Proceedings of COBEM 2009
Copyright © 2009 by ABCM

20th International Congress of Mechanical Engineering
November 15-20, 2009, Gramado, RS, Brazil

TYPE SYNTHESIS OF LOW-DOF PARALLEL ROBOTS BASED ON SCREW THEORY

Roberto Simoni, roberto.emc@gmail.com
Daniel Martins, daniel@emc.ufsc.br
Mechanical Engineering Postgraduate Program (POSMEC), Federal University of Santa Catarina,
Bairro Trindade - Florianópolis - Santa Catarina - Brasil - CEP 88040-970.

Abstract. *In this paper a new procedure for type synthesis of low-DoF parallel robots based on screw theory is presented. The motivation of these work is that for many industrial applications, in particular in the machine-tool field, less than 6 degree of freedom (6-DoF) may be needed to complete the task. For example, for milling operation in the machine-tool domain, the rotation of the platform around its normal is not needed, as the spindle will manage this DoF, hence only 5-DoF are needed. Type synthesis consists basically in finding all possible types of parallel robots capable of generating a specified motion pattern of the moving platform. The type of motion pattern of the moving platform is specified in agreement with the degrees of freedom of the task to be developed. Previous research on parallel manipulators were concentrated mainly on 6-DoF parallel robots, but today parallel robots with fewer than 6-DoF, also called low-DoF parallel robots, are being increasingly studied. In comparison with 6-DoF parallel robots, a low-DoF parallel robot has the advantages of more simpler mechanical design, lower manufacturing cost, simpler controller and, in general, reducing the number of DoF will increase the maximum motion range of the remaining DoF increasing the workspace. The type synthesis procedure present in this paper is developed in five steps. First, the motion pattern is defined in agreement with the DoF necessary for task execution. Second, using reciprocal screws we define the robot twist and wrench system. Third, the legs of parallel robot are enumerated using linear combinations of screws and the assembling of parallel robot is done. Fourth, as twists and wrenches are instantaneous, the identification of instantaneous parallel robots is necessary. And fifth, a method for validation of actuated joints is present. Parallel robots with 4-DoF and 5-DoF are synthesized and tables are presented using a special notation. A functional representation of each parallel robot with 4-DoF and 5-DoF and actuated joints are presented. The advantage this procedure in relation of others procedures based on screw theory, found in literature, is the completeness of type synthesis process.*

Keywords: *low-DoF parallel robots, type synthesis, motion pattern, screw theory, reciprocal screws.*

1. INTRODUCTION

A parallel robot is one in which two or more serial kinematic chains connect the moving platform to the base. Parallel robots can offer advantages over their serial counterparts in terms of rigidity, dynamic performances and accuracy. In recent years, the research and use of parallel robot have evolved from the general 6-DoF parallel robots to the low-DoF parallel robots, which have fewer than 6-DoF. The use of low-DoF parallel robots for many tasks requiring fewer than 6-DoF is attracting interest from research and mainly of industry. The main motivation is that for many applications, less than 6-DoF may be needed. For example, for milling operation in the machine tool domain, the rotation of the platform around its normal is not needed and hence only 5-DoF are needed (Merlet and Daney, 2008). Also, the 6-DoF parallel robots currently used in machining suffer from their complexity since only 5-DoF of the tool need to be controlled (Huang and Li, 2003). Design of new low-DoF parallel robot is a good chance of reducing the costs of design and manufacturing in those applications requiring less mobility.

The characteristics of the motion of the moving platform of a parallel robot with F-DoF may be of several types. For example, a 3-DoF motion may be a 3-DoF translational motion, a 3-DoF spherical motion or a 3-DoF planar motion. Kong and Gosselin (2007) call each type of motion of the moving platform of motion pattern. Thus, the type synthesis consists in finding all the possible types of parallel robots generating a specified motion pattern of the moving platform (Kong and Gosselin, 2007). "Type synthesis is a fundamental and important issue in design of parallel robots (Kong and Gosselin, 2007; Merlet, 2006; Merlet, 2000)".

Up to date, the majority low-DoF parallel robots find in literature has 3-DoF. Fang and Tsai (2004) present a systematic methodology for the structural synthesis of a class of 3-DoF spherical parallel robots and Tsai (1999b) proposed an enumeration of a class of 3-DoF parallel robots based on the structural characteristics. Kong and Gosselin (2004a) presented an systematic methodology based on screw theory for type synthesis of 3-DoF spherical parallel robots. Kong and Gosselin (2004b) presented type synthesis of 3-DoF translational parallel robots based on screw theory. Gogu (2004b) presents a structural synthesis of translational parallel robots with uncoupled motions and revolute actuators situated on the fixed base. Gogu (2007a) presents the structural synthesis of fully-isotropic parallel wrists with three degrees of freedom. Gogu (2008, 2009) presents a structural synthesis approach of parallel robots based on theory of linear transformations and evolutionary morphology (Gogu, 2005).

The number of low-DoF parallel robots with 4-DoF or 5-DoF is fairly small (Kong and Gosselin, 2005; Kong and

Nomenclature

Notation	Description
	Screw theory
$\$$, $\$_0$, $\$_\infty$	screw, screw of 0 pitch and screw of ∞ pitch
$\mathcal{T}$, $\mathcal{W}$	twist and wrench systems
ξ_0, ξ_∞	twist of 0 and ∞ pitch
ζ_0, ζ_∞	wrench of 0 and ∞ pitch
	Mechanical generators
R, P	rotational joint (R joint) and prismatic joint (P joint)
$\underline{\text{R}}$, $\underline{\text{P}}$	rotational actuated joint ($\underline{\text{R}}$ joint) and prismatic actuated joint ($\underline{\text{P}}$ joint)
	Legs of parallel robot
PRRR	leg composed by a P joint and three R joints
Ŕ	R joint with parallel axis within a same leg
R̀	R joint with parallel axis within a same leg
$\overline{\text{R}}$	R joint with parallel axis within a same leg group
PŔŔR̀R̀	PRRRR leg with two R joints with parallel axis in a direction and two R joints with parallel axis in another direction
$\underline{\text{P}}$ŔŔR̀R̀	PŔŔR̀R̀ with $\underline{\text{P}}$ joint
	Parallel robot
3-PŔŔR̀R̀	parallel robot with three PŔŔR̀R̀ legs, the number 3 indicates three legs
3-$\underline{\text{P}}$ŔŔR̀R̀	3-PŔŔR̀R̀ legs parallel robot with $\underline{\text{P}}$ joint

Gosselin, 2004c; Li et al., 2004; Li and Huang, 2003). Recently, Kong and Gosselin (2004c,2005,2007), Huang and Li (2002,2003) and Gogu (2005,2007a,2008,2009) presented systematic methods for type synthesis of parallel robots with 4-DoF or 5-DoF. However, the methods presented are not complete requiring post-processing to complete the type synthesis process and still, a limited number of architectures has been proposed and few were actually implemented. The types of low-DoF parallel robots are deficient and far from the high demand in practice (Huang and Li, 2002).

The methods used for synthesis of parallel robots can be divided into five approaches:

- graph theory; (Simoni et al. , 2007a, 2007b, 2008; Alizade et al. , 2004, 2007; Tsai, 2001)
- group theory; (Simoni et al. , 2009)
- screw theory; (Kong and Gosselin, 2005, 2007; Huang and Li, 2003; Fang and Tsai, 2002)
- Lie subgroups of displacements; (Hervé and Sparacino, 1991; Hervé 1994, 1999; Li et al. , 2004)
- theory of linear transformations and evolutionary morphology; (Gogu, 2005,2008,2009)

The first two approaches are know as number synthesis and the three last approaches are know as type synthesis in robotic literature. Between the last three approaches one that is more appropriate for type synthesis of parallel robots and presents a greater number of references (even poor) is the screw theory approach.

In this paper, a systematic approach is developed for type synthesis of low-DoF parallel robots using screw theory. In type synthesis process we find all possible types of parallel robot arquitetures that generate a specified motion pattern, the next step in conceptual design of parallel robot is the dimensional synthesis (Tsai, 2001). The problem is that some type synthesis procedure found in literature need a post-processing before the dimentional synthesis. Still, a small number low-DoF parallel robot were proposed. In view of this, there are the necessity of investigate new methods for type synthesis and new parallel robots. The procedure for type synthesis is development in five steps. First, the motion pattern is defined in agreement with the DoF necessary for task execution. Them, using reciprocal screws we define the robot twist and wrench systems. The legs of parallel robot are enumerated using linear combinations of screws and the assembling of parallel robot is done. As twists and wrenches are instantaneous, the identification of instantaneous parallel robots is necessary. In finally, a method for validate of actuated joints is present. Parallel robots with 4-DoF and 5-DoF are synthesized and tables are presented using the nomenclature presented above. A functional representation of each parallel robot with 4-DoF and 5-DoF and actuated joints are presented. The difference of proposed procedure in this paper is the completeness of type synthesis procedure of parallel robots via screw theory, also new approaches are introduced in each step of method.

The remainder of this paper is structured as follows. Section 2 introduces the screw theory tools that are used in type synthesis procedure presented in section 3. Section 4 present the type synthesis of 4-DoF and 5-DoF parallel robots. Section 5 present the conclusions and final remarks.

2. SCREW THEORY TOOLS

In this section, some relevant results of screw theory are presented. More details on screw theory can be found in (Hunt, 1990; Ball, 1998; Davidson and Hunt, 2004; Kong and Gosselin, 2007).

2.1 Screw

In screw theory (Hunt, 1990; Ball, 1998; Davidson and Hunt, 2004), a unit screw $\$$ is defined by a pair of vectors

$$\$ = (\,\$_F\,;\,\$_S\,) = \begin{cases} (\,s\,;\,s_0 \times s + hs\,), & \text{if } h \text{ is finite} \\ \\ (\,0\,;\,s\,), & \text{if } h \text{ is infinite} \end{cases} \tag{1}$$

where s is a unit vector along the axis of the screw $\$$, s_0 is a vector directed from origin of the reference frame O-xyz to any point on the axis of the screw, and h is called the pitch. There are two vector components (F-first, S-second) or six scalar components in the above presentation of the screw.

2.2 Screw systems

A screw system of order n $(0 \leq n \leq 6)$ comprises all the screws that are linearly dependent on n given linearly independent screws. A screw system of order n is also called an n-system. Any set of n linearly independent screws within an n-system forms a basis of the n-system. There are many types of screw systems, see for instance (Hunt, 1990; Davidson and Hunt, 2004).

In this paper we uses the canonical base screws (Davidson and Hunt, 2004). For instance,

$$\$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0), \tag{2}$$

represent a 1-system. There are two special cases of note; one when the pitch is $h_\alpha = 0$, *i.e.* $\$_0 = (1\ 0\ 0; 0\ 0\ 0)$, and another when the pitch $h_\alpha = \infty$, *i.e.* $\$_\infty = (0\ 0\ 0; 1\ 0\ 0)$. The mechanical generators of this two 1-systems are respectively the R and P joints.

The canonical base screws

$$\begin{cases} \$_\alpha = (1\ 0\ 0; h_\alpha\ 0\ 0), \\ \$_\beta = (1\ 0\ 0; 0\ h_\beta\ 0), \\ \$_\gamma = (1\ 0\ 0; 0\ 0\ h_\gamma), \end{cases} \tag{3}$$

represent a 3-system.

2.3 Reciprocal Screws

Two screws, $\$_a$ and $\$_b$, are said to be reciprocal if they satisfy the following condition:

$$\$_a \circ \$_b = \$_{a_F} \cdot \$_{b_S} + \$_{b_F} \cdot \$_{a_S} = \$_a^T \Pi \$_b = 0 \tag{4}$$

where

$$\Pi = \begin{bmatrix} 0 & I_3 \\ I_3 & 0 \end{bmatrix}$$

where I_3 is the 3×3 identity matrix and 0 is the 3×3 zero matrix. The operator " $\circ$" is defined as the reciprocal product of two screws.

For example, from 4, we can to see that the screws of two P joints are always reciprocal to each other and the screws of two R joints are reciprocal to each other if and only if their axis are coplanar.

2.4 Reciprocal Screw Systems

Given an n-system, there is a unique reciprocal screw system of order $(6-n)$ which comprises all the screws reciprocal to the original screw system. Let $\mathcal{T}$ and $\mathcal{W}$ denote a screw system and its reciprocal screw system.

Consider, for example, a particular case of the 1-system

$$\mathcal{T} : \left\{ \ \$_0 = (1\ 0\ 0; 0\ 0\ 0) \right. \tag{5}$$

with represent a canonical screw with infinite pitch along to x axis. The reciprocal screw system of this 1-system is the 5-system formed by the following canonical screws:

$$\mathcal{W} : \begin{cases} \$_0 = (1\ 0\ 0; 0\ 0\ 0) & \$_\infty = (0\ 0\ 0; 0\ 1\ 0) \\ \$_0 = (0\ 1\ 0; 0\ 0\ 0) & \$_\infty = (0\ 0\ 0; 0\ 0\ 1) \\ \$_0 = (0\ 0\ 1; 0\ 0\ 0) & \end{cases} \tag{6}$$

Each screw of $\mathcal{W}$ is reciprocal of the screw of $\mathcal{T}$ and vice-versa.

2.5 Twist Systems and Wrench Systems

We call the screw of twist (ξ) if it represents an instantaneous motion of a rigid body. The canonical twist system representing the DoF of a moving platform of a 6-DoF parallel mechanism is

$$\mathcal{T}: \left\{ \begin{array}{ll} \xi_0 = (1\ 0\ 0; 0\ 0\ 0) & \xi_\infty = (0\ 0\ 0; 1\ 0\ 0) \\ \xi_0 = (0\ 1\ 0; 0\ 0\ 0) & \xi_\infty = (0\ 0\ 0; 0\ 1\ 0) \\ \xi_0 = (0\ 0\ 1; 0\ 0\ 0) & \xi_\infty = (0\ 0\ 0; 0\ 0\ 1) \end{array} \right. \tag{7}$$

The first three twists in eq. 7 denote three rotations about the x, y and z axes, respectively. The last three twists in eq. 7 denote three translations along the x, y and z axes, respectively.

The combined constraints of moving platform of parallel robot can also be expressed by a screw system, called wrench system, reciprocal to twist systems of moving platform. In other words, wrench system denotes a system of forces and couples acting on a rigid body or the constrained DoF of the moving platform. For example, if the equation 5 represent a twist system of moving platform them the equation 6 represent their wrench system.

A physical interpretation of reciprocal screws is given by Davidson and Hunt (2004). The relation between a twist system and its wrench system is the virtual power developed by any wrench along any twist is equal to zero (Davidson and Hunt, 2004; Kong and Gosselin, 2007).

According to the prescribed motion pattern of moving platform of the low-DoF parallel robot, a suitable subset of twists can be selected from eq. 7 to form the twist system of moving platform of the low-DoF parallel robot.

3. COMPLETE PROCEDURE FOR TYPE SYNTHESIS OF LOW-DOF PARALLEL ROBOTS

In this section we present a complete procedure for type synthesis of parallel robots when screw theory is used. The procedure is development in five steps presented Tab. 1. Some authors (Huang and Li, 2002; Kong and Gosselin, 2007; Fang and Tsai, 2002) already have proposed methods for type synthesis using screw theory, so in the Tab. 1 we included a observation to indicate what is the difference of our proposed method and the novelty introduced.

Table 1. Type synthesis procedure proposed with differences and novelty introduced.

Step	Description	Sec.	Source	Novelty
1	Derivation of motion pattern of the moving platform	3.1		Complete description of motion pattern of moving platform in terms of twist and wrench systems.
2	Type synthesis of legs	3.2	[1][2][3]	With R and P joints only. Complex joints are obtained by combination of R and P joints in a later stage of design, *e.g.* dimensional synthesis.
3	Assembling of legs	3.3	[1][2][3]	-
4	Identification of full-cycle mobility	3.4	[2]	Identification via CAD simulation
5	Identification of actuated joints	3.5	[1]	Divided into two important sub-steps.
	Selection of actuated joints	3.5.1		Is very important to select the actuated joints on or near the fixed base, see (Tsai, 2001).
	Validation of actuated joints	3.5.2		In just two steps via CAD simulation.

Sources used in this table are shown below. The fifty column shows the original contribution of each step.
[1] : (Kong and Gosselin, 2007)
[2] : (Huang and Li, 2003)
[3] : (Fang and Tsai, 2002)

The difference of proposed method in this paper is the completeness of type synthesis process of parallel robots via screw theory. The main authors about type synthesis of parallel robots via screw theory are; Fang and Tsai, Kong and Gosselin, and Huang and Li. Fang and Tsai (2002, 2004) do not present a systematic procedure for type synthesis but they following a few steps as shown in sources in the column 4 of Tab. 1. Kong and Gosselin (2004a, 2004b, 2005, 2007) present a systematic procedure but in his procedure they do not identify full-cycle parallel robots. Kong and Gosselin (2007) present an equation to identify the mobility of parallel robots generated in type synthesis procedure but the mobility obtained for those equation is usually instantaneous. Also, the validation process of actuated joints proposed here is different from that proposed by them. The procedure for validation of actuated joints proposed by Huang and Li (2003) is distinct of proposed in step 5 on the our procedure. In following subsections (see column 3 in Tab. 1), each step is described in details and examples are shown for a better understanding.

3.1 Step 1: Derivation of motion pattern of the moving platform

This step is to identify the twists and wrenches related by the complete description of motion pattern. The characteristics of the motion of the moving platform of a parallel robot with F-DoF may be of several types. For example, a 3-DoF motion may be a 3-DoF translational motion, a 3-DoF spherical motion, a 3-DoF planar motion or another 3-DoF motion.

Defined the motion pattern the twist and wrench systems are derived by equation 4. For example, for a 3-DoF spherical parallel robot, the twist system consist of three ξ_0 and the wrench system consists of three constraint forces ζ_0 described below

$$\mathcal{T} : \begin{cases} \xi_0 = (1\ 0\ 0; 0\ 0\ 0) \\ \xi_0 = (0\ 1\ 0; 0\ 0\ 0) \\ \xi_0 = (0\ 0\ 1; 0\ 0\ 0) \end{cases} \qquad \mathcal{W} : \begin{cases} \zeta_0 = (1\ 0\ 0; 0\ 0\ 0) \\ \zeta_0 = (0\ 1\ 0; 0\ 0\ 0) \\ \zeta_0 = (0\ 0\ 1; 0\ 0\ 0) \end{cases} \tag{8}$$

The difference of this step, in relation of another found in literature, is the detailed description of motion pattern and description of this motion in terms of canonical base twist and wrench systems.

3.2 Step 2: Type synthesis of legs

We assume that only R and P joints are used in type synthesis of legs. Note that a universal joint (U) is equivalent to two intersecting R joints. A cylindrical joint (C) is equivalent to one P joint and a concentric R joint, and spherical joint (S) is equivalent to three intersecting R joints, and so on. The parallel robot are synthesized with R and P joints, in dimensional synthesis some R and P joints can be contracted forming C, U or S joints depending the length of links.

The legs of parallel robot can be synthesized by linear combination of twists. The type of legs will be represented by a string of characters representing the type of joints from the base to the moving platform in sequence, see nomenclature in pg 2. For example, the string ŔŔŔ (see table of nomenclature in pg 2) denotes a leg with four joins belong to a 3-system. The leg PŔŔŔ belong to a 4-system.

3.3 Step 3: Assembling of legs

Parallel robots can be generated by assembling two or more legs so that the arranging of legs meets the design requirements.

In assembling of the legs the wrench system of each leg ($\mathcal{W}^i$) introduce some constraint on the moving platform. Thus, the wrench system of moving platform ($\mathcal{W}$) is given by linear combination of the wrench systems of all its legs. For example, if a parallel robot is formed by two legs, a leg imposes restriction of rotation about x axis and another leg imposes restriction of translation along z axis, the resulting parallel robot imposes the two leg restrictions in moving platform, for instance, rotation about x axis and translation along z axis.

If the mechanism expected is full-symmetric, *i.e.* it meets the following conditions (Mohamed and Duffy, 1985; Huang and Li, 2003):

- has full-cycle mobility (discuss in next section);
- has identical legs;
- all legs are symmetrical on the base;
- has the same number and mounting position of actuators in each leg;

than, the legs must be assembling symmetrically on the base and to be identical. If the actuators are mounted on the base the number of legs must be equal the mobility of parallel robot.

3.4 Step 4: Identification of full-cycle mobility

One objective of type synthesis is find full-cycle parallel robots. A parallel robot that do not change their motion pattern after a finite motion is said full-cycle parallel robot (Mohamed and Duffy, 1985).

Since twists and wrenches are instantaneous, it is necessary to identify whether the synthesized parallel robots are instantaneous. A parallel robot is said to be instantaneous if its motion pattern changed after an arbitrary feasible finite motion. For example, if a parallel robot has three translational DoF at a moment, and has two translational DoF and one rotational DoF at another moment, the mechanism is instantaneous.

This can be done by CAD simulation of parallel robot motion. If after any feasible finite displacement the twist and wrench systems remains unchanged, the parallel robot is non-instantaneous, *i.e.* full-cycle. This step is crucial in design of parallel robots since the main objective of type synthesis process is to find full-cycle parallel robots.

3.5 Step 5: Identification of actuated joints

This step was divided in two important sub-steps, selection and validation, described bellow.

3.5.1 Step 5.1: Selection of actuated joints

The choice of actuated pairs cannot be arbitrary because the robot can have motion uncontrolled or have over-constrained and consequently the motion pattern desired is not generated. Generally, each leg is controlled by one actuator and all actuators can be mounted on or near the fixed base. As a result of this structural arrangement, parallel manipulators possess the advantages of low inertia, high stiffness, and large payload capacity (Tsai, 2001).

The set of actuated joints must be such that the combination of all twists in the set result on the same twist system of the moving platform. Thus, when the set of actuated joints is blocked the twist system of moving platform becomes a 0-system, *i.e.* the wrench system is a 6-system.

Note that for a low-DoF parallel robots, the set of actuated joints is not unique.

3.5.2 Step 5.2: Validation of actuated joints

For an F-DoF parallel robot, a set of F actuated joints is valid if the DoF of the parallel robot obtained from the mechanism by blocking all the actuated joints is 0 (Kong and Gosselin, 2007). In other works, when all the actuated joints in a parallel robot are blocked, the moving platform loses all DoF and cannot undergo any infinitesimal and finite motion.

The procedure proposed in this paper to validation of actuated joints is summarized in the following steps:

Step 1: Block all joints of set find in step 5.1.

Step 2: Apply loads in all directions an verify if the moving platform can undergo any infinitesimal or finite motion.

If these two steps are valid, than the set of actuated joint is appropriate.

4. TYPE SYNTHESIS OF LOW-DOF PARALLEL ROBOTS

In this section we apply the procedure proposed in above section for type synthesis of 4-DoF and 5-DoF parallel robots. In the end of each section we present a table with several parallel robots and a functional representation of one parallel robot with actuated joints. After the type synthesis procedure proposed here, in design of parallel robot, we consider the dimensional synthesis of each structure obtained.

4.1 Type synthesis of 4-DoF parallel robots

There are many 4-DoF motion patterns, here we apply the method for type synthesis of classical Schönflies motion, generator of three translations and one rotation, also know as SCARA motion (counterparts of serial robots) or 3T1R motion. This example was chosen because the SCARA robots are widely used in the industry (Bonev, 2001; Bonev, 2000; Merlet, 2006), the counterparts 3T1R parallel robot has the advantage of speed, accuracy and stiffness (Tsai, 2001). So, we see great potential for industrial applications, in special in pick-and-place task, and therefore there is a need to explore new 3R1T parallel robots.

The procedure proposed in this section 3 will be apply step by step by synthesis of 3T1R parallel robots.

4.1.1 Step 1: Derivation of motion pattern of the moving platform

The motion pattern is the Schönflies motion or 3T1R motion. We consider that the rotational DoF is about the z axis.

The canonical twist system is easily derived and is composed by four twists and the wrench system is derived of equation 4 and consists of two constraint couples described below

$$\mathcal{T}:\begin{cases}\xi_\infty=(0\ 0\ 0;1\ 0\ 0)\\ \xi_\infty=(0\ 0\ 0;0\ 1\ 0)\\ \xi_\infty=(0\ 0\ 0;0\ 0\ 1)\\ \xi_0=(0\ 0\ 1;0\ 0\ 0)\end{cases}\qquad \mathcal{W}:\begin{cases}\zeta_\infty=(0\ 0\ 0;1\ 0\ 0)\\ \zeta_\infty=(0\ 0\ 0;0\ 1\ 0)\end{cases}\tag{9}$$

4.1.2 Step 2: Type synthesis of legs

As discussed in section 3.2 the legs can be represented by a number of R and P joints in sequence.

As the wrench system of moving platform is given by linear combination of wrench system of legs, the wrench system of each leg of 3T1R parallel robot must be on the maximal a 2-system restricting rotations about x and y axis. The minimal number of 1-DoF joints (R or P) in a serial chain that satisfies the equation 9 is four. Thus, for illustrate our procedure, we consider only legs with four and five joins and with zero, one and two P joints. With these restrictions, there are six possibilities of joints allocation in the legs that satisfies the equation 9: 3R1P, 2R2P, 5R, 4R1P and 3R2P with their combinations. Based in screw reciprocity (eq. 4), we list in Tab. 2 the geometric constraints of joint axis for that each legs generated satisfies the equation 9. The legs were listed with the wrench systems $2\zeta_\infty-$system and $1\zeta_\infty-$system since the wrench system of moving platform is given by linear combinations of wrench systems of legs.

All legs obtained after linear combinations of twists and following the geometric constraints above are listed in Tab. 3.

Table 2. Geometric constraints of legs based on equation 4 and that satisfies the equation 9.

Wrench	Type of joints	Geometric constraints	Notation
$2\zeta_\infty-$system	3R1P	R joints with all parallel axis and the P joint cannot be perpendicular to axis of R joints	(PŔŔŔ)
	2R2P	R joints with all parallel axis and in the minimal one P joint cannot be perpendicular to axis of R joints	(PPŔŔ)
$1\zeta_\infty-$system	5R	four R joints with parallel axis perpendicular to another R joint	(ŔŔŔŔR̀)
		three R joints with parallel axis perpendicular to another two R joints	(ŔŔŔR̀R̀)
	4R1P	three R joints with parallel axis perpendicular to another R joint	(PŔŔŔR̀)
		two R joints with parallel axis perpendicular to another two R joints	(PŔŔR̀R̀)
	3R2P	two R joints with parallel axis perpendicular to another R joint	(PPŔŔR̀)

Table 3. Legs based on geometric constraints listed in Tab. 2.

Wrench	Type of joints	Geometric constraints	Legs
$2\zeta_\infty-$system	3R1P	(PŔŔŔ)	PŔŔŔ, ŔPŔŔ, ŔŔPŔ, ŔŔŔP
	2R2P	(PPŔŔ)	PPŔŔ, PŔPŔ, PŔŔP, ŔPPŔ, ŔPŔP, ŔŔPP
$1\zeta_\infty-$system	5R	(ŔŔŔŔR̀)	ŔŔŔŔR̀, ŔŔŔR̀Ŕ, ŔŔR̀ŔŔ, ŔR̀ŔŔŔ, R̀ŔŔŔŔ
		(ŔŔŔR̀R̀)	ŔŔŔR̀R̀, ŔŔR̀R̀Ŕ, ŔR̀R̀ŔŔ, R̀R̀ŔŔŔ, R̀ŔR̀ŔŔ, R̀ŔŔR̀Ŕ, R̀ŔŔŔR̀, ŔR̀ŔR̀Ŕ, ŔR̀ŔŔR̀, ŔŔR̀ŔR̀
	4R1P	(PŔŔŔR̀)	PŔŔŔR̀, PŔŔR̀Ŕ, PŔR̀ŔŔ, PR̀ŔŔŔ [*]
		(PŔŔR̀R̀)	PŔŔR̀R̀, PR̀ŔŔR̀, PR̀ŔR̀Ŕ [*]
	3R2P	(PPŔŔR̀)	PPŔŔŔ, PŔPŔŔ, PŔŔPŔ, PŔŔŔP, ŔPPŔŔ, ŔPŔPŔ, ŔPŔŔP, ŔŔPŔP, ŔŔPPŔ, ŔŔŔPP

[*] - another combinations with P joints out of base.

4.1.3 Step 3: Assembling of legs

Parallel robots generators of Schönflies motion can be generated by assembling a set of legs shown in Tab. 3.

Due to the large number of Schönflies motion parallel robots, only full-symmetric parallel robots are shown in Tab. 4. In this phase, the type synthesis of full-cycle parallel robots obeys the second and third items on full-cycle as discussed in section 3.3, *i.e.* has identical legs and all legs are symmetrical on the base.

Table 4. Enumeration of 4-DoF parallel robot generators of Schönflies motion from the legs shown in Tab. 3.

Type of joints	Geometric constraints	Parallel robots with $m = 2, 3$ or 4 legs
3R1P	(PŔŔŔ)	m-PŔŔŔ, m-ŔPŔŔ, m-ŔŔPŔ, m-ŔŔŔP
2R2P	(PPŔŔ)	m-PPŔŔ, m-PŔPŔ, m-PŔŔP, m-ŔPPŔ, m-ŔPŔP, m-ŔŔPP
5R	(ŔŔŔŔR̀)	m-ŔŔŔŔR̀, m-ŔŔŔR̀Ŕ, m-ŔŔR̀ŔŔ, m-ŔR̀ŔŔŔ, m-R̀ŔŔŔŔ
	(ŔŔŔR̀R̀)	m-ŔŔŔR̀R̀, m-ŔŔR̀R̀Ŕ, m-ŔR̀R̀ŔŔ, m-R̀R̀ŔŔŔ, m-R̀ŔR̀ŔŔ, m-R̀ŔŔR̀Ŕ, m-R̀ŔŔŔR̀, m-ŔR̀ŔR̀Ŕ, m-ŔR̀ŔŔR̀, m-ŔŔR̀ŔR̀
4R1P	(PŔŔŔR̀)	m-PŔŔŔR̀, m-PŔŔR̀Ŕ, m-PŔR̀ŔŔ, m-PR̀ŔŔŔ
	(PŔŔR̀R̀)	m-PŔŔR̀R̀, m-PR̀ŔŔR̀, m-PR̀ŔR̀Ŕ
3R2P	(PPŔŔR̀)	m-PPŔŔŔ, m-PŔPŔŔ, m-PŔŔPŔ, m-PŔŔŔP, m-ŔPPŔŔ, m-ŔPŔPŔ, m-ŔPŔŔP, m-ŔŔPŔP, m-ŔŔPPŔ, m-ŔŔŔPP

We chose the 4-PŔR̀ŔR̀ parallel robot for application of other steps of procedure. Up to the authors' knowledge this example never been presented in details in the literature.

4.1.4 Step 4: Identification of instantaneous mechanisms

In accordance with Hunt (1990) and Davidson and Hunt (2004) there is only one 4-system that is invariant with finite displacements, this is the given by four twists of the equation 9. After CAD simulation, the 4-P$\mathrm{\acute{R}\grave{R}\acute{R}\grave{R}}$ parallel robot it is shown to be full-cycle mobility, *i.e.* , it do not change their motion pattern by infinitesimal and finite displacements.

4.1.5 Step 5: Validation of actuated joints

Following the procedure to validation of actuated joints proposed in section 3.5. First, we select the set of actuated joints on the base, we select the set of four P joints and their are blocked. With the four P joints blocked, the moving platform cannot undergo any infinitesimal and finite motion. Thus, the set of actuated joint is appropriate and the 4-P$\mathrm{\acute{R}\grave{R}\acute{R}\grave{R}}$ parallel robot is shown in Fig. 1.

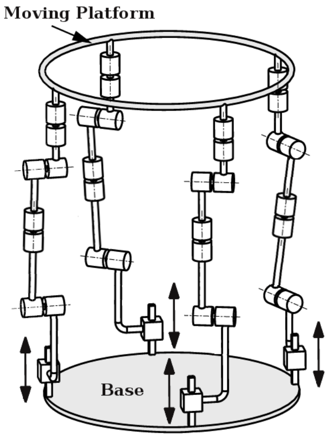

Figure 1. 4-P$\mathrm{\acute{R}\grave{R}\acute{R}\grave{R}}$ parallel robot generator of Schönflies motions.

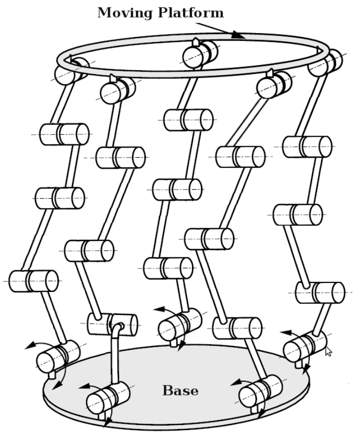

Figure 2. 5-$\mathrm{\underline{\grave{R}}\overline{RRR}\grave{R}}$ parallel robot generator of PPPU motions.

4.2 Type synthesis of 5-DoF parallel robots

There are some 5-DoF motion patterns, we apply here the proposed method for type synthesis of PPPU motion, generator of three translations and two rotations. This example was chosen because PPPU parallel robot is of great interest of machine-tool domain, so called five-axis machining (Refaat et al., 2006; Merlet and Daney, 2008; Merlet, 2006). For example, for milling operation in the machine tool domain, the rotation of the platform around its normal is not needed, as the spindle will manage this DoF, hence only 5-DoF are needed. So, we see great potential for applications in machine-tool domain and therefore there is a need to explore more these PPPU parallel robots.

Applying the same procedure we obtain the parallel robots presented in Tab. 5, where we consider two type joints cases; 5R and 4R1P. Some geometric constraints of legs for 5-DoF parallel robots generators of PPPU motion is given by line of $1\zeta_\infty-$system in Tab. 2. And consequently, the legs are presented in line of $1\zeta_\infty-$system in the Tab. 3. The assembling of legs must consider the axis of all legs in the leg group to be parallel to permit that the moving platform have 5-DoF, differently of 4-DoF that this restriction is not need.

Table 5. Enumeration of 5-DoF parallel robot generators of PPPU motion from the legs shown in Tab. 3 with additional constraints.

Type of joints	Geometric constraints in leg group	Parallel robots with $m = 4$ or 5 legs
5R	$(\mathrm{\overline{RRR}\grave{R}\grave{R}})$	m-$\mathrm{\overline{RRR}\grave{R}\grave{R}}$, m-$\mathrm{\grave{R}\overline{RRR}\grave{R}}$, m-$\mathrm{\grave{R}\grave{R}\overline{RRR}}$, m-$\mathrm{\overline{R}\grave{R}\overline{RR}\grave{R}}$, m-$\mathrm{\overline{R}\grave{R}\grave{R}\overline{RR}}$, m-$\mathrm{\grave{R}\overline{R}\acute{R}\overline{RR}}$, m-$\mathrm{\overline{RR}\grave{R}\overline{R}\grave{R}}$, m-$\mathrm{\overline{RR}\grave{R}\grave{R}\overline{R}}$, m-$\mathrm{\grave{R}\overline{RR}\grave{R}\overline{R}}$
	$(\mathrm{\overline{RR}\grave{R}\grave{R}\grave{R}})$	m-$\mathrm{\overline{RR}\grave{R}\grave{R}\grave{R}}$, m-$\mathrm{\grave{R}\overline{RR}\grave{R}\grave{R}}$, m-$\mathrm{\grave{R}\grave{R}\overline{RR}\grave{R}}$, m-$\mathrm{\grave{R}\grave{R}\grave{R}\overline{RR}}$, m-$\mathrm{\overline{R}\grave{R}\overline{R}\grave{R}\grave{R}}$, m-$\mathrm{\overline{R}\acute{R}\grave{R}\overline{R}}$, m-$\mathrm{\overline{R}\grave{R}\grave{R}\grave{R}\overline{R}}$, m-$\mathrm{\grave{R}\overline{R}\grave{R}\overline{R}\grave{R}}$, m-$\mathrm{\grave{R}\overline{R}\grave{R}\grave{R}\overline{R}}$, m-$\mathrm{\grave{R}\grave{R}\overline{RR}\grave{R}}$,
4R1P	$(\mathrm{P\overline{RR}\grave{R}\grave{R}})$	m-$\mathrm{P\overline{RR}\grave{R}\grave{R}}$), m-$\mathrm{P\grave{R}\overline{RR}\grave{R}}$), m-$\mathrm{P\grave{R}\grave{R}\overline{RR}}$), m-$\mathrm{P\overline{R}\grave{R}\overline{RR}}$), m-$\mathrm{P\overline{R}\grave{R}\overline{R}\grave{R}}$), m-$\mathrm{P\grave{R}\overline{RR}\grave{R}}$), [*]

[*] - another combinations with P joints out of base.

In accordance with Hunt (1990) and Davidson and Hunt (2004) there is not a 5-system that is invariant with finite displacements. Thus the pattern of the particular screw system is destroyed, them either other joint-freedoms previously inactive begin to displace or (in the absence of such joint-freedoms) the parallel robot jams. The simulated parallel robot generators of PPPU motion is instantaneous. Figure 2 shows the 5-$\underline{\grave{R}}\overline{RRR}\grave{R}$ parallel robot. The actuated joint are valid but the parallel robot is instantaneous, when all joints of leg group are aligned the parallel robot jams. The dimensional synthesis can to resolve this problem and can eliminate some singularities in the workspace of robot. Up to the authors' knowledge this robot never been presented in details in the literature.

There is the need of explore more parallel robot that operate in a 5-system, may be that, redundancy of actuation can be improve the performance this type of robots.

5. CONCLUSIONS

This paper present a new procedure, with five steps, for type synthesis of parallel robots based on screw theory. A new procedure for validation of actuated joints also is proposed. The novelty of new procedure is the completeness, *i.e.* the five steps proposed here complete of type synthesis process. After the type synthesis procedure proposed here, in design of parallel robot, we consider the dimensional synthesis of each structure obtained.

The procedure is applied to synthesize two important classes of low-DoF parallel robots for industry applications. The 4-DoF parallel robot generator of Schönflies motions (SCARA motion - counterparts of serial robots) has great potential for industrial applications, in special in pick-and-place task. A new parallel robot is present in details in this text. The 5-DoF parallel robot generator of PPPU motion is of great interest of machine-tool domain, so called five-axis machining. However, none parallel robot presented is full-cycle. In agreement with Hunt (1990) and Davidson and Hunt (2004) there is not a 5-system that is invariant with finite displacements. Thus, the presented parallel robot generators of PPPU motion is instantaneous.

There is the necessity of to investigate others methods for type synthesis of parallel robots generated of kinematic chains obtained in number synthesis processes (Simoni et al. , 2007, 2008, 2009; Alizade et al. , 2004, 2007; Tsai 2001). The kinematic chains obtained in number synthesis are more complex of kinematic chains explored here and for others authors of type synthesis. We believe that this complex chains can solve the problem of instantaneous robots for 5-system case.

6. ACKNOWLEDGEMENTS

The authors would like to acknowledge the financial support of the CAPES and CNPQ.

7. REFERENCES

Alizade, R. and Bayram, Ç. (2004). Structural synthesis of parallel manipulators. *Mechanism and Machine Theory*, 39(8):857–870.

Alizade, R., Bayram, C., and Gezgin, E. (2007). Structural synthesis of serial platform manipulators. *Mechanism and Machine Theory*, 42(5):580–599.

Ball, R. (1998). *A Treatise on the Theory of Screws*. Cambridge University Press.

Bonev, I. (2000). Parallemic. `http://www.parallemic.org/`. Accessed 29-Abr-2009.

Bonev, I. (2001). Delta parallel robot - the story of success. `http://www.parallemic.org/Reviews/Review002.html`. Accessed 29-Abr-2009.

Davidson, J. and Hunt, K. (2004). *Robots and screw theory*. Oxford University Press.

Fang, Y. and Tsai, L. (2002). Structure synthesis of a class of 4-DoF and 5-DoF parallel manipulators with identical limb structures. *The international journal of Robotics Research*, 21(9):799–810.

Fang, Y. and Tsai, L. (2004). Structure synthesis of a class of 3-DOF rotational parallel manipulators. *IEEE Transactions on Robotics and Automation*, 20(1):117–121.

Gogu, G. (2004a). Structural synthesis of fully-isotropic translational parallel robots via theory of linear transformations. *European Journal of Mechanics/A Solids*, 23(6):1021–1039.

Gogu, G. (2004b). Translational Parallel Robots with Uncoupled Motions. In *Product Engineering*, pages 349–378. Springer Netherlands.

Gogu, G. (2005). Evolutionary Morphology. In *Advances in Integrated Design and Manufacturing in Mechanical Engineering*, pages 389–402. Springer.

Gogu, G. (2007a). Fully-isotropic Three-degree-of-freedom Parallel Wrists. *2007 IEEE International Conference on Robotics and Automation*, pages 895–900.

Gogu, G. (2007b). Structural synthesis of fully-isotropic parallel robots with Schönflies motions via theory of linear transformations and evolutionary morphology. *European Journal of Mechanics/A Solids*, 26(2):242–269.

Gogu, G. (2008). *Structural Synthesis of Parallel Robots: Part 1: Methodology*. Springer Verlag.

Gogu, G. (2009). *Structural Synthesis of Parallel Robots: Part 2: Translational Topologies with Two and Three Degrees of Freedom*. Springer.

Hervé, J. (1994). The mathematical group structure of the set of displacements. *Mechanism and machine theory*, 29(1):73–81.

Hervé, J. (1999). The Lie group of rigid body displacements, a fundamental tool for mechanism design. *Mechanism and Machine Theory*, 34(5):719–730.

Hervé, J. and Sparacino, F. (1991). Structural synthesis ofparallel'robots generating spatialtranslation. In *Advanced Robotics, 1991.'Robots in Unstructured Environments', 91 ICAR., Fifth International Conference on*, pages 808–813.

Huang, Z. and Li, Q. (2002). General methodology for type synthesis of symmetrical lower-mobility parallel manipulators and several novel manipulators. *The International Journal of Robotics Research*, 21(2):131–145.

Huang, Z. and Li, Q. (2003). Type synthesis of symmetrical lower-mobility parallel mechanisms using the constraint-synthesis method. *The International Journal of Robotics Research*, 22(1):59–79.

Hunt, K. (1990). Kinematic geometry of mechanisms. *Cambridge University Press, Cambridge*.

Kong, X. and Gosselin, C. (2004a). Type synthesis of 3-DOF spherical parallel manipulators based on screw theory. *Journal of Mechanical Design*, 126:101–109.

Kong, X. and Gosselin, C. (2004b). Type synthesis of 3-DOF translational parallel manipulators based on screw theory. *Journal of Mechanical Design*, 126:83–93.

Kong, X. and Gosselin, C. (2004c). Type synthesis of 3T1R 4-DOF parallel manipulators based on screw theory. *IEEE transactions on robotics and automation*, 20(2):181–190.

Kong, X. and Gosselin, C. (2005). Type synthesis of 5-DOF parallel manipulators based on screw theory. *Journal of Robotic Systems*, 22(10):535–547.

Kong, X. and Gosselin, C. (2007). *Type synthesis of parallel mechanisms*. Springer.

Li, Q. and Huang, Z. (2003). Type synthesis of 4-DOF parallel manipulators. *IEEE International Conference on Robotics and Automation, 2003. Proceedings. ICRA'03*, 1:755–760.

Li, Q., Huang, Z., and Hervé, J. (2004). Type synthesis of 3R2T 5-DOF parallel mechanisms using the Lie group of displacements. *IEEE transactions on robotics and automation*, 20(2):173–180.

Merlet, J. (2000). Parallel robots- Open problems. *Robotics research*, pages 27–32.

Merlet, J. (2006). *Parallel robots*. Kluwer Academic Publishers.

Merlet, J. and Daney, D. (2008). Appropriate Design of Parallel Manipulators. In *Smart Devices and Machines for Advanced Manufacturing*, pages 1–25. Springer London.

Mohamed, M. and Duffy, J. (1985). A direct determination of the instantaneous kinematics of fully parallel robot manipulators. *ASME Journal of Mechanisms, Transmissions, and Automation in Design*, 107(2):226–229.

Refaat, S., Herve, J., Nahavandi, S., and Trinh, H. (2006). High-Precision Five-Axis Machine for High-Speed Material Processing Using Linear Motors and Parallel-Serial Kinematics. *IEEE Conference on Emerging Technologies and Factory Automation, 2006. ETFA'06*, pages 501–506.

Simoni, R., Carboni, A., and Martins, D. (2008). Enumeration of parallel manipulators. *Robotica, doi: 10.1017/S0263574708004979*, pages 1–9.

Simoni, R. and Martins, D. (2007). Criteria for Structural Synthesis and Classification of Mechanism. *Proceedings of the 19th International Congress of Mechanical Engineering–COBEM*.

Simoni, R., Martins, D., and Carboni, A. (2007). Mãos Robóticas: Critérios para Síntese Estrutural e Classificação. *XV Jornadas de Jovenes Investigadores da Asociacion de Universidades do Grupo Montevideo (AUGM), Campus de la UNA-Paraguay*.

Simoni, R., Martins, D., and Carboni, A. (2009). Enumeration of kinematic chains and mechanisms. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 223(4):1017–1024.

Tsai, L. (1999a). *Robot analysis: the mechanics of serial and parallel manipulators*. Wiley-Interscience.

Tsai, L. (1999b). The enumeration of a class of three-dof parallel manipulators. In *The Tenth World Congress on the Theory of Machines and Mechanisms, Finland*, volume 1, pages 1121 –1126.

Tsai, L. (2001). *Mechanism design: enumeration of kinematic structures according to function*. CRC Press.

8. Responsibility notice

The author(s) is (are) the only responsible for the printed material included in this paper.