THE ULTRASONIC DETERMINATION OF ELASTIC CONSTANTS OF COMPOSITE MATERIALS USING A DIFFRACTION FREE RECEIVER

Julio Cezar Adamowski, jcadamow@usp.br
Marco Aurélio Brizzotti Andrade, marcobrizzotti@gmail.com
Flávio Buiochi, fbuiochi@usp.br
Nicolás Pérez Alvarez, nicoperez@usp.br
Department of Mechatronic and Mechanical Systems Engineering – University of São Paulo
Av. Prof. Mello Moraes, 2231, 05508-900, São Paulo-SP, Brazil

Abstract. *The determination of elastic constants of anisotropic materials by measurement of the density and ultrasonic velocities has been studied by several researchers in the last four decades. The accuracy of elastic constants is highly dependent on the precision of the velocity measurement. The diffraction effect that occurs when using limited size ultrasonic transducers introduces error when measuring velocities. It can be shown that an infinite-plane receiver with uniform sensitivity yields a plane wave-only measurement, avoiding the diffraction effect. In practice this receiver is obtained by using a piezoelectric PVDF thin-film receiver, sufficiently large to intercept the entire propagating wave. Large-aperture PVDF receivers have broadband frequency response and are very useful for measurements of pressure amplitude and velocity without diffraction effect. This effect increases as the frequency decreases for the same size of transducer. On the other hand, the velocity dispersion effect, present in composite materials, increases with frequency. This work describes the use of a large aperture PVDF receiver in angle beam through-transmission method of velocity measurement in fiber reinforced composites. This large-aperture receiver is a 52-µm thick PVDF polymer membrane with gold electrodes. The PVDF membrane is bonded to a matched backing material, stiff enough to prevent low frequency bending vibration. The effective diameter of the receiver is 80-mm, and it was chosen in order to intercept the entire field produced by the lowest frequency transmitter used in this work. The experiments have been made in a goniometer which has an angular resolution of 0.01 degree in two rotation axes. To analyze the diffraction and dispersion effects, measurements where conducted in a 9.5 mm thick aluminum plate, a 4.5 mm acrylic plate and a 2.12 mm unidirectional carbon-fiber/epoxy plate. It was used a set of 5 pairs of 19 mm diameter ultrasonic transducers (1.0, 2.25, 3.5, 5.0 and 10.0 MHz). Experiments were conducted using a pair of transducers and a 19 mm diameter transducer as emitter and the large-aperture receiver. The temperature was kept constant (21.0 ±0.1 degree C). Analyzing the diffraction effect, it was observed that the longitudinal velocity in the aluminum plate increases more than 1 % when using a pair of 1MHz transducers. That effect disappears when using the large-aperture receiver and it is negligible when using a pair of 10 MHz, 19 mm diameter, transducers. On the other hand, it was observed that in the acrylic plate the longitudinal velocity increases 0.7%, and increases more than 0.8% in the carbon-fiber/epoxy plate (density = 1585 kg/m^3), from 1 to 10 MHz. As a compromise between axial resolution and velocity dispersion, the elastic constants of a 2.115 mm thick unidirectional carbon fiber/epoxy composite plate were determined using a focused transducer with nominal frequency of 5 MHz and diameter of 10mm, as emitter and the large-aperture PVDF receiver. The elastic constants is compared with the tensile test results, showing good agreement.*

Keywords: *elastic constants, composite, phase velocity, diffraction effect, ultrasonic*

1. INTRODUCTION

The determination of elastic constants of anisotropic materials by measurement of the density and ultrasonic velocities has been studied by several researchers in the last four decades (Markham, 1970), (Hosten, 2001), (Wang *et al*., 2003) and (Bouzidi and Schmitt, 2006). The accuracy in the determination of the elastic constants is highly dependent on the precision of the velocity measurement. The acoustic diffraction effect introduces errors when measuring the velocities.

A good physical understanding of the acoustical field pattern is provided by considering the plane and edge waves radiated by a circular transducer. A circular source radiates a plane wave into the geometrical region straight ahead of the source, and a spreading edge wave from the periphery of the source. It is clear that diffraction effects can be considered to be caused by the edge wave component of the field. This effect increases as the frequency decreases for the same transducer diameter. Mathematical correction of the diffraction effect has been successfully applied when using piston-like transducers and calculating its effective radiating aperture (Zeqiri, 1996). In practical transducers, discrepancies from piston-like behavior can be observed, such as a head-wave (Hayman and Weight, 1979) which originates from a plate wave propagating laterally across the face of the transducer. There are further complications when the field is produced by focusing transducers.

It can be shown that an infinite-plane receiver with uniform sensitivity yields a plane-wave-only measurement (Leeman *et al*., 1985). It acts as a spatial filter, responding only to those field components that are perpendicularly incident In practice this receiver is obtained by using a piezoelectric PVDF (Polyvinylidene Fluoride) or its copolymer

(P(VDF-TrFE)) thin-film receiver, sufficiently large to intercept the entire propagating wave and electroded throughout its entire extent (Costa, 1989). This technique avoids the beam diffraction effect that occurs when using limited size ultrasonic transducers. This is also valid for fields generated by focused transducers, given a distance invariant output when the medium has very low attenuation coefficient. This technique was successfully used to measure density of liquids (Adamowski *et al*., 1995).

The effects of diffraction and velocity dispersion are experimentally analyzed using a large aperture PVDF receiver together with a piezoelectric ceramic emitter in through-transmission method of ultrasonic velocity measurement in solid material plates immersed in water.

The elastic constants of a unidirectional CFRP (carbon fiber reinforced polymer) plate are determined using phase velocities measured with an angle beam through-transmission assembly equipped with a diffraction-free PVDF receiver.

2. ELASTIC CONSTANTS

2.1. Determination of elastic constants of anisotropyc materials

Figure 1 show the coordinate system attached to a unidirectional laminate. The fibers are placed parallel to axis x_3. The plane x_1-x_2 can be considered isotropic (Hosten, 2001).

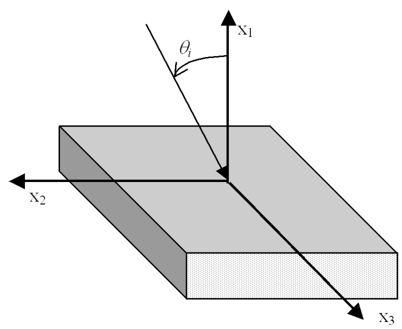

Figure 1. Coordinate system of a unidirectional laminate.

The determination of elastic constants from a set of bulk ultrasonic wave phase velocities in an arbitrary direction of a measured sample of composite material is based on the Christoffel's equation (Auld, 1990), Eq. (1). Equation (1) allows the calculation of the phase velocity at a given direction from a set of the elastic constants. The inverse problem consists in the determination of the elastic constants from the phase velocity measurement in several specific directions.

$$\det\begin{vmatrix} \Gamma_{11}-\rho v^2 & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12} & \Gamma_{22}-\rho v^2 & \Gamma_{23} \\ \Gamma_{13} & \Gamma_{23} & \Gamma_{33}-\rho v^2 \end{vmatrix} = 0 \qquad (1)$$

where, ρ is the density of material, v is the phase velocity, $\Gamma_{il}=C_{ijkl}n_i n_k$ is the Christoffel's tensor, n_i is the unit vector in the direction of wave propagation, and C_{ijkl} the stiffness tensor.

The immersion through-transmission method is based on the measurement of a time delay between the time spent by the wave traveling in the absence of the sample material and the time with the sample material. The time delay is measured for a set of angular directions in two or more planes of symmetry using a goniometer device shown in the diagram of Fig. 2. Each time delay is used to calculate the phase velocity and the corresponding refraction angle in the sample (Rokhlin and Wang, 1992). The goniometer uses a wide band piezoelectric transducer as an emitter and a large aperture PVDF receiver which allows the measurement of plane wave in a wide range of frequencies, eliminating the need for diffraction correction. Besides that, the PVDF thin-film has a relatively flat frequency response in a wideband of frequencies. A photograph of the goniometer is shown in Fig. 3.

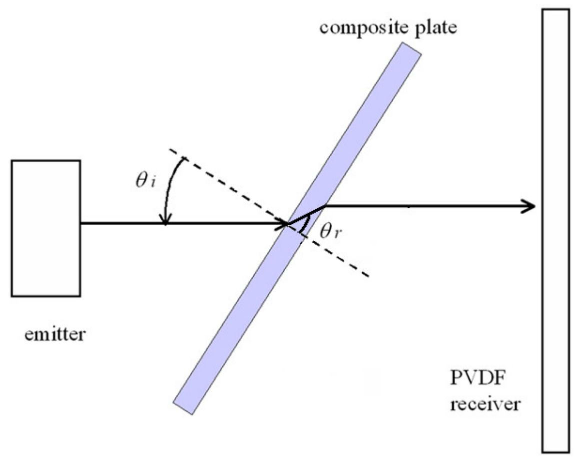

Figure 2. Schematic of the goniometer

The phase velocity inside the sample is obtained by measuring the time delay Δt of the ultrasonic wave traveling with and without the composite plate at a known temperature. The acoustic velocity in water, v_w, is tabulated and can be obtained from the temperature. The phase velocity for an incidence angle θ_i in a composite plate with thickness h is (Rokhlin and Wang, 1992):

$$v(\theta_r) = \left(\frac{1}{{v_w}^2} - \frac{2\Delta t \cos\theta_i}{h v_w} + \left(\frac{\Delta t}{h} \right)^2 \right)^{-1/2} \tag{2}$$

where the refraction angle θ_r is:

$$\theta_r = sin^{-1}\left(\frac{v(\theta_r) sin\theta_i}{v_w} \right) \tag{3}$$

The large-aperture receiver is a 52µm-thick PVDF membrane with gold electrodes. The PVDF membrane is bonded to a matched backing material, stiff enough to prevent low frequency bending vibration. The backing material has almost the same acoustic impedance of water around 20°C. The PVDF membrane is slightly stretched by using two concentric metallic rings. Each electrode is electrically connected to the corresponding terminal by contact rings. The external ring is grounded and the internal ring is connected to the signal.

The effective diameter of the receiver was chosen in order to intercept the entire ultrasonic field produced by the transmitter. The PVDF receiver used in this work, shown in Fig. 4, has 80mm active diameter.

2.2. Diffraction and dispersion on velocity measurement

Due to the high attenuation behavior of composite materials the velocity dispersion introduces error that increases with the frequency and the sample thickness. On the other hand, the acoustic diffraction effect decreases with the frequency for the same transducer's effective area. The diffraction effect is analyzed, in the range of 1 to 10 MHz, using aluminum which is a low dispersion material. The velocity dispersion is measured in a PMMA (polymetilmethacrylate) and an unidirectional CFRP samples.

The experiments have been made in a goniometer device immersed in distilled water. The water inside the goniometer was kept at 21.5 ± 0.05°C with the aid of a thermostatic bath. The goniometer device allows changing the emitter and the receiver transducers. To analyze the diffraction effect, measurements of longitudinal velocity in a 9.5 mm thickness aluminum plate where conducted with 5 pairs (emitter and receiver) of 19 mm diameter NDE, Panametrics model Videoscan, transducers of 1.0, 2.25, 3.5, 5.0 and 10.0 MHz. The receiver transducer is placed in the position of the PVDF receiver shown in Fig. 3.

Figure 3. Photo of goniometer.

The measurements of longitudinal velocity in the aluminum plate were repeated using the same set of emitter transducers but using a large aperture PVDF receiver as shown in Fig 3.

The emitter is excited with one cycle sinusoidal wave (center frequency of transducer) using a function generator and a broadband power amplifier, and the electrical signals of the received waves are amplified (Panametrics 5072PR) and digitized by an oscilloscope (HP 500MHz bandwidth, 2Gs/s) connected to a computer via Ethernet. The echoes are then stored and processed in the computer using MATLAB.

Figure 4. Large-aperture PVDF receiver.

Figure 5 shows the measured longitudinal velocities in the aluminum plate. The time delay between the two echoes, with and without sample, is measured using the Hilbert transform of the cross-correlation between them (Higuti and Adamowski, 2002). The results show that the diffraction effect may produce more than 1% error when using pair of transducers with 19 mm diameter and frequency under 2 MHz. This effect is eliminated with the large-aperture PVDF receiver as shown by the results of the dotted line and the result shown by the square mark.

Figure 6 and Fig. 7 show the results of the longitudinal velocity measurement in a acrylic 4.5 mm thickness plate and a 2.115 mm thickness CFRP plate using a set of five NDT transducers of 1.0, 2.25, 3.5, 5.0, and 10.0 MHz, 19 mm diameter, as emitters and the diffraction-free PVDF receiver.

The results show that the longitudinal velocity increases with the frequency, producing errors of about 1% in the acrilic and the CFRP plates when the frequency inceases from 1 to 10 MHz.

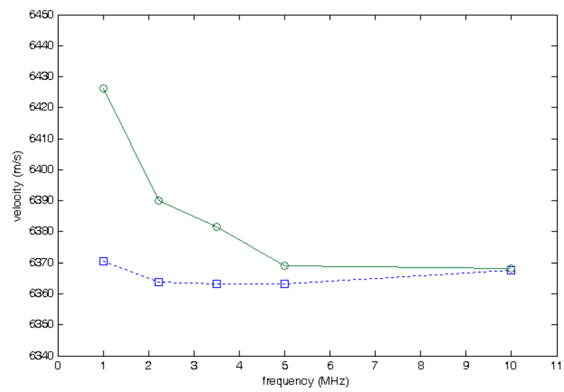

Figure 5. Measurements of longitudinal velocity in a 9.5 mm thick aluminum plate.

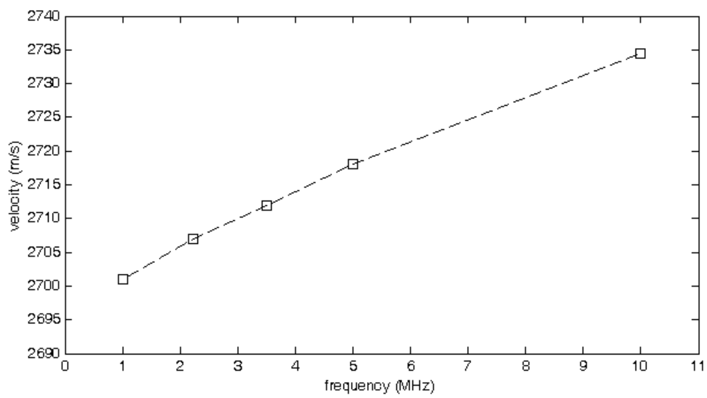

Figure 6. Measurements of longitudinal velocity in a 4.5mm thick acrylic plate

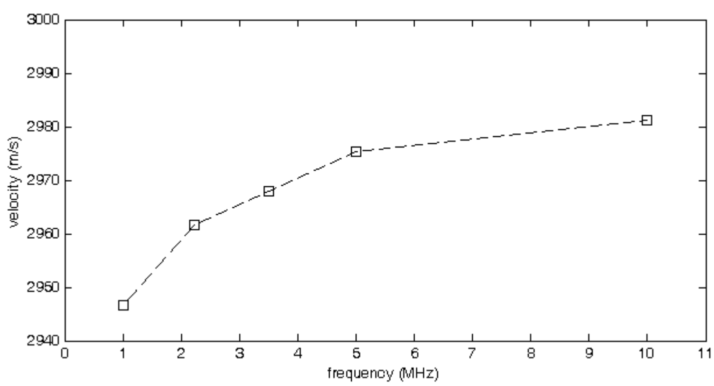

Figure 7. Measurements of longitudinal velocity in a 2.115 mm thick CFRP plate.

2.3. Experimental results of elastic constants

The experimental results of the phase velocities were obtained from a 2.11-mm-thick unidirectional carbon-fiber/epoxy square plate (80 x 80mm) with density ρ=1585 kg/m^3 using a goniometer shown in Fig. 2 and Fig. 3. The emitter is a focused transducer with nominal frequency of 5 MHz and diameter of 10mm, and the receiver is the diffraction-free PVDF transducer shown in Fig. 4. The velocities are measured in the plane x_1-x_2 and x_1-x_3, changing the incidence angle from 0 to 40°, spaced by 0.5°, and in the plane x_1-x_3 changing the incidence angle from 0 to 30°, spaced by 0.02°. The distilled water inside the goniometer was kept at 21.5 ± 0.05^{o}C with the aid of a thermostatic bath. Additionally, a pair of reference transducers, operating in transmission mode, mounted face to face in a mechanical frame which allows a precise adjusting of axial distance, is used as reference in the measurement of the propagation velocity in water. The results of the phase velocities, experimentally obtained, are shown in Fig. 8.

The unknown elastic constants are found by minimizing an objective function F, which is the sum of the squares of the deviations between the experimental and calculated phase velocities, given by:

$$F = \sum_{i=1}^{N} (v_i^{\exp} - v_i^{calc})^2 \tag{4}$$

where $v_i^{\exp}$ is obtained by Eq. (2), and N is the number of measured velocities. The minimization of F is implemented using *fminisearch* function of MATLAB.

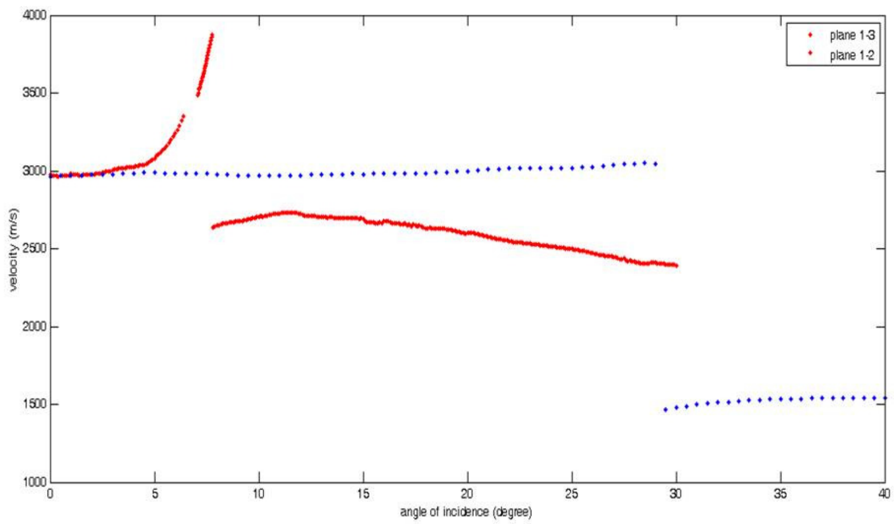

Figure 8. Phase velocities in planes 1-2 and 1-3.

A successful implementation of optimization requires reasonable estimates of the elastic constants as initial values to feed into the algorithm.

Table 1 shows the calculated results of the elastic constants obtained by minimizing function F from Eq. 5. Table 2 shows the engineering constants (Young and shear modulus and Poisson's ratio) calculated from the elastic constants of Tab. 2 and those obtained by a tensile test of samples of the same CFRP plate.

Table 1. Elastic Constants.

Elastic constant	**(GPa)**
C_{11}	14.1
C_{12}	6.8
C_{13}	6.2
C_{33}	135
C_{44}	6.8

Table 3. Engineering Constants.

Constant	Calculated	Tensile test
E_1 (GPa)	10.7	10
E_3 (GPa)	132	140
G_{13} (GPa)	6.8	7
ν_{31}	0.30	0.31

3. CONCLUSION

Large-aperture PVDF receivers have broadband frequency response and are very useful for measurements of pressure amplitude and velocity without diffraction effects. Gathering signal processing techniques and carefully designed large-aperture receivers, resulted in a phase velocity measurement with an accuracy around 0.1% under stabilized temperature conditions. Experimental results of diffraction effects on velocity measurement, in a low attenuation material, show that this error can be more than 1% when using a pair of 19-mm diameter transducers under 2 MHz. On the other hand, the measurements with the large-aperture PVDF receiver show the elimination of the diffraction effect, even when using a 5-MHz 10-mm diameter focused transducer. The dispersion effect is shown for the measurement of longitudinal velocity in an acrylic plate (medium attenuation) with the diffraction-free receiver. The fiber/plastic reinforced composites generally have high dispersive effect. The results of the elastic constant measurement with the large-aperture PVDF receiver show good agreement with the tensile test.

4. ACKNOWLEDGEMENTS

Work supported by FAPESP, CAPES, CNPq and PETROBRAS/ANP.

5. REFERENCES

Adamowski, J.C., Buiochi, F., Simon, C., Silva, E.C.N., 1995, and R.A. Sigelmann, "Ultrasonic measurement of density of liquids", Journal of Acoustical Society of America, vol. 97, pp. 354-361.

Auld, B.A., 1990, "Acoustics fields and waves in solids", vol. 1, EUA, John Wiley & Sons, 2nd. ed., 1990.

Bouzidi, Y., Schmitt, D.R., 2006, "A large ultrasonic bounded acoustic pilse transducer for acoustic transmission goniometry: Modeling and calibration", Journal of Acoustical Society of America, vol. 119, pp. 54-64.

Costa, E.T., 1989, "Development and application of a large-aperture PVDF hydrophone for measurement of linear and non-linear ultrasound fields", PhD. Thesis, University of London.

Hayman, A.J., Weight, J.P., 1979, "Transmission and reception of short ultrasonic pulses by circular and square transducers", J. Acoust. Soc. Am. 66(4), pp. 945-951.

Higuti, R.T., Adamowski,J.C., 2002, "Ultrasonic densitometer using a multiple reflection technique," IEEE Trans. Ultras., Ferroelec., Freq. Contr., vol. 49(9), pp.1260-1268.

Hosten, B., 2001, "Ultrasonic through-transmission method for measuring the complex stiffness moduli of composite materials", Handbook of elastic properties of solids, liquids and gases, vol. 1, pp. 67-86, 2001.

Leeman, S., Seggie, D. A., Ferrari, L. A., Sankar, P. V., Doherty, M. 1985,"Diffraction-free attenuation estimation," Ultrasonics International Conf. Proc., Butterworth, Guildford, pp. 128-132.

Markham, M.F., 1970, " Measurement of the elastic constants of fibre composites by ultrasonics", Composites, pp. 145-149.

Rokhlin, S.I., Wang, W., 1992, "Double through-transmission bulk wave method for ultrasonic phase velocity measurement and determination of elastic constants of composite materials", Journal of Acoustical Society of America, vol. 91, pp. 3303-3312.

Zeqiri, B., 1996, "Validation of a diffraction correction model for throughtransmission substitution measurements of ultrasonic absorption and phase velocity", J. Acoust. Soc. Am. 99(2), PP. 996-1001.

Wang, W., Lavrentyev, A.I., Rokhlin, S.I., 2003, "Beam and phase effects in angle-beam-through-transmission method of ultrasonic velocity measurement", Journal of Acoustical Society of America, vol. 103, pp. 1551-1559.

5. RESPONSIBILITY NOTICE