PARAMETRIC EVALUATION FOR FREE CONVECTION IN A TYPICAL GEOMETRY FROM INTEGRATED COLLECTOR STORAGE SOLAR WATER HEATERS

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Abstract. *Solar energy is always treated as the energy of the future and several efforts have been made to broaden its use in new equipment. Some recent papers studied systems with Integrated Collector Storage Solar Water Heaters (ICSSWH). In these systems, solar collectors and storage tanks were integrated. These systems were used in earlier stages of Solar Energy studies conducted at the beginning of last century. The main problem of these first integrated systems was substantial heat loss to the environment, especially at night and non-collection periods. Several improvements were proposed involving glazing system, methods of insulation, reflector configurations, use of evacuation, internal and external baffles and phase change materials for minimizing the problem. It was partially solved using devices that prevent reverse circulation at night. The main idea for this kind of system is the construction of low cost solar water heaters but, some of these enhancements, make the system more expensive. Nevertheless, in most cases, this system is cheaper than others. In this study, free convection in a water tank geometry used in some ICSSWH was analyzed and some parameters were changed to study its influence on overall efficiency of the system.*
Keywords*: CBS scheme, Artificial Compressibility, Finite Element Method, Integrated collector-storage solar devices, Free convection on cylindrical cavities.*

1. INTRODUCTION

Several studies have been conducted with devices that can capture solar energy in an integrated collector / storage device. Hence, these studies are only a resumption of previous devices presented in the beginning of solar energy history. Numerical simulation and computational modeling can be widely used for solar collector investigations in these kinds of devices with strong reductions in costs for experimental tests. Although numerical simulation applications in engineering saw a large increase over the past 30 years, highlighting the problems involving fluid mechanics and heat transfer, this was not true in solar energy.

Numerical simulations for typical geometries of energy storage/solar collector devices and the free convection of the water in the tank are still unusual in scientific literature. Furthermore, experimental analyses are often present in recent works introducing new ideas and possibilities. A basic literature for studies in solar energy complains the book of Duffie & Beckman (1991), which is a full review of the solar devices technology. Another important basic reference is the work by Kalogirou (2006), which deals with the main recent advances and technologies related to solar energy. For experimental analysis with several important results, one can cite Chaurasia and Twidell (2001), Kaptan and Klic (1996), Smyth et al. (2004). Several different geometries for ICS systems are presented by Tripanagnostopoulos et al. (1999), Tripanagnostopoulos & Yanoulis (1992), Mohamad (1997) e Joudi et al. (2004).

A large number of numerical procedures can be used in fluid flow problems. A classical problem for testing this procedure was proposed by Ghia, Ghia and Shin (1982) and was known as lid driven cavity. Different geometries for this classical problem can be found in works of Erturk (2007). Nithiarasu and Liu (2005) used this problem to verify the Characteristic Based Split (CBS) procedure solution for different Reynolds' numbers, with good agreement with the results of Ghia, Ghia and Shin (1982). The CBS procedure was also used to solve several geometries, as one can find in Codina et al (2006), Massarotti et al. (2006), Nithiarasu and Liu (2006), Nithiarasu and Zienkiewicz (2006), Thomas and Nithiarasu (2005), Rojek et al. (2006) and Kulkarni et al (2006) with good agreement results for fluid flow analysis.

Thus, this work intends to implement and solve a numerical algorithm for simulating free convection of an incompressible fluid in cylindrical geometries. The mathematical model is solved using a mesh composed of bi-linear elements and the CBS method proposed by Zienkiewicz and Codina (1996) and Nithiarasu (2003). First of all, a validation procedure was developed to verify the accuracy of the numerical implementation. Then, the proposed geometry for the Integrated Collector Storage (ICS) system was solved and discussed for different external-internal wall spacings. The reservoir scheme with azimuthal symmetry, used to simplify the geometry, was shown in Figure 1. The main function of the internal wall, shown in the figure, is to increase flow and, consequently, free convection in this region. Details of implementation and the assumptions adopted will be presented with the mathematical model.

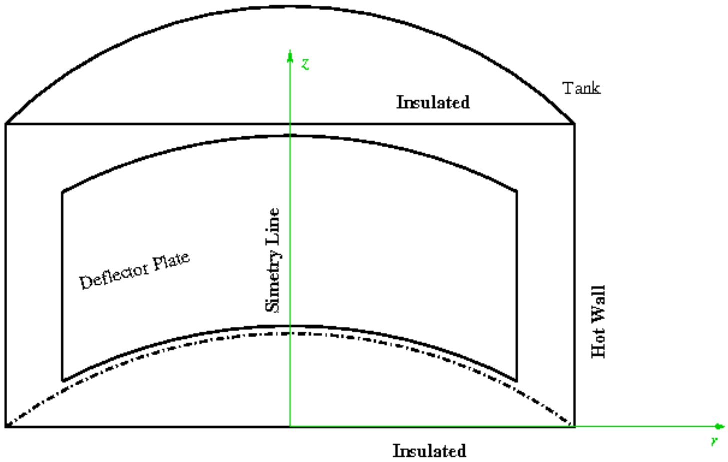

Figure 1: Hot water reservoir scheme .

2. FORMULATION

As shown previously, there are several methodologies for the numerical solution of Navier-Stokes equations. In this study, a fractional step method, adapted from the CBS method described by Zienkiewicz and Codina (1996), for cylindrical coordinates will be used. For the current analysis, the solution, including the body forces, need to be implemented to obtain free convection inside cavities. Thus, the dimensionless equations form of mass (compressibility form), momentum and energy conservation, as shown in expressions (1) to (4), should be solved.

$$\frac{1}{c^2}\cdot\frac{\partial p}{\partial t} = -\frac{1}{r}\frac{\partial(rU)}{\partial r} - \frac{\partial V}{\partial z} \tag{1}$$

$$\frac{\partial U}{\partial t} + \frac{\partial(uU)}{\partial r} + \frac{\partial(vU)}{\partial z} = -\frac{\partial p}{\partial r}\Pr\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rU)}{\partial r}\right) + \frac{\partial^2 U}{\partial z^2} - \frac{u}{r^2}\right] \tag{2}$$

$$\frac{\partial V}{\partial t} + \frac{\partial(vU)}{\partial r} + \frac{\partial(vV)}{\partial z} = -\frac{\partial p}{\partial z}\Pr\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{\partial^2 V}{\partial z^2}\right] + \mathrm{Ra}\cdot\Pr\cdot\Theta\cdot\vec{i}_g \tag{3}$$

$$\frac{\partial \Theta}{\partial t} + \frac{\partial(u\Theta)}{\partial r} + \frac{\partial(v\Theta)}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(\Theta)}{\partial r}\right) + \frac{\partial^2 \Theta}{\partial z^2} \tag{4}$$

The dimensionless variables, shown in the previous equations, can be calculated by the expressions:

$$r = \frac{r*}{H};\quad U = \frac{uH}{\alpha};\quad V = \frac{vH}{\alpha};\quad P = \frac{P - P_{ref}}{\rho}\left(\frac{H}{\alpha}\right)^2; \Theta = \frac{T - T_f}{\delta T};\quad Pr = \frac{\nu}{\alpha};\quad Ra = \frac{g\,\beta\,\overline{\delta T}\,H^3}{\nu^2}Pr;\quad t = \frac{\alpha t^*}{H^2}; \tag{5}$$

A major problem associated with transport equations, similar to energy transport equations, is associated with the nonlinearity of the solution plus the advective terms. For a general treatment of these equations it is common to use an arbitrary property φ, so::

$$\frac{\partial \phi}{\partial t} + U_i\frac{\partial \phi}{\partial x_i} - \frac{\partial}{\partial x_i}\left(k\frac{\partial \phi}{\partial x_i}\right) + Q = 0 \tag{6}$$

In this equation, there are several alternatives to treat non-linear terms. In this case, the characteristics method using the streamline, as seen Figure 2, will be used. This method was first published by Zienkiewicz et al. (1984) and changes equation (6) with the change in the independent variable to follow the streamline. This allows this equation to be rewritten as:

$$\frac{\partial \phi}{\partial t}(x'(t), t) - \frac{\partial}{\partial x_i'}\left(k\frac{\partial \phi}{\partial x_i'}\right) + Q(x_i') = 0 \tag{7}$$

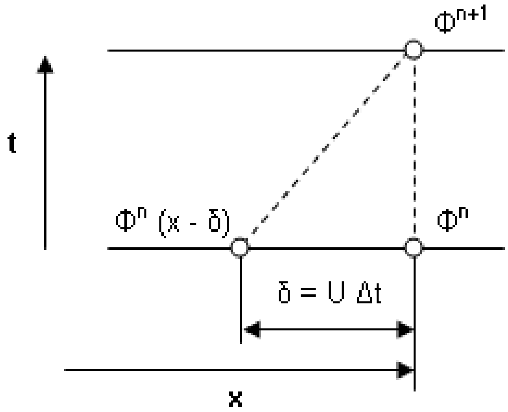

Figure 2: The pseudo-characteristics streamline.

This equation can be approximated, even according to Figure 2, as:

$$\frac{1}{\Delta t}(\phi^{n+1}-\phi^{n}\,|_{(x-\delta)})\approx\theta\left[\frac{\partial}{\partial x}\left(k\frac{\partial\phi}{\partial x}\right)-Q\right]^{n+1}+(1-\theta)\left[\frac{\partial}{\partial x}\left(k\frac{\partial\phi}{\partial x}\right)-Q\right]^{n}_{(x-\delta)}\tag{8}$$

where $\theta=0$ for an explicit discretization and $0<\theta\leqslant 1$ for an implicit one. Length δ, as described in work Zienkiewicz and Codina (1996), according to average velocity is calculated by:

$$\delta=\Delta t\left(U^{n+\frac{1}{2}}-\frac{\Delta t}{2}U^{n}\frac{\partial U^{n}}{\partial x}\right)\tag{9}$$

A general equation can be obtained by expanding the Taylor series for the various terms evaluated at x-δ. Further details of this procedure can be found in reference Zienkiewicz et al. (1984). Replacing the value of δ obtained by the expression (9) and rearranging the equation, one can obtain:

$$\begin{aligned}\Delta\phi=\langle\phi^{n+1}-\phi^{n}\rangle=\Delta t&\left\{k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{\partial^{2}\phi}{\partial z^{2}}\right]^{n+\theta}-Q^{n+\theta}-\left(u^{n}\frac{\partial\phi^{n}}{\partial r}+v^{n}\frac{\partial\phi^{n}}{\partial z}\right)\right\}\\&+(1-\theta)\Delta t^{2}\cdot\left\{\left(u^{n}\cdot\frac{\partial Q^{n}}{\partial r}+v^{n}\cdot\frac{\partial Q^{n}}{\partial z}\right)-k\cdot\frac{u^{n}}{r}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial r}\right)\right]^{n}\right\}\\&+\frac{\Delta t^{2}}{2}\left[u^{n}\frac{\partial}{\partial r}\left(u^{n}\frac{\partial\phi^{n}}{\partial r}\right)+v^{n}\frac{\partial}{\partial z}\left(v^{n}\frac{\partial\phi^{n}}{\partial z}\right)+u^{n}\frac{\partial v^{n}}{\partial r}\frac{\partial\phi^{n}}{\partial z}+v^{n}\frac{\partial u^{n}}{\partial z}\frac{\partial\phi^{n}}{\partial r}\right]\end{aligned}\tag{10}$$

The procedure for the momentum equation

The conservation of momentum equations, given by expressions (2) and (3), may be appropriate to fit in with the general transport equation (10). Thus, the equation adapted to the equations of characteristics can be written as:

$$\begin{aligned}\Delta\tilde{U}=\langle\tilde{U}^{n+1}-\tilde{U}^{n}\rangle=\Delta t&\left\{\Pr\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\tilde{U}}{\partial r}\right)+\frac{\partial^{2}\tilde{U}}{\partial z^{2}}\right]^{n}-Q_{r}^{n}-\left(u^{n}\frac{\partial\tilde{U}^{n}}{\partial r}+v^{n}\frac{\partial\tilde{U}^{n}}{\partial z}\right)\right]\\&+(1-\theta)\cdot\Delta t^{2}\cdot\left[\left(u^{n}\cdot\frac{\partial Q_{r}^{n}}{\partial r}+v^{n}\cdot\frac{\partial Q_{r}^{n}}{\partial z}\right)-\Pr\cdot\frac{u^{n}}{r}\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\tilde{U}}{\partial r}\right)\right)^{n}\right]\\&+\frac{\Delta t^{2}}{2}\left[u^{n}\frac{\partial}{\partial r}\left(u^{n}\frac{\partial\tilde{U}^{n}}{\partial r}\right)+v^{n}\frac{\partial}{\partial z}\left(v^{n}\frac{\partial\tilde{U}^{n}}{\partial z}\right)+u^{n}\frac{\partial v^{n}}{\partial r}\frac{\partial\tilde{U}^{n}}{\partial z}+v^{n}\frac{\partial u^{n}}{\partial z}\frac{\partial\tilde{U}^{n}}{\partial r}\right]\end{aligned}\tag{11}$$

for the radial coordinate. The expression for velocity in *z* direction uses the $\tilde{V}$ and Q_z terms. It is quite similar and won't be demonstrated. Using discretization by finite elements with the Galerkin method, one can obtain::

$$[M]\Delta\tilde{U}=\Delta t\left[\Pr[H]-[C]+\Delta t\left(\frac{1}{2}[K_{u}]-\Pr\cdot(1-\theta)\cdot[K_{k}]\right)\right]\cdot\tilde{U}^{n}+\Delta t(\Delta t(1-\theta)\cdot[C]-[M])Q_{r}^{n}+[F_{s}]\tag{12}$$

with the matrices calculated by:

- Matrix *[M_u]*:

$$[M_u]=\int_\Omega N_{u,i}\cdot N_{u,j}\,d\Omega \tag{13}$$

- Matrix *[H]*:

$$[H]=-\int_\Omega\left[\left(\frac{1}{r}\frac{\partial N_{u,i}}{\partial r}-\frac{1}{r^2}N_{u,i}\right)\left(r\frac{\partial N_{u,j}}{\partial r}\right)+\frac{\partial N_{u,i}}{\partial z}\frac{\partial N_{u,j}}{\partial z}\right]d\Omega \tag{14}$$

- Matrix *[C]*:

$$[C]=\int_\Omega\left[N_{u,i}\cdot u_n\cdot\frac{\partial N_{u,j}}{\partial r}+N_{u,i}\cdot v_n\cdot\frac{\partial N_{u,j}}{\partial z}\right]d\Omega \tag{15}$$

- Matrix *[K_k]*:

$$[K_k]=-\int_\Omega\frac{1}{r}\left(u^n\frac{\partial N_{u,i}}{\partial r}+N_{u,i}\frac{\partial u^n}{\partial r}\right)\cdot\left(\frac{\partial N_{u,j}}{\partial r}\right)\cdot d\,\Omega \tag{16}$$

- Matrix *[K_u]*:

$$[K_{ur}]=\int_\Omega u\frac{\partial u}{\partial r}\cdot N_{u,i}\cdot\frac{\partial N_{u,j}}{\partial r}d\,\Omega+\int_\Omega u\frac{\partial v}{\partial r}\cdot N_{u,i}\frac{\partial N_{u,j}}{\partial z}d\,\Omega-\int_\Omega\left[u^2\cdot\frac{\partial N_{u,i}}{\partial r}\cdot\frac{\partial N_{u,j}}{\partial r}+2\cdot u\cdot\frac{\partial u}{\partial r}\cdot N_{u,i}\cdot\frac{\partial N_{u,j}}{\partial r}\right]d\,\Omega \tag{17}$$

and the $[K_{uz}]$ matrix can be calculated by analogy. If a global correction matrix $[K_u]=[K_{ur}]+[K_{uz}]$ is used the boundary surface integral can be expressed by:

$$[F_S]=[f_u]+\mathrm{Pr}\cdot([f_H]-[f_k]) \tag{18}$$

The source term Q is often used to represent all the other terms which don't have the equation’s main variables. This one may represent different quantities such as force, pressure field, energy generation and several others.

Momentum equations can be represented using the pressure gradient, buoyancy and turbulence effects in source term. For pressure terms, there are different approaches according to the method worked as can be seen in several papers that address the issue. For a general treatment of pressure terms, one can make a discretization explicit or implicit. Considering the treatment only for pressure:

$$Q^{n+\theta_2}=\frac{\partial p^{n+\theta_2}}{\partial x_i} \tag{19}$$

where $\Delta p=p^{n+1}-p^n$:

$$\frac{\partial p^{n+\theta_2}}{\partial x_i}=\theta_2\frac{\partial p^{n+1}}{\partial x_i}+(1-\theta_2)\frac{\partial p^n}{\partial x_i}=\frac{\partial p^n}{\partial x_i}+\theta_2\frac{\partial\Delta p}{\partial x_i} \tag{20}$$

Although this procedure is possible, for increasing method stability is not usual to include the pressure term in the first step of the solution, as suggested by the "A" split proposed by Zienkiewicz et al. (1984). According to this model, the velocity correction can be estimated by:

$$U_i^{*,n+\theta}=U^n+\theta\left(\Delta\tilde{U}-\frac{\partial p^{n+\theta_2}}{\partial r}\cdot\Delta t\right) \quad \text{and} \quad V_i^{*,n+\theta}=V^n+\theta\left(\Delta\tilde{V}-\frac{\partial p^{n+\theta_2}}{\partial z}\cdot\Delta t\right) \tag{21}$$

For natural convection, source terms are given by buoyancy forces. Thus, considering this force on the *z* direction:

$$Q_r^{*,n+\theta}=\mathrm{Pr}\cdot\frac{u^{n+\theta}}{r^2} \quad \text{e} \quad Q_z^{*,n+\theta}=\mathrm{Ra}\cdot\mathrm{Pr}\,\Theta^{n+\theta} \tag{22}$$

The pressure correction

The pressure field for this method can be calculated using artificial compressibility for an incompressible flow, using a fake compressibility coefficient. Because of this, the method receives the name of “Artificial Compressibility”. It is based on equation (1), using an artificial compressibility coefficient β instead of sound speed on medium *c*.

$$\Delta\rho=\left(\frac{1}{\beta^2}\right)\Delta p=-\Delta t\left[\frac{1}{r}\frac{\partial}{\partial r}(r\,u^{n+\theta})+\frac{\partial v^{n+\theta}}{\partial z}\right] \tag{23}$$

Using a velocity profile estimated by equation (21), one can obtain:

$$\Delta\rho=\left(\frac{1}{\beta^2}\right)\Delta p=-\Delta t\left\{\frac{1}{r}\frac{\partial}{\partial r}r\left[U^n+\theta\left(\Delta\tilde{U}-\frac{\partial p}{\partial r}^{n+\theta_2}\Delta t\right)\right]+\frac{\partial}{\partial z}\left[V^n+\theta\left(\Delta\tilde{V}-\frac{\partial p}{\partial z}^{n+\theta_2}\Delta t\right)\right]\right\}=$$
$$-\Delta t\cdot\left\{\left[\frac{1}{r}\frac{\partial}{\partial r}r\cdot(U^n+\theta\Delta\tilde{U})+\frac{\partial}{\partial z}(V^n+\theta\Delta\tilde{V})\right]-\Delta t\cdot\theta\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\cdot\frac{\partial p}{\partial r}^{n+\theta_2}\right)+\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial z}^{n+\theta_2}\right)\right]\right\} \tag{24}$$

and substituting $\partial p^{n+\theta_2}/\partial x_i$ expressed in equation (20):

$$\left(\frac{1}{\beta^2}\right)\Delta p=-\Delta t\cdot\left[\frac{1}{r}\frac{\partial}{\partial r}r\cdot(U^n+\theta\Delta\tilde{U})+\frac{\partial}{\partial z}(V^n+\theta\Delta\tilde{V})\right]$$
$$+\Delta t^2\cdot\theta\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\cdot\frac{\partial p^n}{\partial r}\right)+\frac{\partial^2 p^n}{\partial z^2}+\theta_2\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\cdot\frac{\partial\Delta p^n}{\partial r}\right)+\frac{\partial^2\Delta p^n}{\partial z^2}\right]\right\} \tag{25}$$

where $\Delta\tilde{U}_i$ is calculated by using equation (12). It is important to note that when the pressure term is included in the first step calculation, this procedure must be adapted

Equation (25) can be discretized according to nodal approximations resulting in:

$$([M_p]-\Delta t^2\cdot\theta\cdot\theta_2\cdot[H_p])\Delta p^*=-\Delta t\cdot\{[G_r]\cdot(U^n+\theta\Delta\tilde{U})+[G_z]\cdot(V^n+\theta\Delta\tilde{V})-\Delta t\cdot\theta\cdot[H_p]\cdot P^n\} \tag{26}$$

where the matrices can be calculated by the weighted residual method associated with Galerkin discretization:

- Matrix *[M_p]*:

$$[M_p]=\int_\Omega N_{p,i}\cdot\frac{1}{\beta^2}\cdot N_{p,j}\,d\Omega \tag{27}$$

- Matrix *[H_p]* :

$$[H_p]=-\int_\Omega\left[\frac{\partial}{\partial r}\left(\frac{N_{p,i}}{r}\right)\left(r\frac{\partial N_{p,j}}{\partial r}\right)+\frac{\partial N_{p,i}}{\partial z}\frac{\partial N_{p,j}}{\partial z}\right]d\Omega \tag{28}$$

- Matrix *[G_r]*:

$$[G_r]=\int_\Omega N_{p,i}\frac{\partial N_{u,j}}{\partial r}\,d\Omega+\int_\Omega\frac{N_{p,i}}{r}\cdot N_{u,j}\,d\Omega \tag{29}$$

- Matrix *[G_z]*:

$$[G_z]=\int_\Omega N_{p,i}\frac{\partial N_{u,j}}{\partial z}\,d\Omega \tag{30}$$

The velocity correction equations

Velocity correction equations are used to correct the velocity profile based on the pressure correction. This equation is a generalization of expression (10), considering the inclusion of the pressure gradient term in Q (equation (19)):

$$\Delta U_i^*=\Delta\tilde{U}-\frac{\partial p}{\partial r}^{n+\theta}\cdot\Delta t+(1-\theta)\left(u^n\cdot\frac{\partial}{\partial r}\frac{\partial p^n}{\partial r}+v^n\cdot\frac{\partial}{\partial z}\frac{\partial p^n}{\partial r}\right)\cdot\Delta t^2$$
$$\Delta V_i^*=\Delta\tilde{V}-\frac{\partial p}{\partial z}^{n+\theta}\cdot\Delta t+(1-\theta)\left(u^n\cdot\frac{\partial}{\partial r}\frac{\partial p^n}{\partial z}+v^n\cdot\frac{\partial}{\partial z}\frac{\partial p^n}{\partial z}\right)\cdot\Delta t^2 \tag{31}$$

that could be simplified substituting equation (20) in (31) :

$$\Delta U_i^*=\Delta\tilde{U}-\Delta t\cdot\left(\frac{\partial p}{\partial r}^n+\theta_2\cdot\frac{\partial\Delta p}{\partial r}^n\right)+(1-\theta)\cdot\left(u^n\cdot\frac{\partial}{\partial r}\frac{\partial p}{\partial r}^n+v^n\cdot\frac{\partial}{\partial z}\frac{\partial p}{\partial r}^n\right)\cdot\Delta t^2$$
$$\Delta V_i^*=\Delta\tilde{V}-\Delta t\cdot\left(\frac{\partial p}{\partial z}^n+\theta_2\cdot\frac{\partial\Delta p}{\partial r}^n\right)+(1-\theta)\cdot\left(u^n\cdot\frac{\partial}{\partial r}\frac{\partial p}{\partial z}^n+v^n\cdot\frac{\partial}{\partial z}\frac{\partial p}{\partial z}^n\right)\cdot\Delta t^2 \tag{32}$$

Again, applying the weighted residual method associated with a Galerkin discretization, these equations can be represented by:

$$[M_u]\cdot\Delta U_i^*=[M_u]\cdot\Delta\tilde{U}-\Delta t\cdot[G_{pr}]\cdot(P^n+\theta_2\cdot\Delta p)+(1-\theta)\cdot\Delta t^2[P_r]\cdot P^n+[F_{cvr}]$$
$$[M_u]\cdot\Delta V_i^*=[M_u]\cdot\Delta\tilde{V}-\Delta t\cdot[G_{pz}]\cdot(P^n+\theta_2\cdot\Delta p)+(1-\theta)\cdot\Delta t^2[P_z]\cdot P^n+[F_{cvz}] \tag{33}$$

where the matrix *[M_u]* was previously defined and the others are:

- Matrix *[G_{pr}]*:

$$[G_{pr}]=\int_{\Omega} N_{u,i}\frac{\partial N_{p,j}}{\partial r}d\,\Omega \tag{34}$$

- Matrix *[G_{pz}]* (if N_u=N_p then *[G_{pz}]=[G_z]*):

$$[G_{pz}]=\int_{\Omega} N_{u,i}\frac{\partial N_{p,j}}{\partial z}d\,\Omega \tag{35}$$

- Matrix *[P_r]* :

$$[P_r]=-\int_{\Omega} u^n\cdot\frac{\partial N_{u,i}}{\partial r}\frac{\partial N_{u,j}}{\partial r}d\,\Omega-\int_{\Omega}\frac{\partial u^n}{\partial r}\cdot N_{u,i}\cdot\frac{\partial N_{p,j}}{\partial r}d\,\Omega-\int_{\Omega} v^n\cdot\frac{\partial N_{u,i}}{\partial z}\frac{\partial N_{p,j}}{\partial r}d\,\Omega-\int_{\Omega}\frac{\partial v^n}{\partial z}\cdot N_{u,i}\cdot\frac{\partial N_{p,j}}{\partial r}d\,\Omega \tag{36}$$

- Matrix *[P_z]* :

$$[P_z]=-\int_{\Omega} u^n\cdot\frac{\partial N_{u,i}}{\partial r}\frac{\partial N_{p,j}}{\partial z}d\,\Omega-\int_{\Omega}\frac{\partial u^n}{\partial r}\cdot N_{u,i}\cdot\frac{\partial N_{p,j}}{\partial z}d\,\Omega-\int_{\Omega} v^n\cdot\frac{\partial N_{u,i}}{\partial z}\frac{\partial N_{p,j}}{\partial z}d\,\Omega-\int_{\Omega}\frac{\partial v^n}{\partial z}\cdot N_{u,i}\cdot\frac{\partial N_{p,j}}{\partial z}d\,\Omega \tag{37}$$

and the boundary terms:

$$[F_{cvr}]=\int_{\Gamma}\vec{u}_n N_{u,i}\frac{\partial N_{p,j}}{\partial r}d\,\Gamma \qquad \text{e} \qquad [F_{cvz}]=\int_{\Gamma}\vec{u}_n N_{u,i}\frac{\partial N_{p,j}}{\partial z}d\,\Gamma \tag{38}$$

The time step is an important parameter for solution convergence and must be as high as possible, decreasing the number of steps to achieve a steady state solution. Optimize time steps can significantly reduce the computation time and techniques for this implementations can be fond in Nithiarasu (2003).

Artificial compressibility

The artificial compressibility factor value, equivalent to sound speed in equation (1), can lead to restrictions in the time step. Thus, for steady state solutions this factor is used to quickly reach this state without compromising results.

According to , the value used for artificial compressibility can be calculated as:

$$\beta_{\tau}=max(\varepsilon, v_{conv}, v_{diff}) \tag{39}$$

where $\varepsilon=0{,}5$, v_{conv} is the convective velocity and v_{diff} is the diffusive velocity, which can be calculated as:

$$v_{conv}=(u_i u_i)^{1/2} \qquad \text{and} \qquad v_{diff}=\frac{2}{h\cdot\mathrm{Pr}} \tag{40}$$

with h as the smallest length scale for the finite element.

In order to keep the convergence in schemes using the semi-implicit method it is recommended to use high values for the artificial compressibility coefficient. For solutions like this, it is fundamental to establish a reference value for pressure inside the domain.

3. RESULTS AND DISCUSSIONS

Validation

The model validation is based on a solution usually used as a reference when free convection problems are treated. This original problem was proposed by Vahl Davis and solutions were obtained in different meshes and the benchmark solution was then presented through the use of Richardson extrapolation. In this work, a posterior study by Ismail and Scalon (2000) is used for comparison.

The value of Ra=10^6 was used to compare results. In this case, the circumferential effect was reduced by enlarging the curvature radius to 100. The hot and cold walls are lateral and some results can be seen in Table 1. The comparison shows that the presented results are quite close to the benchmark solution proposed by Vahl Davis. For the Nusselt number the results have the same behavior and are very close too. Therefore, the proposed formulation and methodology can be considered appropriate for treating free convection inside cavities.

Simulation of the collector storage device

After validation of the methodology, the simulation will now be considered for the proposed problem. The simulation was done to evaluate development of free convection inside the cavity and the influence of deflector plate spacing on the wall. For this preliminary analysis some geometric parameters were determined that can be seen in Table 2.

Table 1: Comparison of obtained results with previous simulations.

Reference	$V_{n,max}$	Nu_{max}	Nu_{med}
Vahl Davis (Benchmark)	219.36 (y=0.0379)	17.925 (y=0.0378)	8.8
Este Trabalho	231.65 (y=0.041)	16.81 (y=0.06)	8.74
Vahl Davis	195.44 (y=0.0447)	14.22 (y=0.124)	9.03
Ismail & Scalon	220.48 (y=0.04454)	15.2 (y=0.09)	8.93

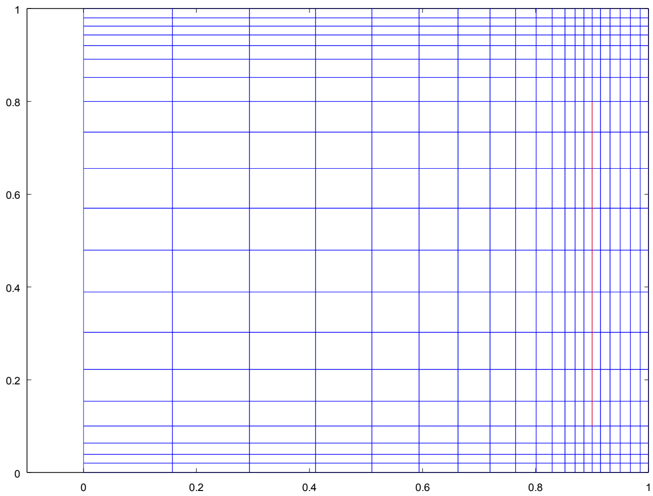

Figure 3: Mesh grid used for solution.

The computer code used for the routines was implemented on a math platform: the GNU-Octave (2008), free software similar to MATLAB. All subroutines are written as functions called from the main program. For post processing and plots, the tool that best fits the needs of generating isocurves is the Kelley and Galbraith, GRI (2008), free software like the previous one. The mesh used is not regular, but generated from the spline approximation for defined points of interest. Use of this type of approach does not generate significant differences in size between neighboring elements and sets the highest standards of performance. The mesh used for the proposed problem uses problem symmetry and can be seen in Figure 3.

The first analysis was done for aspect ratio $d/H = 0.06$ and the evolution velocity and temperature profiles can be seen in Figure 4. These results show the relationship between the two profiles. The hot surface of the solar collector generates upward-flow that is critical for the formation of temperature gradients. The upward movement causes a temperature increase at the top of the tank. This is very adequate for heat storage systems and is known as thermal stratification.

The deflector plate plays an important role to direct fluid to upward Thus, other spacing between the deflector plate and the $d/H = 0.1$ and $d/H = 0.15$ wall are shown in Figures 5 and 6, respectively. Some important differences in the velocity and temperature profile could be noticed.

Table 2: Parameters used in collector-storage device simulation.

Parameter	Mesh size	Aspect Ratio	Deflector spacing	Deflector height	Space deflector ground	Fluid	Raleigh Number
Value	21 x 21	$R/H=1$	$d/H=0.1$ e 0.15	$L/H=0.7$	$z_f/H=0.1$	Water (Pr=3.5)	$Ra=10^6$

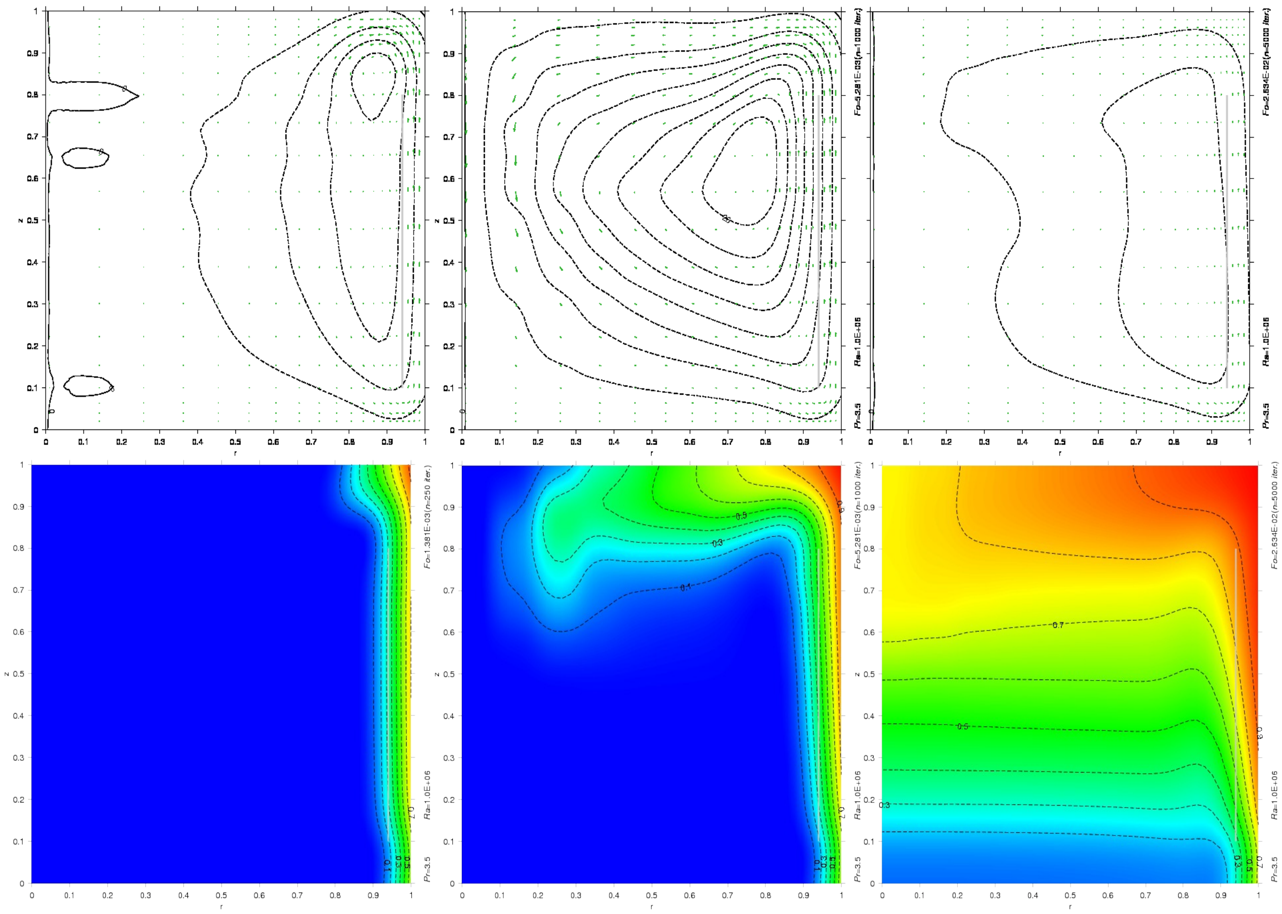

Figure 4: Time development for velocity, stream function and velocity profiles for *d/H=0.06*.

The main difference is the change in velocity profile at early stages. One can see that the deflector plate limits the upstream fluid region and, consequently, its acceleration. However for very small spacing between the deflector plate and tank wall, the effects of viscosity within the boundary layer become more significant and reduce upward movement. This can be clearly noticed by the stream function graphics and velocity profiles shown in Figures 4, 5 and 6. The scale for the vector fields and stream functions in this figures was the same for all cases

Another expected effect that can be observed is the decrease of convection as time and mixed temperature in the tank rises. However, an interesting fact to note is that the deflector causes a bottleneck in the tank and it virtually determines the time point where the collector will load. This effect can be seen in the figure of the end-time simulation with 5000 steps in all cases where it notes that although the upper tank is not fully charged, the channel output is hindering the free movement of fluid in the region near the wall into the tank.

4. CONCLUSIONS

The proposed method for solution of fluid flow on radial coordinates works fine for the cartesian lid driven cavity, what could be done leading $r \rightarrow \infty$. One can see the good agreement from present results with previous works. Another important aspect is the respect the role of mass conservation, which is verified by the appropriate value of stream functions on closed surfaces.

Using this method for a solution on the ICS geometry, several changes in streamlines can be observed by the presence of the deflector. Regarding the geometry, the deflector plate has a great influence in storage charge operating conditions and, when in correct position, increase the amount of water at high temperatures. New tests, including the variation of deflector plate height and position should be made to complement this study and determine the position and size that increases the availability from the system.

5. ACKNOWLEDGEMENTS

The authors acknowledge *CAPES - Coordenação de Aperfeiçoamento do Pessoal de Nível Superior* for financing the Master's of Science for the student Alice Jordam, making this study possible.

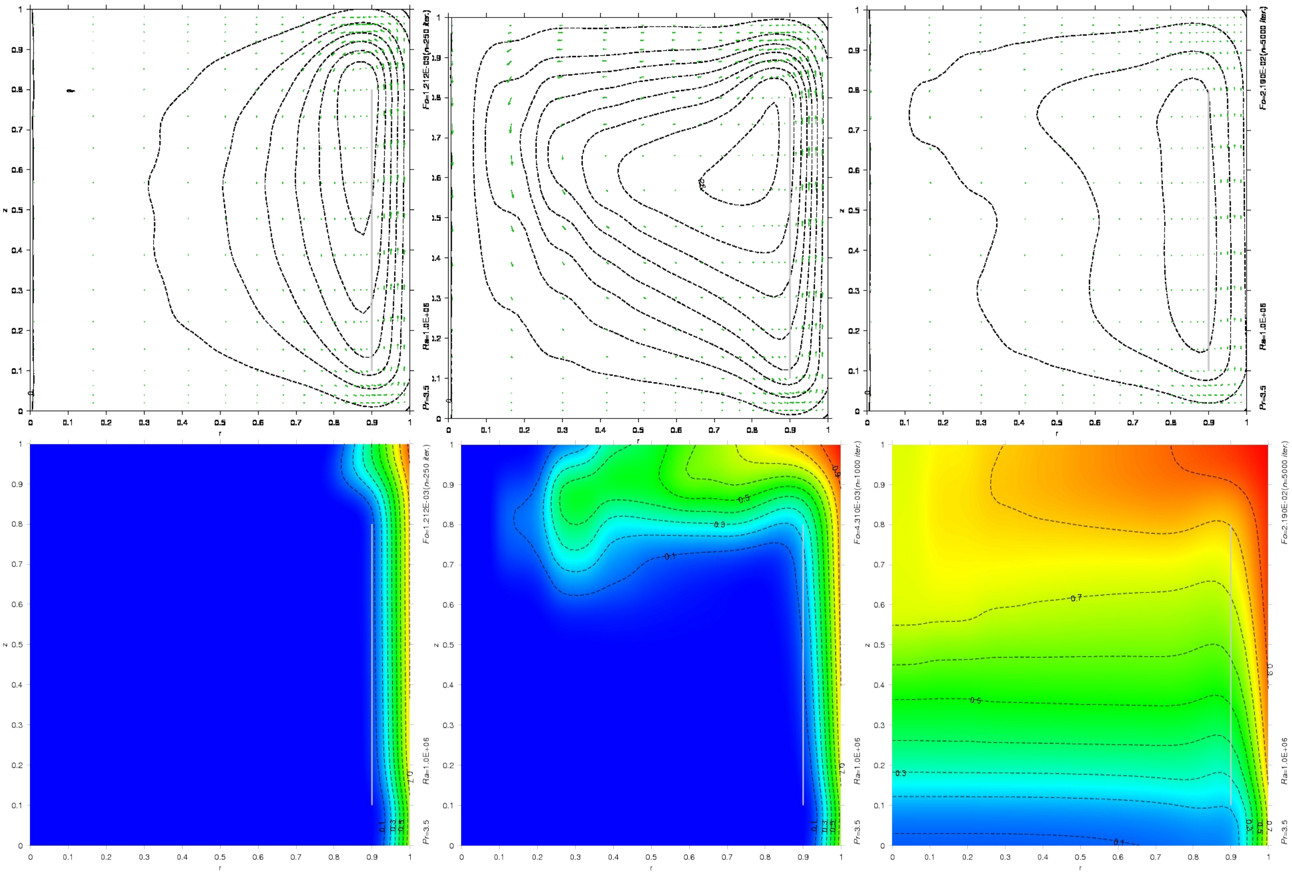

Figure 5: Time development for velocity, stream function and velocity profiles for *d/H=0.1*.

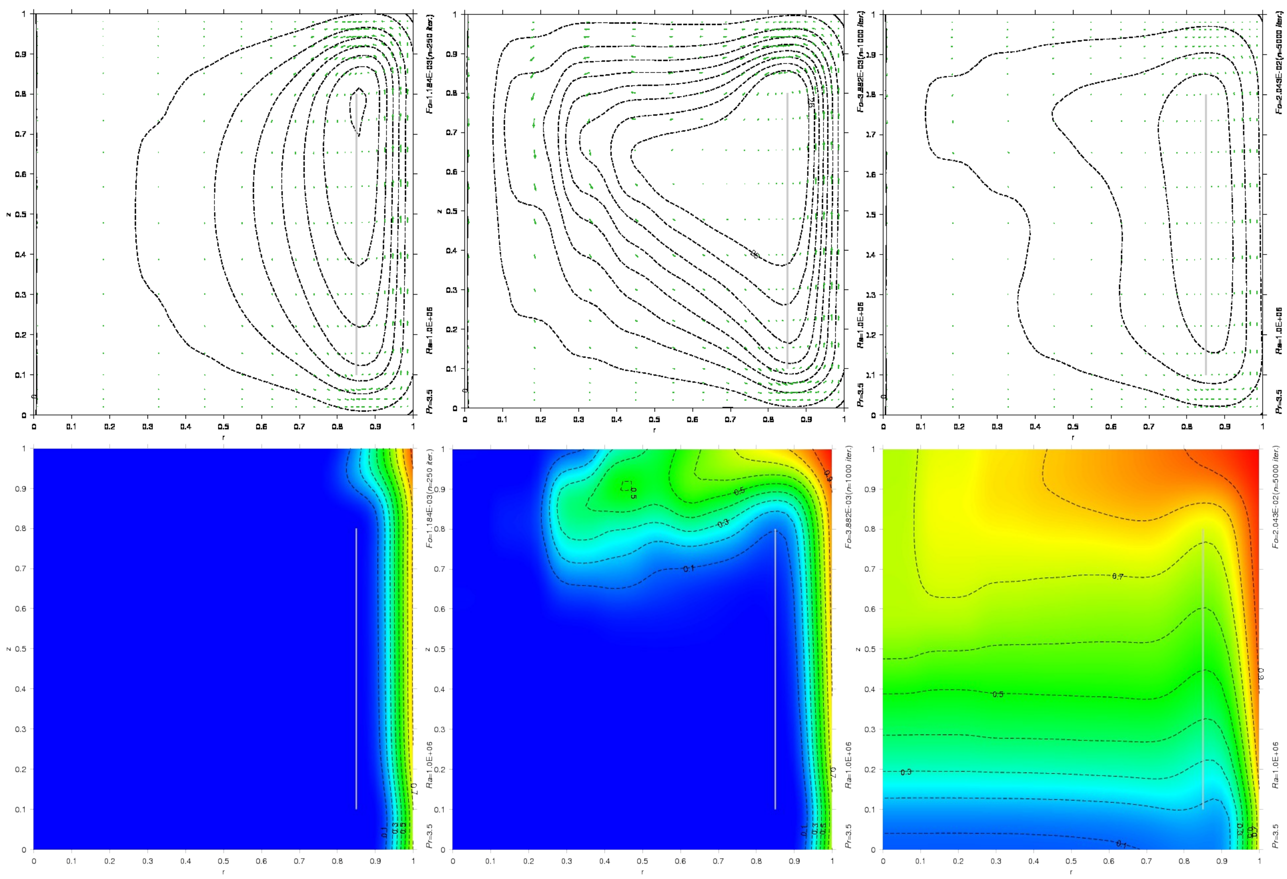

Figure 6: Time development for velocity, stream function and velocity profiles for *d/H=0.15*.

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