group G. The set of orbits of the set X under the action of the group G form a partition of the set X. The associated equivalence relation is defined by $x \approx y$ if and only if there exists an element g in the set G such that $g \cdot x = y$. The orbits are equivalence classes under this relation; two elements x and y are equivalent if and only if their orbits are the same, *i.e.* $\mathcal{O}_x = \mathcal{O}_y$.

The action of the group of automorphisms of a graph permutes the graph vertices. If a graph vertex of the label x is moved by the action of an element of the group of automorphisms to a vertex of the label y, then x and y are in the same orbit, *i.e.* $\mathcal{O}_x = \mathcal{O}_y$. For graphs, the equivalence relation is associated with the symmetry of their vertices, if the vertices of labels x and y are in the same orbit they have the same properties of symmetry in the graph. The orbit of a graph vertex corresponds to the set of vertices for which the vertex is moved by the action of the group of automorphisms of the graph.

For the metamorphic robot or graph shown in Fig. 1 the orbits are:

- $\mathcal{O}_1 = \{1, 2, 6, 7\}$;
- $\mathcal{O}_2 = \{3, 5, 8, 10\}$;
- $\mathcal{O}_3 = \{4, 9\}$ and
- $\mathcal{O}_4 = \{11\}$.

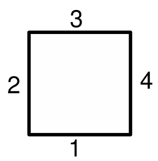
Isomorphisms (automorphisms) avoidance is a recurrent problem in topological synthesis of kinematic chains, mechanism and manipulator, see *e.g.* Simoni et al. (2009). This problem is based on graph algorithms and, unless for special cases, they are non-polynomial-time algorithms (NP-hard). The McKay algorithm (McKay, 1998; McKay, 1990; McKay, 2007) is considered the fastest generic graph algorithm to avoid isomorphisms available today (Jain and Wysotzki, 2005; Foggia et al., 2001; Miyazaki, 1997).

3. STANDARD MODULES AND BINARY ORBITS

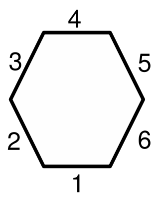
In this section, we present the standard modules of metamorphic robots and discuss symmetries of these modules. We also introduce the fundamental concepts of our technique of enumeration of planar metamorphic robots configurations: binary inversions and binary orbits.

As discussed in Section 1, two standard modules applied to planar metamorphic robots are:

- square modules (Pamecha et al., 1996; Dumitrescu et al., 2002; Chiang and Chirikjian, 2001), Fig. 3(a), and
- hexagonal modules (Pamecha et al., 1996; Abrams and Ghrist, 2004; Walter et al., 2004; Dumitrescu et al., 2002; Walter et al., 2002), Fig. 3(b).



(a)



(b)

Figure 3. Two standard modules of planar metamorphic robots

Metamorphic robot system with square modules are represented by a four-bar kinematic chain as shown in Fig. 3(a). Similarly, the hexagonal module is represent by a six-bar kinematic chain as shown in Fig. 3(b). Also, other issues of the

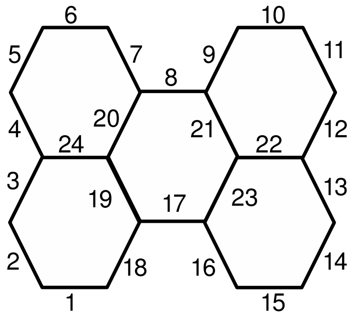


Figure 4. Configuration of hexagonal metamorphic robot.

metamorphic robot design, such as the polarity (Pamecha et al., 1996), were not considered during the enumeration of metamorphic robot configurations.

Figs. 3(a) and 3(b), representing the modules, have internal symmetries which may be identified by the orbits of the automorphisms group. In these modules, all links (edges) have the same properties; therefore, there is a single orbit for each module:

- square module: $\mathcal{O}_1 = \{1, 2, 3, 4\}$ and
- hexagonal module: $\mathcal{O}_1 = \{1, 2, 3, 4, 5, 6\}$.

A general metamorphic robot have multiple orbits. For example, the robot shown in Fig. 4 has the following symmetries identified by the orbits of automorphisms group:

- $\mathcal{O}_1 = \{1, 6, 10, 15\}$;
- $\mathcal{O}_2 = \{2, 5, 11, 14\}$;
- $\mathcal{O}_3 = \{3, 4, 12, 13\}$;
- $\mathcal{O}_4 = \{22, 24\}$;
- $\mathcal{O}_5 = \{7, 9, 16, 18\}$;
- $\mathcal{O}_6 = \{8, 17\}$ and
- $\mathcal{O}_7 = \{19, 20, 21, 23\}$.

In kinematic terms, there are two types of links in the metamorphic robot system shown in Fig. 4: binary and quaternary. Binary links 1-18 are connected to two other links while the quaternary links 19-24 are connected to four other links. Thus, the orbits $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$ and $\mathcal{O}_5$ are composed by binary links and $\mathcal{O}_4$, $\mathcal{O}_6$ and $\mathcal{O}_7$ are composed by quaternary links.

Planar metamorphic robots may have other types of links, but they must have a subset of binary links since all "external" links are binary. These binary provide means for the movement of the metamorphic robot. Hence, all links of a metamorphic robot may be divided into two sets: binary and non-binary links.

Definition 8 (Binary orbits) *Binary orbits are orbits composed only by binary links.*

A property derived from the concept of binary orbits and directly derived from the definition 7 is:

Lemma 9 (Element of binary orbits) *Every binary link is an element of a binary orbit.*

Using the concepts above, binary links can be classified into binary orbits. Links in the same binary orbit have identical symmetry properties in the metamorphic robot configuration. Therefore, when a new module is connected to any link of a binary orbit, the resulting configurations are isomorphic.

For planar metamorphic robots, a new module can only be connected to links that belong to binary orbits. The binary orbits for the configuration shown in Fig. 4 are; $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$ and $\mathcal{O}_5$.

In Section 4. the configurations of metamorphic robot with "$n + 1$" modules generated by configurations of metamorphic robot with "n" modules are explored.

4. ENUMERATION OF PLANAR METAMORPHIC ROBOTS CONFIGURATIONS

The enumeration process follows a tree structure. In the root of the tree, a first module is placed. The following modules are added, one at a time, selecting just one representative for each binary orbit. See definition 8 in section 3.

Orbits are equivalence classes and capture the internal symmetry of a structure (metamorphic robot). The module elements (links) in the same orbit when connected to other module elements result in isomorphic configurations due to the symmetry that the orbit represents. For example, Fig. 5 shows a metamorphic robot with two square modules and another square module will be connected.

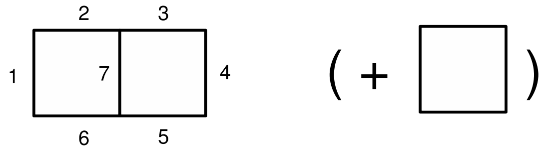


Figure 5. Metamorphic robot with two square modules and another module for connection.

The orbits of automorphisms group of metamorphic robot with two square modules are:
• $\mathcal{O}_1 = \{1, 4\}$;
• $\mathcal{O}_2 = \{2, 3, 5, 6\}$ and
• $\mathcal{O}_3 = \{7\}$.

Since link 7 is quaternary, there are just two binary orbits
• $\mathcal{O}_1 = \{1, 4\}$ and
• $\mathcal{O}_2 = \{2, 3, 5, 6\}$.

The connection of a new module with links belonging to a common orbit results in kinematically isomorphic configurations as shown in Figs. 6 and 7. Figure 6 shows that the connection of a new module to the configuration of metamorphic robot on elements from the orbit $\mathcal{O}_1 = \{1, 4\}$ results in isomorphic configurations.

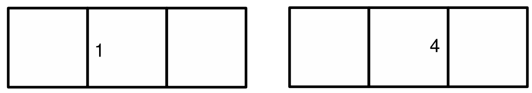


Figure 6. Kinematically isomorphic configurations, obtained from Fig. 5, by connecting another module in orbit $\mathcal{O}_1 = \{1, 4\}$.

Similarly, Fig. 7 shows that the connection of a new module with elements from the orbit $\mathcal{O}_2 = \{2, 3, 5, 6\}$ also results in isomorphic configurations.

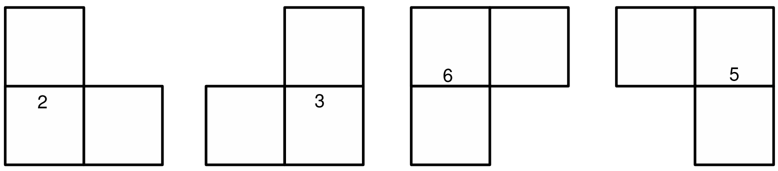


Figure 7. Kinematically isomorphic configurations, obtained from Fig. 5, by connecting another module in in orbit $\mathcal{O}_2 = \{2, 3, 5, 6\}$.

Summing up, there are only two ways of connecting the new module to the current configuration, as shown in Fig. 8. Thus, the basic set of all non-isomorphic configurations of a planar metamorphic robotic system are obtained. This set if formed by kinematically non-isomorphic metamorphic robots.

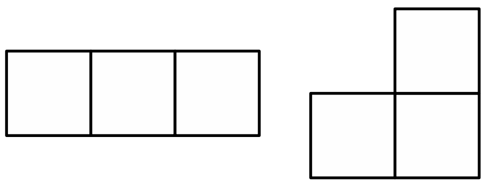


Figure 8. Kinematically distinct (non-isomorphic) configurations of a metamorphic robot with three square modules identified by the orbits of the automorphisms group.

4.1 METAMORPHIC ROBOT CONFIGURATIONS WITH SQUARE MODULES

The technique will be presented by a example using square modules to facilitate the understanding of as the tools are applied. In section 4.2 we present metamorphic robot configurations with hexagonal modules.

Without loss of generality, for identification of symmetries of metamorphic robot system with square modules by group theory, we represent this module by a four-bar kinematic chain as shown in Fig. 3(a).

Consider a example with a set of five square modules as shown in Fig. 9. We start with a module in the root of the tree (level 1) and identify all the ways to connect another module, for this we enumerate the binary orbits through the group theory tools. In the example there are only one binary orbit. Figure 9 marks one representative from each binary orbit with small inclined parallel lines.

The next step is to enumerate configurations of metamorphic robots with three square modules adding another module from the second level of the tree. For this, we enumerate the binary orbits of configuration metamorphic robot of the root. In this case are two as was illustrated in the Figs. 6, 7 and 8. The configurations metamorphic robot with three square modules are obtained in the third level of the tree (see Fig. 9).

The configurations metamorphic robot with four square modules are obtained in the fourth level of the tree. In this level, there are two isomorphic configurations to be eliminated. This isomorphisms elimination is applied in every level of the tree (see Fig. 9).

Finally, to enumerate the configurations of metamorphic robot with five square modules, all non-isomorphic configurations of metamorphic robot with four square modules generated in the fourth level of the tree become roots for the fifth level. The process repeats: identification of the binary orbits, connection of a new module to single representative from each binary orbit, and elimination of the isomorphic configurations. At the end, of the process, all non-isomorphic metamorphic robot configurations with five square modules are obtained in the fifth level of the tree.

The numbers of all non-isomorphic planar metamorphic robot configurations with up to five square modules are: (see Fig. 9):

- 1: with a single module (level 1);
- 1: with two modules (level 2);
- 2: with three modules (level 3);
- 5: with four modules (level 4);
- 12: with five modules (level 5).

4.1.1 Procedure in algorithmic form

In algorithm form, the procedure is summarized as:

Step 1: Calculate the binary orbits of the metamorphic robot configuration of the root.

Step 2: Assemble a new module with one element from each binary orbit, identified in the previous step, of the current metamorphic robot configuration.

Step 3: Run an (efficient) isomorphisms test to eliminate the possible isomorphic configurations in each level of the tree.

Group theory allows reducing the number of isomorphisms drastically by preventing symmetries during the assembling procedure. However, as the number of modules increases, the number of isomorphisms increases almost combinatorially and the process becomes computationally expensive. Hence, there is still a need of a more efficient isomorphism detection.

4.2 METAMORPHIC ROBOT CONFIGURATIONS WITH HEXAGONAL MODULES

Let the enumeration of all non-isomorphic planar metamorphic robot configurations with up to four hexagonal modules. The procedure is presented in the Fig. 10. Besides each arrow is located the number of binary orbits. The module of the first level of the tree has only one binary orbit. The metamorphic robot in second level has three binary orbits. The third level, from left to right, has 2, 7 and 4 binary orbits, respectively.

The numbers of all non-isomorphic planar metamorphic robot configurations with up to four hexagonal modules are: (see Fig. 10):

- 1: with a single module (level 1);
- 1: with two modules (level 2);
- 3: with three modules (level 3);
- 8: with four modules (level 4);

5. MOTION PLANNING

The motion planning problem for a self-reconfigurable metamorphic robotic system is to determine a sequence of robot motions required to go from a given initial configuration to a desired goal configuration.

It would be desirable to design an optimal algorithm that minimizes the number of steps required to reach the final configuration. However, there is no simple solution for computing the optimal sequence of moves required to reconfigure. The reason is that the search space, *i.e.* the number of possible sequences of configurations, grows exponentially with the number of modules in the system. It is a combinatorial optimization problem that bears the hallmarks of a NP-complete problem, although no formal proof has been published yet (Walter et al., 2005).

This paper solved the fundamental problem related on question 1 which is the determination of all possible non-isomorphic kinematically configurations with a determined number of modules. After known the set of possible solutions to the questions 2 and 3 are solved with the algorithms proposed in literature (Walter et al., 2005; Walter et al., 2004; Chiang and Chirikjian, 2001; Casal and Yim, 1999; LaValle, 2006; Kamimura et al., 2003). A interesting algorithm is proposed by Casal and Yim (1999) where the basic operations in the configurations with few modules are pre-calculated,

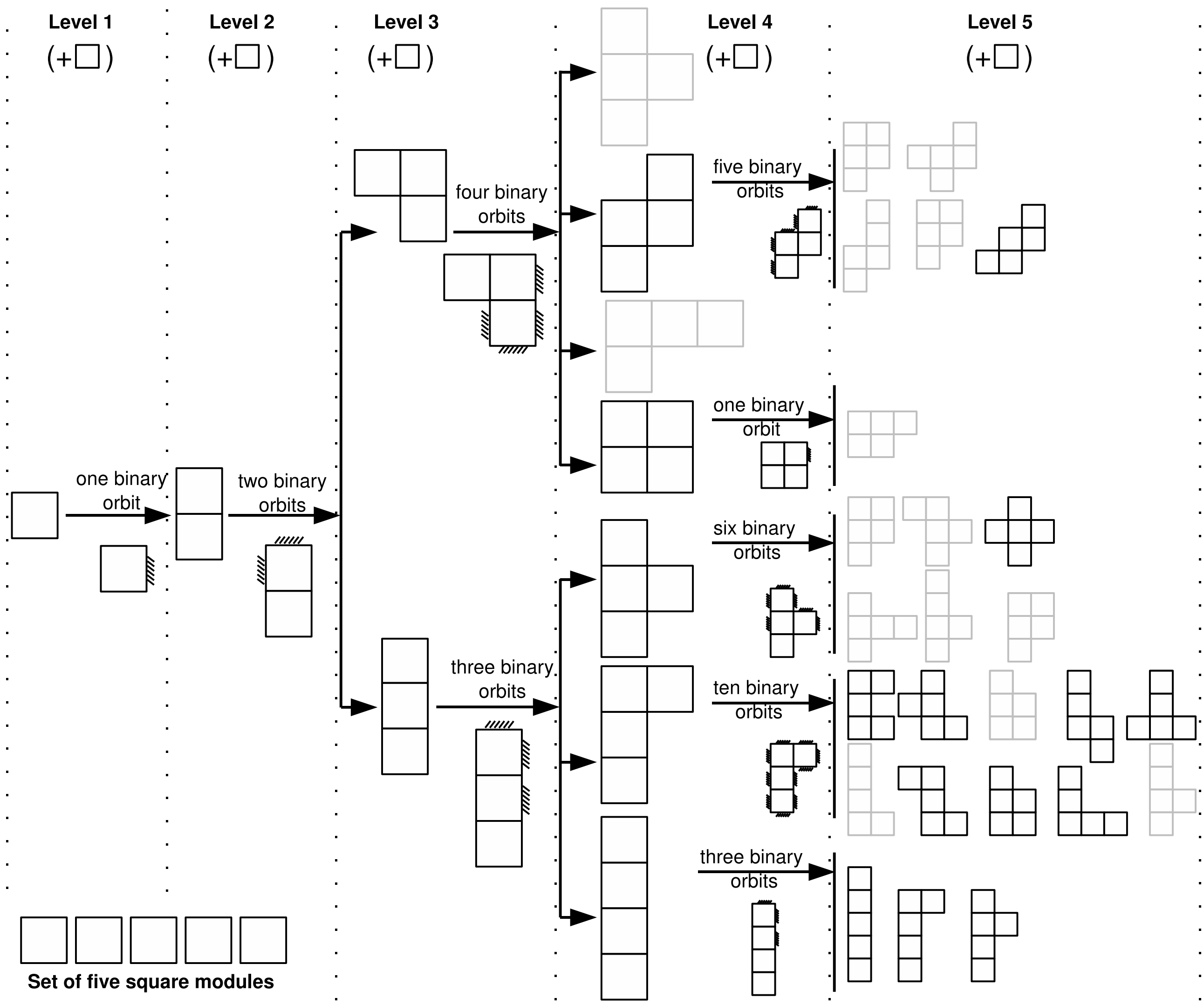


Figure 9. Enumeration of all non-isomorphic metamorphic robot configurations (bold lines) with up to five square modules. Configurations with thin lines are those discarded due to isomorphism with previously generated kinematic chains.

optimized and the number of moves stored on tables. This algorithm can be successfully apply when all configurations are known, *i.e.* can be apply in our case. The manner of determine a sequence of module movements required to go from a given initial position to a desired goal configuration, already known, consists of an ordered series of simple, precomputed sub-reconfigurations.

6. DISCUSSIONS AND IMPLEMENTATION

To implement our algorithm, we used two freely available software: nauty (McKay, 1990; McKay, 1998; McKay, 2007) and the Boost Graph Library (Siek et al., 2002; BGL, 2000).

Nauty (No AUTomorphisms, Yes?) (McKay, 1990; McKay, 1998; McKay, 2007), is a set of quite efficient C language procedures for determining the group of automorphisms of a graph with colored vertices. Nauty is also able to generate a canonically-labeled isomorphic of the graph to assist in isomorphism testing and is considered by some authors as the fastest graph isomorphism algorithm available today (Jain and Wysotzki, 2005; Foggia et al., 2001; Miyazaki, 1997).

The program nauty (McKay, 1990; McKay, 2007) is used for calculate the binary orbits of automorphisms group of each robot configuration and for the test of isomorphisms in each level in the tree of generation as shown in Fig. 9. On the other hand, the interface that generates configurations of metamorphic robots is based on graph structures components provided by the Boost Graph Library (Siek et al., 2002; BGL, 2000).

The complexity of our enumeration of metamorphic robot configurations is limited by complexity of the test of isomorphisms, *i.e.* it is exponential time ($O(e^n)$) (Jain and Wysotzki, 2005; Foggia et al., 2001; Miyazaki, 1997). The manipulation of graphs by the Boost Graph Library such as adding edges, adding vertices, vertices iterator among other routines used in process of generation and manipulation of graphs is of polynomial-time complexity ($O(n)$).

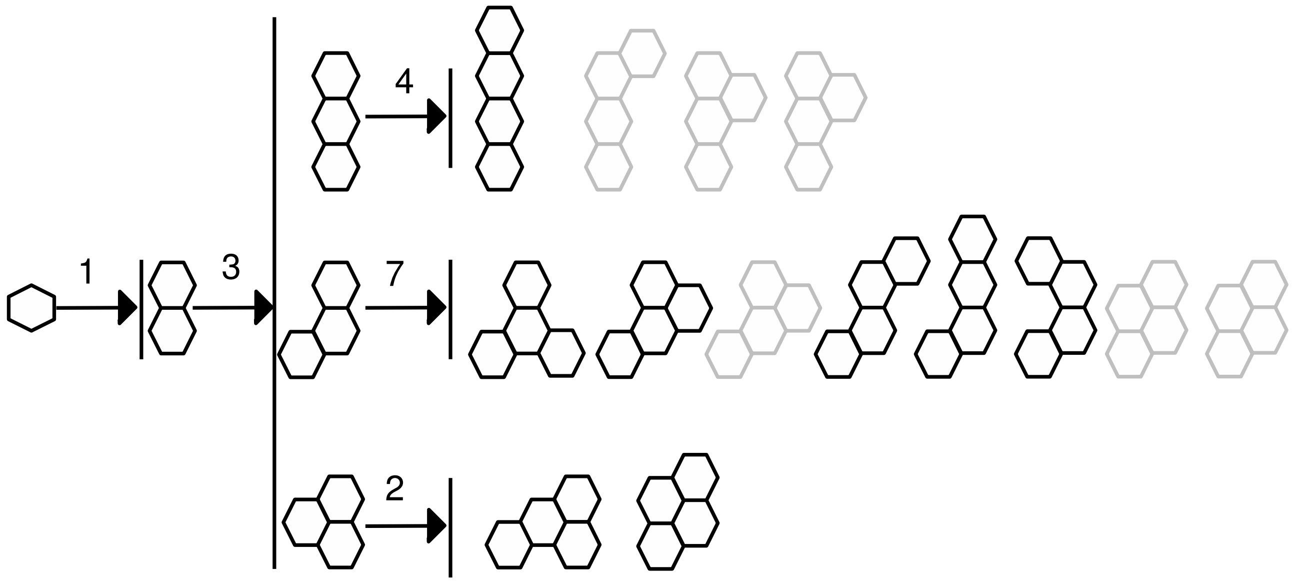


Figure 10. Enumeration of all non-isomorphic metamorphic robot configurations (bold lines) with up to four hexagonal modules. Configurations with thin lines are those discarded due to isomorphism with previously generated kinematic chains.

Our next step is to implement the algorithm proposed by Casal and Yim (1999) where basic reconfiguration operations between the configurations, in the set of all possible configurations, are precomputed, optimized and stored in a table.

7. CONCLUSIONS

This paper introduced a technique for enumeration of all kinematically non-isomorphic planar metamorphic robot configurations. This technique was applied to the most common planar metamorphic robots, namely square and hexagonal modules. However, the technique may be easily extended to enumerate non-planar metamorphic robot configurations based on other types of modules with only minor changes.

This technique may provide a first answer to question 1 of the Introduction: *how many possible configurations a finite set of metamorphic robotic system can assume?*

The second related question – *"how many modules one must buy/make to have enough kinematic flexibility to perform task X or Y?"* – is only partially answered in this paper, since the correlation between tasks and metamorphic robotic system configurations is not addressed here. Conjugating such correlations with an efficient isomorphism-free enumeration method may possibly yield an interesting topic of research.

Another subject discussed in this paper was the motion planning of a metamorphic robot system, *i.e.* how to determine a sequence of module movements required to go from a given initial configuration to a desired goal configuration. The most relevant algorithms in the literature rely on the extensive enumeration of all metamorphic robot configurations which can be obtained by our method in a more efficient way.

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9. REFERENCES

Abrams, A. and Ghrist, R. (2004). State Complexes for Metamorphic Robots. *INT J ROBOT RES*, 23(7):811–830.

Alperin, J. and Bell, R. (1995). *Groups and Representations*. Springer, New York.

BGL (2000). The boost graph library. `http://www.boost.org/doc/libs/1_37_0/libs/graph/doc/index.htm`. Accessed 03-Abr-2009.

Bi, Z., Gruver, W., and Zhang, W. (2003). Adaptability of reconfigurable robotic systems. In *P IEEE International Conference on Robotics and Automation,*, volume 2, pages 2317–2322.

Burrow, M. (1993). *Representation theory of finite groups*. Academic Press, New York.

Casal, A. and Yim, M. (1999). Self-reconfiguration planning for a class of modular robots. In *Proc. SPIE, Sensor Fusion and Decentralized Control in Robotic Systems II*, pages 246–257.

Chen, I. and Burdick, J. (1995). Determining task optimal modular robot assembly configurations. In *P IEEE International*

Conference on Robotics and Automation,, volume 1, pages 132–137.

Chen, I. and Burdick, J. (1998). Enumerating the Non-Isomorphic Assembly Configurations of Modular Robotic Systems. *INT J ROBOT RES*, 17(7):702–719.

Chiang, C. and Chirikjian, G. (2001). Modular robot motion planning using similarity metrics. *Autonomous Robots*, 10(1):91–106.

Chirikjian, G. (1994). Kinematics of a metamorphic robotic system. In *P IEEE International Conference on Robotics and Automation,*, pages 449–455.

Chirikjian, G. and Pamecha, A. (1996). Bounds for self-reconfiguration of metamorphic robots. In *P IEEE International Conference on Robotics and Automation,*, volume 2, pages 1452–1457.

Chitta, S. and Ostrowski, J. (2006). Enumeration and motion planning for modular mobile robots. *Department of Computer and Information Science, University of Pennsylvania, Technical report No. MS-CIS-01-08*.

Dumitrescu, A., Suzuki, I., and Yamashita, M. (2002). High speed formations of reconfigurable modular robotic systems. In *P IEEE International Conference on Robotics and Automation,*, volume 1, pages 123–128.

Dumitrescu, A., Suzuki, I., and Yamashita, M. (2004). Formations for Fast Locomotion of Metamorphic Robotic Systems. *INT J ROBOT RES*, 23(6):583–593.

Foggia, P., Sansone, C., and Vento, M. (2001). A performance comparison of five algorithms for graph isomorphism. In *In Proc. 3rd IAPR TC-15 Workshop on Graph-based Representations in Pattern Recognition*, pages 188–199.

Jain, B. and Wysotzki, F. (2005). Solving inexact graph isomorphism problems using neural networks. *NEUROCOMPUTING*, 63:45–67.

Kamimura, A., Kurokawa, H., Toshida, E., Tomita, K., Murata, S., and Kokaji, S. (2003). Automatic locomotion pattern generation for modular robots. In *P IEEE International Conference on Robotics and Automation,*, pages 714–720.

LaValle, S. (2006). *Planning algorithms*. Cambridge University Press.

McKay, B. (1990). Nauty Users Guide (version 1.5) Technical Report TR-CS-90-02. *Department of Computer Science, Australian National University*.

McKay, B. (1998). Isomorph-Free Exhaustive Generation. *J ALGORITHM*, 26(2):306–324.

McKay, B. (2007). Nauty website. `http://cs.anu.edu.au/∼bdm/nauty`. Accessed 03-Abr-2009.

Miyazaki, T. (1997). The complexity of McKayâĂŹs canonical labeling algorithm. In *Groups and Computation II: Workshop on Groups and Computation*, pages 239–256. American Mathematical Society.

Pamecha, A., Chiang, C., Stein, D., and Chirikjian, G. (1996). Design and implementation of metamorphic robots. In *P ASME Design Engineering Technical Conference and Computers in Engineering Conference*, pages 1–10.

Pamecha, A., Ebert-Uphoff, I., and Chirikjian, G. (1997). Useful metrics for modular robot motion planning. *IEEE T ROBOTIC AUTOM*, 13(4):531–545.

Rotman, J. (1995). *An Introduction to the Theory of Groups*. Springer, New York.

Rus, D. and Vona, M. (2001). Crystalline Robots: Self-Reconfiguration with Compressible Unit Modules. *AUTON ROBOT*, 10(1):107–124.

Scott, W. (1964). *Group Theory*. Prentice-Hall, New Jersey.

Siek, J., Lee, L., and Lumsdaine, A. (2002). *The Boost Graph Library: User Guide and Reference Manual*. Addison-Wesley, New York.

Simoni, R., Carboni, A., and Martins, D. (2008). Enumeration of parallel manipulators. *Robotica, doi: 10.1017/S0263574708004979*, pages 1–9.

Simoni, R., Martins, D., and Carboni, A. (2009). Enumeration of kinematic chains and mechanisms. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 223(4):1017–1024.

Walter, J., Tsai, E., and Amato, N. (2005). Algorithms for fast concurrent reconfiguration of hexagonal metamorphic robots. *IEEE transactions on Robotics*, 21(4):621–631.

Walter, J., Welch, J., and Amato, N. (2002). Concurrent metamorphosis of hexagonal robot chains into simple connected configurations. *IEEE T ROBOTIC AUTOM*, 18(6):945–956.

Walter, J., Welch, J., and Amato, N. (2004). Distributed reconfiguration of metamorphic robot chains. *DISTRIB COMPUT*, 17(2):171–189.

Yim, M., Lamping, J., Mao, E., and Chase, J. (1997). Rhombic dodecahedron shape for self-assembling robots. *Xerox PARC, SPL TechReport P9710777*.

Yim, M., Zhang, Y., Lamping, J., and Mao, E. (2001). Distributed Control for 3D Metamorphosis. *AUTON ROBOT*, 10(1):41–56.

Yoshida, E., Murata, S., Kurokawa, H., Tomita, K., and Kokaji, S. (1998). A distributed reconfiguration method for 3D homogeneous structure. In *P IEEE/RSJ International Conference on Intelligent Robots and Systems,*, volume 2, pages 852–859.

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