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NUMERICAL SIMULATION OF WATER TRANSPORT IN BANANA USING GENERALIZED COORDINATES AND CAUCHY BOUNDARY CONDITION

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***Abstract**: This work proposes a numerical solution for the diffusion equation applied to solids obtained through the revolution of arbitrary bi-dimensional geometries, using generalized coordinates and non-orthogonal grids. For such, the diffusion equation was discretized and solved using the finite volume method, with fully implicit formulation, for the boundary condition of the third kind. The proposed solution exploits symmetry conditions and it is justified by the reduction of the computational effort demanded in comparison to the traditional method with the use of three-dimensional grids. The proposed solution was applied to describe the drying of banana and, for the drying conditions, it was obtained a diffusivity of 4.48×10^{-6} m^2h^{-1} and a convective mass transfer coefficient of $5.53 \times 10^{-4} mh^{-1}$.*

***Keywords**: Diffusion, Complex geometries, Banana drying, Discretization, Non-orthogonal grid.*

Nomenclature

A, B	coefficients of algebraic equation from discretized equation
α	parameter of the diffusion equation in the transformed domain
Γ^{Φ}	transport coefficient (dimension depends on the process under study) *nn*
λ	transport coefficient (dimension depends on the process under study)
J	Jacobian of the transformation (m^{-3})
S	source term (dimension depends on the process under study)
Φ	dependent variable of the diffusion equation (dimension depends on the process under study)
ξ, η, γ	axes of the system of generalized coordinates (dimensionless)
t, τ	times in the physical and transformed domains, respectively (s)
h	convective transfer coefficient (dimension depends on the process under study)
D	mass diffusivity (m^2s^{-1})
M	moisture content (kg/kg dry matter)
T	temperature (K)
ρ	density (kg m^{-3})
c_p	specific heat (J mol^{-1} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
x, y, z	axes of the Cartesian coordinates system

Subscripts

∞	external neighborhood
i	initial
N, n	north
S, s	south
E, e	east
W, w	west
p	constant pressure
P	nodal point

i, j represent numbers for η and ξ lines of the grid in the transformed domain

Superscripts
o previous time
P nodal point

1. INTRODUCTION

The drying of a wet body is important because enables to minimize losses of the product during its storage. Naturally, a mathematical model which describes the mechanism of the drying must be adopted for its study. Various theories and consequent mathematical models are reported in the literature. One of these assumes that the water transfer from the interior of the product to its surface occurs by liquid diffusion. Then, the mathematical model to describe the process involves the diffusion equation.

The diffusion equation has analytical solutions for several simple geometries, such as an infinite slab, infinite cylinder, and a sphere, among others. In these solutions, it is normally supposed that the medium has constant thermo-physical properties as, for example, in Luikov (1968) and Crank (1992). Analytical and numerical solutions for diffusion of water are also reported for parallelepipeds (Nascimento, 2002), prolated (Lima, 1999; Jia et al., 2001) and oblated spheroids (Carmo, 2004). However, only few works are available for arbitrary geometries, particularly using the finite volume method and generalized coordinates, with variable thermo-physical parameters and non-orthogonal grids. Thus, the study here presented is motivated by the lack of works involving problems about water transient diffusion in solids of arbitrary geometry, which is necessary for a rigorous description of the drying process of a solid of any shape. In this case, the commonly used Cartesian, cylindrical or spherical coordinates are not appropriate. Even some more flexible coordinate systems, as defined for prolate (Lima, 1999) and oblate spheroids (Carmo, 2004) or still other ellipsoidal systems (Li et al., 2004) are limited to only some specific geometric shapes.

Our study proposes a numerical solution of the diffusion equation for solids which can be obtained by revolution of arbitrary two-dimensional plane surfaces about a fixed axis in the same plane, thereby exploring symmetry conditions. The proposed numerical solution involves boundary condition of the third kind, using the finite volume method, with a fully implicit formulation, and generalized coordinates. This study may be justified by a significant reduction of computational effort in relation to the traditional numerical solutions by three-dimensional grids, as in Wu et al. (2004), assuming constant thermo-physical parameters, and orthogonal grid. The proposed numerical solution was applied to the drying of banana, which was considered as a solid obtained by the revolution of an ellipse.

2. MATHEMATICAL MODELING

2.1. Diffusion Equation

The diffusion equation in Cartesian coordinates is given by (Tannehill et al., 1997; Maliska, 2004)

$$\frac{\partial(\lambda\Phi)}{\partial t}=\frac{\partial}{\partial x}\left(\Gamma^{\Phi}\frac{\partial\Phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma^{\Phi}\frac{\partial\Phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\Gamma^{\Phi}\frac{\partial\Phi}{\partial z}\right)+S \tag{1}$$

where t is the time, x, y and z are the Cartesian coordinates of position, λ and Γ^{Φ} are transport coefficients, S is a source term and Φ is the dependent variable to be determined. Equation (1) is frequently named diffusion equation of the physical domain, in contrast to the transformed domain.

In general, Cartesian coordinates are not appropriate to solve diffusion problems for solids of arbitrary shape. Thus, a coordinate system whose axes coincide with the borders of the control volumes of the studied solid will be used. This means that the new axes, denoted by ξ, η *e* γ, defining a curvilinear, non-orthogonal coordinate system must be used, as shown in Fig. 1. The curvilinear coordinates given by ξ, η and γ can be expressed as functions of x, y and z through transformations of the type (Tannehill et al., 1997; Maliska, 2004):

$$\xi = \xi\,(x,y,z),\quad \eta = \eta\,(x,y,z),\quad \gamma = \gamma\,(x,y,z). \tag{2}$$

Then, the diffusion equation can be written in the new coordinate system as (Maliska, 2004; Wu et al., 2004):

$$\frac{\partial}{\partial\tau}\left(\frac{\lambda\Phi}{J}\right)=\frac{S}{J}+\frac{\partial}{\partial\xi}\left(\alpha_{11}J\Gamma^{\Phi}\frac{\partial\Phi}{\partial\xi}+\alpha_{12}J\Gamma^{\Phi}\frac{\partial\Phi}{\partial\eta}+\alpha_{13}J\Gamma^{\Phi}\frac{\partial\Phi}{\partial\gamma}\right)+$$

$$\frac{\partial}{\partial \eta}\left(\alpha_{21} J \Gamma^{\Phi} \frac{\partial \Phi}{\partial \xi} + \alpha_{22} J \Gamma^{\Phi} \frac{\partial \Phi}{\partial \eta} + \alpha_{23} J \Gamma^{\Phi} \frac{\partial \Phi}{\partial \gamma}\right) +$$

$$\frac{\partial}{\partial \gamma}\left(\alpha_{31} J \Gamma^{\Phi} \frac{\partial \Phi}{\partial \xi} + \alpha_{32} J \Gamma^{\Phi} \frac{\partial \Phi}{\partial \eta} + \alpha_{33} J \Gamma^{\Phi} \frac{\partial \Phi}{\partial \gamma}\right) \tag{3}$$

where τ is the time, and J is the Jacobian of the transformation to be defined below, together with the coefficients α_{ij} for the type of solid under study. Equation (3), written in generalized coordinates ξ, η and γ, is frequently called diffusion equation in the transformed domain. Note that the structured grid to be used in Eq. (3) is fixed in time, e.g., the volume of the solid is constant.

2.2. Diffusion Equation for revolution solids

The proposed numerical solution in this work for solids of revolution is similar to the solution for diffusion in long solids obtained by extrusion, which is a typical two-dimension problem (Maliska, 2004). But a solution via the finite volume method, for two-dimensional non-orthogonal structured grids in an arbitrary domain, using generalized coordinates was not found in the consulted literature for revolution solids. The basic idea departs from a control volume generated by an elementary cell of a two-dimensional non-orthogonal structured grid in the (x, y)-plane through rotation by an angle θ about y, as sketched in Fig. 1. Since a symmetric diffusion in relation to the y-axis is assumed, there is no flux in the direction of γ perpendicular to the generating cell of the control volume.

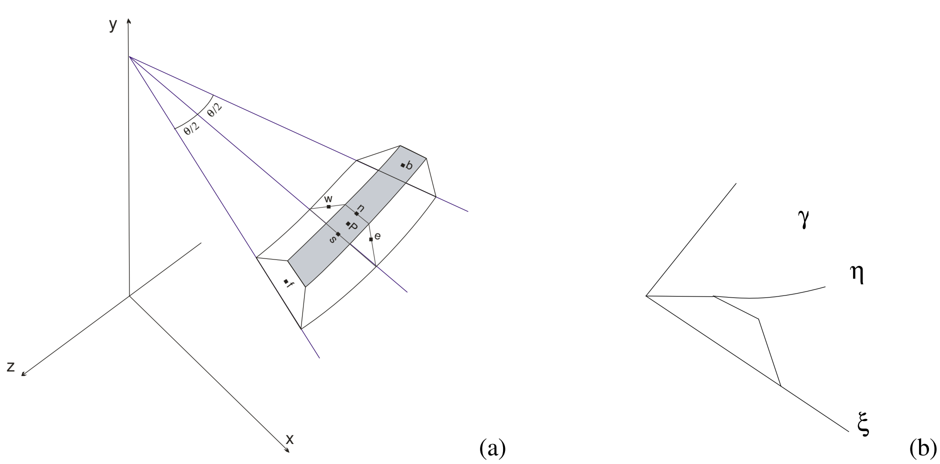

Figure 1. (a) Control volume with a nodal point P obtained by rotation about y of an elementary cell of a two-dimensional non-orthogonal structured grid in a vertical plane. The faces "f" and "b" refer to front and back. (b) System of generalized coordinates defined by the axes ξ, η and γ along the borders of the control volume.

The derivatives of x and y with respect to γ and the derivatives of z with respect to ξ and η are zero for the control volume shown in Fig. 1. In this case, the generating cell is contained in the vertical (ξ,η)-plane, while γ and z are located in a horizontal plane. Thus, the Jacobian of the transformation is given by the determinant:

$$\frac{1}{J} = \begin{vmatrix} x_{\xi} & x_{\eta} & 0 \\ y_{\xi} & y_{\eta} & 0 \\ 0 & 0 & z_{\gamma} \end{vmatrix}, \quad \text{or} \qquad \frac{1}{J} = z_{\gamma}\begin{vmatrix} x_{\xi} & x_{\eta} \\ y_{\xi} & y_{\eta} \end{vmatrix} \tag{4}$$

where the symbol g_m means the partial derivative of g with respect to m.

By hypothesis, there is no flux in the direction of γ for the solid of revolution under study. So, the last term of the right hand side of Eq. (3) becomes zero, because a derivative with respect to γ is involved. Again, by hypothesis, the derivative of Φ with respect to γ is also zero. Thus, besides the knowledge of the Jacobian determined by Eq. (4), the following expressions must be known for the numerical solution of Eq. (3):

$$\alpha_{11} = z_\gamma^2\,(x_\eta^2 + y_\eta^2), \quad \alpha_{12} = \alpha_{21} = -z_\gamma^2\,(x_\xi x_\eta + y_\xi y_\eta), \quad \alpha_{22} = z_\gamma^2\,(x_\xi^2 + y_\xi^2) \tag{5a-c}$$

For the type of solid under investigation, Eq. (3) can be written in the following way:

$$\frac{\partial}{\partial\tau}\left(\frac{\lambda\Phi}{J}\right) = \frac{\partial}{\partial\xi}\left(\alpha_{11}J\Gamma^\Phi\frac{\partial\Phi}{\partial\xi} + \alpha_{12}J\Gamma^\Phi\frac{\partial\Phi}{\partial\eta}\right) +$$

$$\frac{\partial}{\partial\eta}\left(\alpha_{21}J\Gamma^\Phi\frac{\partial\Phi}{\partial\xi} + \alpha_{22}J\Gamma^\Phi\frac{\partial\Phi}{\partial\eta}\right) + \frac{S}{J}. \tag{6}$$

3. NUMERICAL SOLUTION

With a fully implicit formulation, the integration of Eq. (6) with respect to $\Delta\xi\Delta\eta$ and time (from τ up to $\tau+\Delta\tau$), for a revolution solid gives for a control volume with elementary cell in the (ξ,η)-plane and unit length in γ for a time interval $\Delta\tau$ the following result:

$$\frac{\lambda_P\Phi_P - \lambda_P^0\Phi_P^0}{J_P}\Delta\xi\Delta\eta = \left[\alpha_{11e}J_e\Gamma_e^\Phi\Delta\eta\Delta\tau\left.\frac{\partial\Phi}{\partial\xi}\right|_e + \alpha_{12e}J_e\Gamma_e^\Phi\Delta\eta\Delta\tau\left.\frac{\partial\Phi}{\partial\eta}\right|_e\right] -$$

$$\left[\alpha_{11w}J_w\Gamma_w^\Phi\Delta\eta\Delta\tau\left.\frac{\partial\Phi}{\partial\xi}\right|_w + \alpha_{12w}J_w\Gamma_w^\Phi\Delta\eta\Delta\tau\left.\frac{\partial\Phi}{\partial\eta}\right|_w\right] +$$

$$\left[\alpha_{21n}J_n\Gamma_n^\Phi\Delta\xi\Delta\tau\left.\frac{\partial\Phi}{\partial\xi}\right|_n + \alpha_{22n}J_n\Gamma_n^\Phi\Delta\xi\Delta\tau\left.\frac{\partial\Phi}{\partial\eta}\right|_n\right] -$$

$$\left[\alpha_{21s}J_s\Gamma_s^\Phi\Delta\xi\Delta\tau\left.\frac{\partial\Phi}{\partial\xi}\right|_s + \alpha_{22s}J_s\Gamma_s^\Phi\Delta\xi\Delta\tau\left.\frac{\partial\Phi}{\partial\eta}\right|_s\right] + \frac{S_P}{J_P}\Delta\xi\Delta\eta\Delta\tau, \tag{7}$$

where the terms without superscript are evaluated at time $\tau+\Delta\tau$, while the terms with superscript zero are evaluated at a previous time τ. The subscripts *"e", "w", "n"* and *"s"* mean the east, west, north and south borders, respectively, of an elementary generating cell of a control volume of unit length, while *P* is the nodal point of this volume. All the elements described above are shown in Fig. 1.

In order to complete the discretization of Eq. (7), it should be noted that for a two-dimensional non-orthogonal structured grid created in the generating plane surface of the solid of revolution, there are 9 different types control volumes in the transformed domain, as shown in Fig. 2.

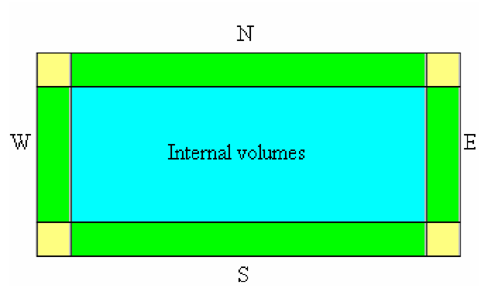

Figure 2. Regions with 9 different types of control volumes in the transformed domain for a structured grid: internal volumes, and boundary volumes in the north (N), in the south (S), in the east (E) and in the west (W).

3.1. Internal control volume and symmetry

The discretization for the internal control volumes (Fig. 2) is presented by Silva et al. (2007), which solved a similar problem for the boundary condition of the first kind. In order to exploit possible simplifications caused by symmetries, as presented in this article, a boundary condition without flux may be useful, and the discretization is presented in Silva et al. (2008a).

3.2. Boundary condition of the third kind

With a similar procedure as that presented by Silva et al. (2007) for the internal control volumes of a grid, algebraic equations can also be determined for each control volume located in the boundaries of the grid defined in the generating area of the solid in study. As example, for the convective boundary condition, the discretization of the diffusion equation will be presented for a control volume located in the east boundary, shown in the fragment of grid in the transformed domain, in Fig. 3.

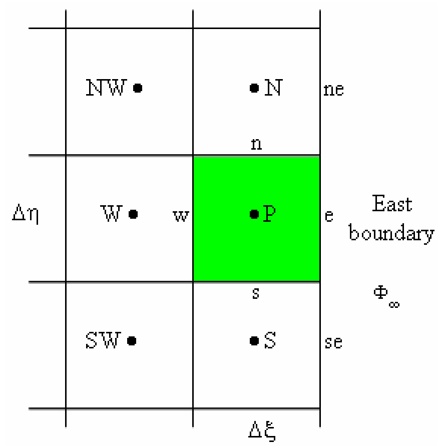

Figure. 3. Control volume P at the east boundary and its neighbors in the transformed domain.

For the east interface of the control volume with node *P*, as shown in Fig. 3, the boundary condition of the third kind can be expressed by

$$-\Gamma_e^{\Phi}\frac{\Phi_e-\Phi_P}{\Delta n_e}=h_e(\Phi_e-\Phi_{\infty e}), \tag{8}$$

where Γ_e^{Φ} and $\Phi_{\infty e}$ respectively are the transport parameter and the Φ value for the neighborhood on the east boundary. The symbols Φ_e, h_e Δn_e, represent the dependent variable, the convective mass (or heat) transfer coefficient, and the distance from the nodal point *P* up to east boundary while Φ_P is the value of Φ at the nodal point *P*.

For a nodal point P in the control volume at the east boundary shown in Fig. 3, and assuming boundary condition of the third kind, one obtains:

$$A_p\Phi_P=A_w\Phi_W+A_s\Phi_S+A_n\Phi_N+A_{sw}\Phi_{SW}+A_{nw}\Phi_{NW}+B, \tag{9}$$

where:

$$A_P=\frac{\lambda_P}{J_P}\cdot\frac{\Delta\xi\Delta\eta}{\Delta\tau}+\frac{\sqrt{\alpha_{11e}}\,\Delta\eta}{\dfrac{1}{h_e}+\dfrac{\Delta n_e}{\Gamma_e^{\Phi}}}+\alpha_{11w}J_w\Gamma_w^{\Phi}\frac{\Delta\eta}{\Delta\xi}-\frac{1}{4}\alpha_{21n}J_n\Gamma_n^{\Phi}\frac{1-f_e}{1+f_e}+$$

$$\alpha_{22n} J_n \Gamma_n^{\Phi} \frac{\Delta\xi}{\Delta\eta} + \frac{1}{4}\alpha_{21s} J_s \Gamma_s^{\Phi} \frac{1-f_e}{1+f_e} + \alpha_{22s} J_s \Gamma_s^{\Phi} \frac{\Delta\xi}{\Delta\eta},$$

$$A_w = \alpha_{11w} J_w \Gamma_w^{\Phi} \frac{\Delta\eta}{\Delta\xi} + \frac{1}{4}\alpha_{21s} J_s \Gamma_s^{\Phi} - \frac{1}{4}\alpha_{21n} J_n \Gamma_n^{\Phi},$$

$$A_e = 0,$$

$$A_s = \alpha_{22s} J_s \Gamma_s^{\Phi} \frac{\Delta\xi}{\Delta\eta} + \frac{1}{4}\alpha_{12w} J_w \Gamma_w^{\Phi} - \frac{1}{4}\alpha_{21s} J_s \Gamma_s^{\Phi} \frac{1-f_{se}}{1+f_{se}},$$

$$A_n = \alpha_{22n} J_n \Gamma_n^{\Phi} \frac{\Delta\xi}{\Delta\eta} + \frac{1}{4}\alpha_{21n} J_n \Gamma_n^{\Phi} \frac{1-f_{ne}}{1+f_{ne}} - \frac{1}{4}\alpha_{12w} J_w \Gamma_w^{\Phi},$$

(10a-j)

$$A_{sw} = \frac{1}{4}\alpha_{12w} J_w \Gamma_w^{\Phi} + \frac{1}{4}\alpha_{21s} J_s \Gamma_s^{\Phi},$$

$$A_{se} = 0,$$

$$A_{nw} = -\frac{1}{4}\alpha_{12w} J_w \Gamma_w^{\Phi} - \frac{1}{4}\alpha_{21n} J_n \Gamma_n^{\Phi},$$

$$A_{ne} = 0,$$

$$B = \frac{\lambda_P^0 \Phi_P^0}{J_P} \cdot \frac{\Delta\xi\Delta\eta}{\Delta\tau} + \frac{S_P}{J_P}\Delta\xi\Delta\eta + \frac{\sqrt{\alpha_{11e}}\,\Phi_{\infty e}\Delta\eta}{\dfrac{1}{h_e} + \dfrac{\Delta n_e}{\Gamma_e^{\Phi}}} + \frac{1}{2}\alpha_{21n} J_n \Gamma_n^{\Phi}\left(\frac{f_e}{1+f_e}\Phi_{\infty e} + \frac{f_{ne}}{1+f_{ne}}\Phi_{\infty ne}\right) -$$

$$\frac{1}{2}\alpha_{21s} J_s \Gamma_s^{\Phi}\left(\frac{f_e}{1+f_e}\Phi_{\infty e} + \frac{f_{se}}{1+f_{se}}\Phi_{\infty se}\right).$$

where the factor f is given, in a generic way, as follows:

$$f = \frac{h\,\Delta n}{\Gamma^{\Phi}}. \tag{11}$$

In Equations (10) and (11), h is the convective mass (or heat) transfer coefficient in the east interface of the control volume, and Φ_∞ is the value of the variable Φ for the air in the external neighborhood of the solid in study. On the other hand, Δn is the distance of the nodal point P to its east border, while the indexes *"e", "ne"* and *"se"* mean east, northeast and southeast (Fig. 3).

In the same way that the discretized equations were obtained for the control volumes of the east boundary, they also can be obtained for the control volumes of the north, south and west, and for the volumes to northeast, southeast, northwest and southwest of the grid. Thus, it is obtained a system of equations in Φ that it can be solved, for example, using the Gauss-Seidel method.

3.3. Average of Φ

Once Φ is numerically determined in each position and time, the average value at a time t may be calculated by (Silva, 2007; Silva, 2008a; Silva, 2008b; Hadrich, 2008):

$$\overline{\Phi} = \frac{1}{V}\sum_{i=1}^{N}\Phi_i V_i \tag{12}$$

with

$$V = \sum_{i}^{N} V_i \tag{13}$$

where Φ_i and V_i are the value of Φ and the volume of the control volume *"i"*, *N* is the number of control volumes, and *V* is the volume of the solid.

4. GENERAL CONSIDERATIONS

The proposed numerical solution can be used to study the conduction of heat if we impose: $\Phi = T$ (temperature), $\Gamma^{\Phi} = k$ (conductivity) and $\lambda = \rho c_p$ (ρ is the density and c_p is the specific heat). On the other hand, establishing $\Phi = M$ (moisture content), $\Gamma^{\Phi} = D$ (water diffusivity), $\lambda = 1$, and $S = 0$ the proposed numerical solution can be used to study the water diffusion in solids.

The validation of the proposed numerical solution was presented in Silva et al., (2008a). On the other hand, the developed software for the numerical solution (http://zeus.df.ufcg.edu.br/labfit/diffusion.htm), including the graphic interface, was developed in Compaq Visual Fortran Professional Studio Edition V. 6.6.0 (Fortran 95) using a programming language option called QuickWin Application, under Windows XP platform. An application using experimental data of the literature will be made for drying of bananas.

4.1. Optimizations

In order to determine optimal values for the parameters *D* and *h*, during the drying of bananas, the objective function was defined as the chi-square of the fits. The expression for the chi-square involving the fit of a simulated curve to the experimental data is given by (Bevington and Robinson, 1992; Taylor, 1997)

$$\chi^2 = \sum_{i=1}^{N_p} \left(M_i^{\exp} - M_i^{sim}\right)^2 \frac{1}{\sigma_i^2} \tag{14}$$

where $M_i^{\exp}$ is the moisture content measured in the experimental point *"i"*, M_i^{sim} is the correspondent simulated moisture content, N_p is the number of experimental points, $1/\sigma_i^2$ is the statistical weight referring to the point *"i"*. In general, in the absence of information, the statistical weights are made equal to 1. The parameter σ_i is the standard deviation of $M_i^{\exp}$. In Eq. (13), the chi-square depends on M_i^{sim}, which depends on D and *h*. So, changing the values of D and *h*, the value of χ^2 is also changed and this fact was used in order to determine the minimum chi-square.

If there is availability of experimental data, the calculations for the optimization obey to the following steps:

1) Informing the initial values for the parameters "D" and *"h"*. Solving the diffusion equation and determining the chi-square;
2) Informing the value for the correction of *"h";*
3) Correcting the parameter *"h"*, maintaining the parameter "D" with constant value. Solving the diffusion equation and calculating the chi-square;
4) Comparing the last calculated value of the chi-square with the previous one. If the latest value is smaller, return to the step 2; otherwise, proceed to step 5;
5) Informing the value for the correction of "D";
6) Correcting the parameter "D", maintaining the parameter *"h"* with constant value. Solving the diffusion equation and calculating the chi-square;
7) Comparing the last calculated value of the chi-square with the previous one. If the latest value is smaller, return to the step 5; otherwise, proceed to step 8;
8) Return to the step 2 until the stipulated convergence for the parameters D and h is reached.

5. RESULTS AND DISCUSSION

The data analyzed by Silva et al. (2008b) referring to the thin-layer drying of bananas (R = 0.01522 m) were used for the application purpose in drying. The process happened in the following conditions: temperature of 50 °C and relative humidity of 20%. The initial moisture content is 3.21 kg water/kg dry matter and the equilibrium moisture content is 0.0559 kg water/kg dry matter. Silva et al. (2008b) described the drying process supposing the banana shape as an infinite cylinder with constant volume. The authors admitted a boundary condition of the first kind, and the variable diffusivity. However, they also mention that such a boundary condition may not be accurate enough to describe drying process. Hence they reported the diffusivity should only be interpreted as an expression that fits the numerical simulation to the experimental data.

In the present work, the boundary condition of the third kind was considered. In addition, the banana was considered as an ellipsoid obtained by the revolution of a plane area defined by an ellipse as shown in Fig. 4. Due to symmetry, only half a banana is studied.

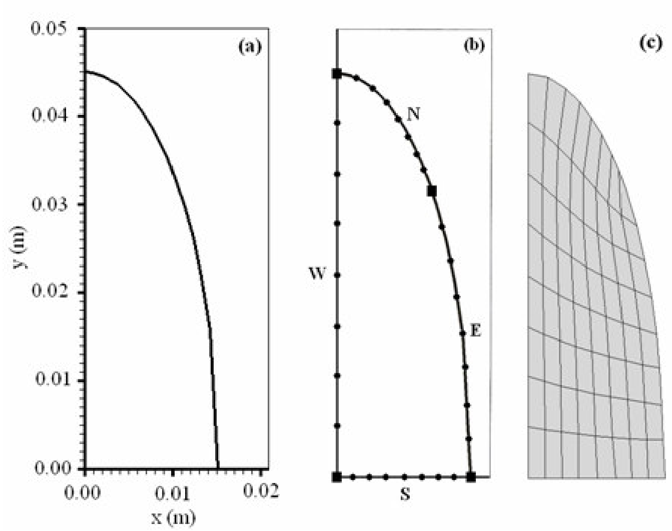

Figure. 4. Generation of the initial mesh for half banana.

The initial mesh was refined until a new mesh with 64x64 elements was obtained, and the time drying (40.1 h) was divided in 1000 time steps. Previous study indicates that one hundred control volumes and one thousand time steps should guarantee an adequate refinement (time and mesh) for the problem in study. The diffusivity was admitted constant because the drying temperature is low, and the shrinkage was not considered. So, for the third kind boundary condition, after the optimization process, the water diffusivity and the convective mass transfer coefficient was determined. The drying kinetics can also to be represented as is shown in Fig. 5.

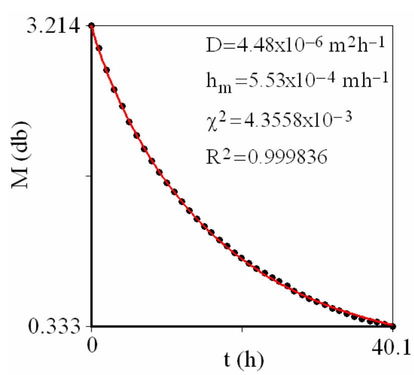

Figure. 5. Drying kinetics of banana given by the developed numerical solution and optimizer: fit to experimental data.

Figure 5 also presents the obtained values for the water diffusivity *D*, given by $D = 4.48x10^{-6}\ m^2\,h^{-1}$ (or $D = 1.24x10^{-9}\ m^2\,s^{-1}$) and the convective mass transfer coefficient *h*, given by $h = 5.53x10^{-4}\ m\,h^{-1}$ (or $h = 1.54x10^{-7}\ m\,s^{-1}$). An inspection in Fig. 5 makes it possible to affirm that the obtained results are good because chi-square is only $4.3558x10^{-3}$ and the determination coefficient is 0.999836.

The moisture distribution in predetermined times can be seen through the contour plots present in Fig. 6 in the following times: 2; 4; 6; 12; 24; and 40.1 h.

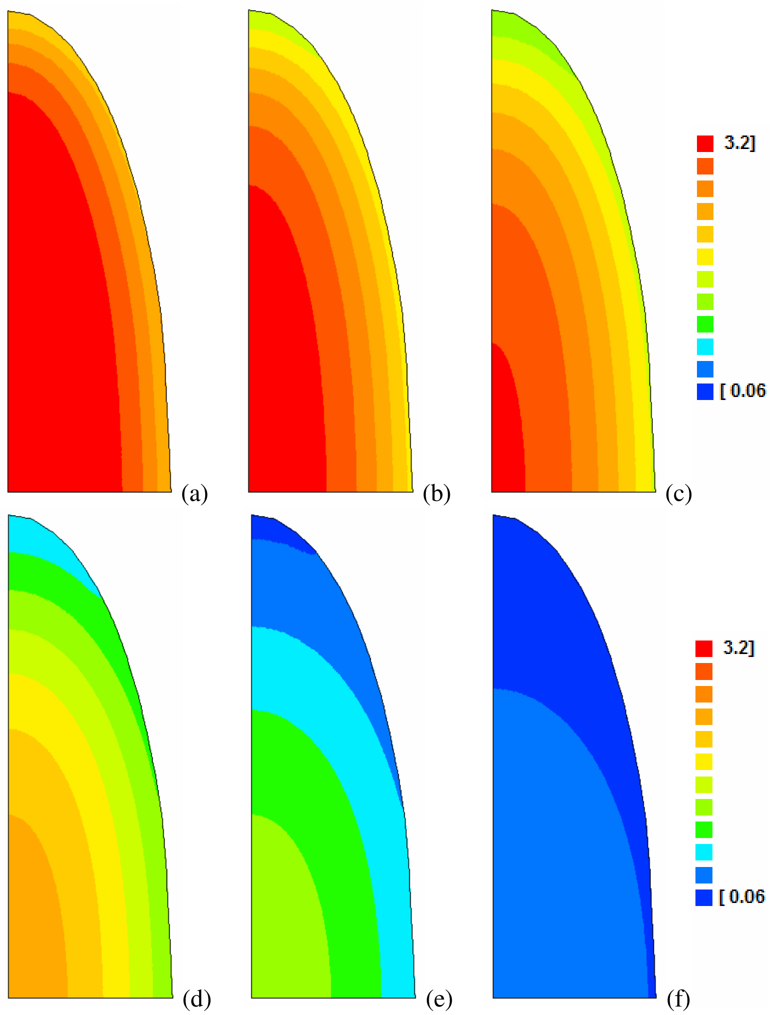

Fig. 6. Contour plot showing moisture distribution within the banana in t = :
(a) 2 h; (b) 4 h; (c) 6 h; (d) 12 h; (e) 24 h; (f) 40.1 h.

6. CONCLUSIONS

The proposed numerical solution of the diffusion equation with boundary condition of the third type, for revolution solids, produced compatible results with the expected one. This means that diffusion in such solids can be studied starting from two-dimensional grids, which simplifies the numerical solution of this type of problems and reduces the computational effort in comparison with typical three-dimensional solutions. The proposed numerical solution in this work can not only be applied to mass transfer, but also to diffusion of heat and, obviously, in the description of the drying kinetics of porous solids that involve the simultaneous diffusion of heat and mass.

The proposed numerical solution, coupled to the described optimizer, appropriately determined the process parameters relative to the drying of bananas.

7. ACKNOWLEDGEMENTS

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