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Dispersion and inertia effects analysis on dynamic mechanical material characterisation

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Abstract.. *The dynamic mechanical characterisation of a given material is usually performed via the Split Hopkinson Pressure Bar, SHPB. In this device, a sample material is sandwiched between two long bars. One of the bars is hit by a striker, setting in motion the propagation of elastic waves. To avoid superposition of the pulse reflectedin the bar-sample interface, the deformation readings of the bar cannot be made in the contact interface. Hence, it is necessary to shift in space the measured pulse, which then causes a distortion of the signal. This paper aims to correct such a distortion, the so called dispersion, for the pulse signal measured in tests with the polymer PVC. It is also considered the influence of the radial inertia on the measured dynamic stress-strain curve. It is found that the PVC dnyamic stress-strain curve curve is affected by dispersion effects and that inertia plays a minor role.*

Keywords*: SPHB, dispersion, inertia effects, PVC dynamic properties*

1. INTRODUCTION

Thermoplastics are important polymers used in the manufacturing of many components, as the ones used in the automotive industry and packaging devices. These applications usually require a detailed structural analysis which, in turn, can only be reliably performed if the material behaviour is well understood.

There has been a substantial research effort during the last decades to understand the behavior of polymers (Arruda and Boyce, 2000), including its dynamic response (Du Bois *et al*., 2006). It is clear that thermoplastics constitutive models rely on material parameters that must be measured. Hence, any knowledge of the response of polymers to load, temperature, environment, etc... is important. In the case of structures undergoing large strains and dynamic loads, the material model should contemplate strain rate effects on its response (Rittel and Dorogoy, 2008 and Bouix *et al.*, 2009).

To evaluate strain effects in a material, one should perform tensile or compressive tests at different speeds. This is most simply accomplished by using the so called Split Hopkinson Pressure Bar (SHPB). This device, shown in Figure 1, consists of a striker which is launched against a long cylindrical input bar. The material sample is sandwiched between two long bars. Strain gages positioned in the middle of both bars are able to capture the incident, reflected and transmitted strain waves. From these signals, it is possible to calculate the stress-strain curve by simple formulae (Zhao, 2003) which are derived using three major assumptions: (i). propagating of waves in the bar is described by one-dimensional wave propagation theory. (ii). the stress and strain fields in the specimen are uniform in its axial direction. (iii). inertia effects and friction effects in compression test are negligible.

A 3D theory of propagation of pulses in cylindrical bars was developed by Pocchammer (1876) and Chree (1941). They found a relation between phase velocity and frequency of a wave travelling in a long bar. This relation is essential to be understood and used if dispersion effects are to be taken into consideration. This task is pursuit here. Also, based on the work of Davies and Hunter (1963), lateral inertia expansion of the sandwiched tested material is explored. It is possible then to infer how both effects, wave attenuation and material radial and axial inertias, affect the measured curve. This theory is here applied to a single test of a PVC polymer.

2. SPHB CORRECTIONS

2.1. Pochhammer-Chree frequency equation

The SPHB in Fig. 1 consists of a striker and two long bars with a short specimen sandwiched between them. The striker is accelerated by a gas gun and impacts the input bar.

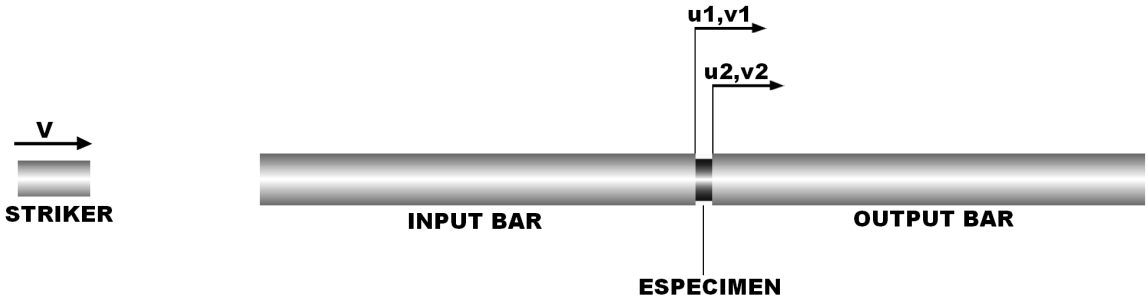

Figure 1. SPHB and operation scheme.

This impact generates a compressive pulse in the input bar that travels along its length until it reaches the interface bar-specimen. The pulse is partially transmitted through the specimen to the output bar and partially reflected in the input bar. The time history of these pulses, incident, ε_i, reflected, ε_r, and transmitted, ε_t, are measured by strain gages located in the input and output bas. A Wheatstone bridge is used to read this signal originated in the strain gages and it is amplified by a high speed transducer amplifier. The stress, σ_s, strain, ε_s, and strain rate, $\dot{\varepsilon}_s$, are simple to calculate from these pulses by

$$\sigma_s = E\frac{A}{2A_s}(\varepsilon_i + \varepsilon_r + \varepsilon_t), \qquad \varepsilon_s = \frac{c_0}{L_s}\int_0^t (\varepsilon_i - \varepsilon_r - \varepsilon_t)dt \qquad \text{and} \qquad \dot{\varepsilon}_s = \frac{c_0}{L_s}(\varepsilon_i - \varepsilon_r - \varepsilon_t) \tag{1}$$

where $c_0 = \sqrt{E/\rho}$ is the velocity of the elastic wave, E is the Young's modulus, ρ is the density, A is the cross section area of the bars, L_s and A_s are the length of the specimen and its cross section area. The strain gauges used to measure the strain pulses are located far from the specimen interfaces in order to avoid superposition of the incident and reflected pulses. It is usually assumed that these waves will not change shape as they travel to the station measurement points. However, they do change their amplitude due to the radial restriction of the bars, so that energy is consumed for radial expansion of the bars. This is what one calls wave dispersion.

The equations of motion in polar coordinates are Kolsky (1963),

$$\rho\frac{\partial^2 u_r}{\partial t^2} = (\lambda + 2\mu)\frac{\partial \Delta}{\partial r} - \frac{2\mu}{r}\frac{\partial \omega_z}{\partial \theta} + 2\mu\frac{\partial \omega_\theta}{\partial z} \tag{2}$$

$$\rho\frac{\partial^2 u_\theta}{\partial t^2}=(\lambda+2\mu)\frac{1}{r}\frac{\partial\Delta}{\partial\theta}-2\mu\frac{\partial\omega_r}{\partial z}+2\mu\frac{\partial\omega_z}{\partial r} \tag{3}$$

$$\rho\frac{\partial^2 u_z}{\partial t^2}=(\lambda+2\mu)\frac{\partial\Delta}{\partial z}-\frac{2\mu}{r}\frac{\partial}{\partial r}(r\omega_\theta)+\frac{2\mu}{r}\frac{\partial\omega_r}{\partial\theta} \tag{4}$$

where Δ is the dilatation in cylindrical coordinates given by:

$$\Delta=\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial\theta}+\frac{\partial u_z}{\partial z} \tag{5}$$

and λ , μ are Lamé's constants. $\varpi_r, \varpi_\theta, \varpi_z$ are the rotational components about the three orthogonal directions and u_r, u_θ, u_z are the displacements. If the propagation is axi-symmetric, u_θ is null and so are the two stress components at the bar surface. The relation between these and the strains are given by

$$\sigma_{rr}=\lambda\Delta+2\mu\frac{\partial u_r}{\partial r} \tag{6}$$

$$\sigma_{rz}=\mu\left[\frac{\partial u_r}{\partial z}+r\frac{\partial u_z}{\partial r}\right] \tag{7}$$

A relation between material properties, phase velocity, cylinder radius and frequency can be found as

$$\frac{\left[2\mu\frac{\partial^2}{\partial a^2}J_0(h'a)-\frac{\lambda}{\lambda+2\mu}p^2\rho J_0(h'a)\right]}{2\mu\gamma\frac{\partial}{\partial a}J_1(\kappa'a)}=\frac{2\gamma\frac{\partial}{\partial a}J_0(h'a)}{\left(2\gamma^2-\frac{p^2\rho}{\mu}\right)J_1(\kappa'a)} \tag{8}$$

where a is the radius of the rod, $\left.\frac{\partial}{\partial r}\right|_{r=a}=\frac{\partial}{\partial a}$, J_0 is the Bessel function of first kind of order zero, J_1 is the Bessel function of first kind of order one, p is the angular frequency, $\Lambda=2\pi/\gamma$ is the wavelength and h' and κ' are:

$$h'^2=\frac{\rho p^2}{(\lambda+2\mu)}-\gamma^2 \tag{9}$$

$$\kappa'^2=\frac{\rho p^2}{\mu}-\gamma^2 \tag{10}$$

For any chosen p , many γ will satisfy the so called Pochhammer-Chree Frequency equation, each one representing a vibration mode shape of the bar. The phase velocity can be easily found by $V_p=p/\gamma$. Although for a single frequency there can be many vibration shape modes present, each one with a different phase velocity, the correction method can allow only one phase velocity per frequency. So, it is necessary to find out which is the predominant shape mode for each frequency. To determine the rate at which the energy of a pulse is propagated, the group velocity, V_g should be calculated. The group velocity is defined as the propagation speed of a packet of waves, with the wavelengths of the component waves of the package being close to Λ. In other words, the mode with higher group velocity will be predominant. Mathematically, the group velocity can be calculated as

$$V_g=V_p-\Lambda\frac{\partial V_p}{\partial\Lambda} \tag{11}$$

Figure 2 shows the phase and group velocity as a function of the frequency for the bar in Figure 1, with 25.4 mm diameter, 214.11GPa Young´s modulus, 0.29 Poisson and 7896 kg/m^3 density. Figure 3 shows the phase velocity used in the correction method.

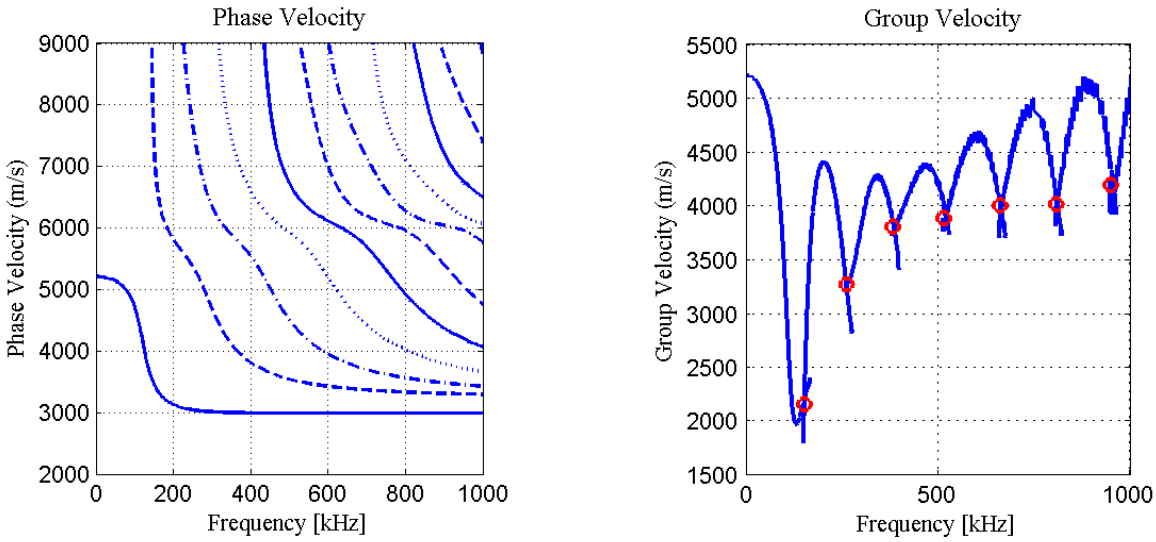

Figure 2. Phase and group velocities as a function of frequency

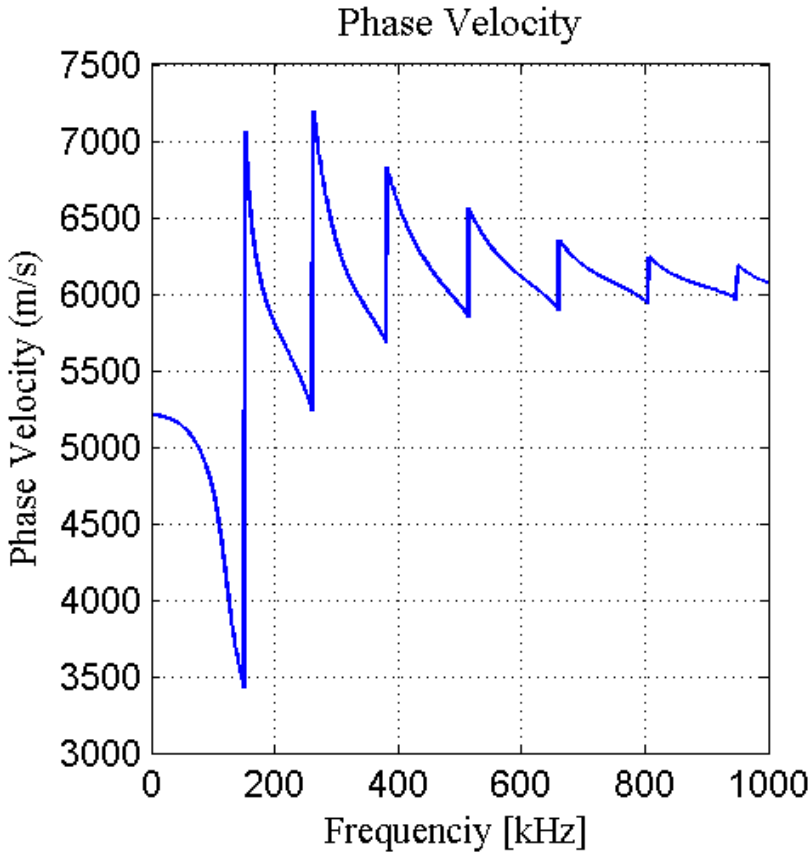

Figure 3. Phase velocity used for the dispersion correction

2.2. Dispersion correction

The Fast Fourier Transform, FFT, is the tool used for the correction of dispersion. One of its properties, the time shifting, can be described as

$$f(t+\tau) \Leftrightarrow F(w)e^{-jw\tau} \tag{12}$$

where $f(t)$ is a function in the time domain and $F(w)$ is its Fourier Transform. The time τ can be calculated by

$$\tau(w)=\frac{z}{V_p(w)} \tag{13}$$

where z is the distance from the stain gauge to the interface bar/specimen. Therefore, the strain in the interface, ε^*, can be obtained from the strain measured by the strain gauges, ε, as

$$\varepsilon^*(t)=FFT^{-1}\left(FFT(\varepsilon(t))e^{\frac{jzw}{V_p(w)}}\right) \tag{14}$$

It can be seen that the phase velocity, obtained from Figure 3 for the bar under study, is essential for the dispersion correction according to equation (14).

2.3. Inertial effects correction

According to Davies and Hunter (1963), radial and axial inertia of the sample material lead to an overestimation and underestimation of the actual material stress response, respectively. They developed a relation between the stress measured in the bar/specimen interfaces, σ_b , and the actual specimen stress, σ_s, given by

$$\sigma_s=\sigma_b+\rho_s\left(\frac{l^2}{6}-\nu_s\frac{d^2}{8}\right)\frac{d^2\varepsilon}{dt^2} \tag{15}$$

where ρ_s and ν_s are the density and Poisson for the material of the specimen, l and d are the length and diameter of the specimen and ε is the axial strain.

3. EXPERIMENTAL SET-UP AND RESULTS

The bars of the SHPB apparatus in Figure 1 are made of high-strength steel, with 7896 Kg/m^3 of density. Both bars are 1.4 m long, 25.4 mm diameter. Two diametrically opposed Excel PA-13-062AB-120L strain gauges are glued close to the middle of each bar, at 731mm in the input bar and at 739 mm in the output bar. The strain gauges are connected to a Fylde FE-H395-TA high speed transducer amplifier.

The electrical voltage is digitalized by a National Instruments PCI-6110, 12-Bit, 5 MS/s/ch, Multifunction data acquisition system. Both the control of the acquisition PCI board and the data processing are performed using Matlab. The steel striker used in all the tests is 214 mm long and diameter 25.4 mm. From the PVC sample material, disks of 20 mm diameter and 4 mm thick were manufactured and tested statically and dynamically. According to the manufacture of the PVC, its density is 1430 Kg/m^3 and the Young´s modulus is 3 GPa.

Figure 4 shows the quasi-static and one dynamic curve at engineering strain rates of 0.001 and 3000/s, respectively. The dynamic curve is corrected due to dispersion and inertia effects.

4. CLOSURE

It is clear from Figure 4 a large discrepancy between the stress-strain curves with and without dispersion effects. This is specially true for small strains, up to around 10%. It is interesting to note that the Young's modulus of the PVC is expected to increase with the strain rate since this material is visco-elastic. This is the case shown in Figure 4 when correction for the dispersion is applied to the measured data. If the dispersion correction was overlooked, then visco-elastic effects in this polymer would not be captured by this test.

Although the dispersion correction is quite important, Figure 4 also shows that inertial effects can be neglected, at least for this polymer at this strain rate of 3000/s. This is so partially due to the low density of this material, which then does not require much energy to be radially expanded and axially compressed.

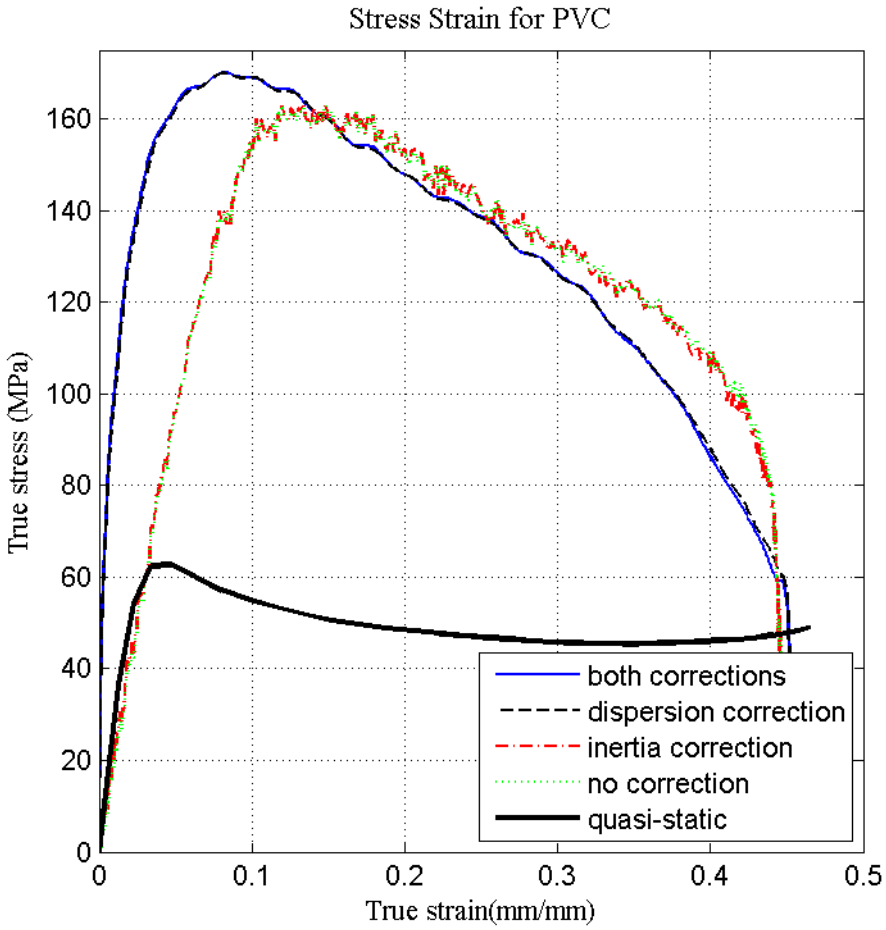

Figure 4. Dynamic stress strain curves for PVC.

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