NUMERICAL AND EXPERIMENTAL MODEL CORRELATION OF EXTERNAL STORES INTEGRATED IN FIGHTER AIRCRAFTS

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Abstract. *This work details a numerical modeling of external loads to be integrated on an aircraft wing. New stores added to aircraft change its structure and aerodynamic characteristics, what modifies the aircraft aeroelastic behavior. In order to evaluate the influence of external loads on the aircraft structure, the pylon/store set is modeled, initially, as a multibody in free condition. An eigenvalue problem is solved in this condition estimating the natural frequencies and mode shapes. The model is updated using the correlation of dynamic parameters of this coupling extracted from results of an experimental modal analysis. All these procedures aim the application of representative models in fighter aircrafts modernization programs that include the utilization of new external loads.*

Keywords: *multibody systems, experimental modal analysis, dynamic model*

1. INTRODUCTION

The Brazilian Air Force has been upgrading its F-5E fighters, manufactured by Northrop in the seventies. The modernization process considers the integration of new weapon system such as in-house developed external stores. By this way, the structural dynamic characteristics of the aircraft are modified with this modification in its mass distribution.

A comprehensive structural model for the aircraft wing combination is used by (Gern and Librescu, 1998). They have obtained the structural dynamic equations of motion via Hamilton's Variational Principles and the application of generalized function theory to exactly consider the span wise location and properties of attached stores.

A numerical methodology to examine fluid-structure interaction of a wing/pylon/store system has been developed by Cattarious (1999). The wing, pylon, and store data considered in this analysis are based on an F16 configuration that was identified to induce flutter in flight at subsonic speeds. The wing structure is modeled as an elastic plate. In contrast, the pylon and store are taken for rigid bodies. The store body is connected to the pylon through an elastic joint with pitch and yaw degrees of freedom.

Traditionally, the information about the dynamic modification, such as natural frequencies, mode shapes and damping factors are obtained from a ground vibration test (GVT) using all dynamic system (aircraft and its external loads). Sometimes, the aircraft is not available for test. A substructure model using results from an experimental modal analysis (EMA) is proposed by Karpel and Raveh (1996). Only clamped pylon and external store is considered on the model. In order to incorporate the substructure model on the global one, the author adjusts fictitious modal masses.

When the external load has low inertia properties, the pylon can be clamped on the ground, following the methodology presented by (Marto and Alonso, 2005), in order to guarantee a sufficient boundary condition. However, when the inertial properties increase, new mechanisms appear. A rigid structure needs to be built in order to extract appropriate modal parameters as shown Kiessling and Schwochow (2006) by DLR or in Dupuis (2003).

Due to difficulties for obtaining a support with enough stiffness so that it can be considered a rigid assembly, Fernandes and Marto (2008) proposed to substitute the complex rig for a simple one to extract modal parameters from a free-free boundary condition ground vibration test (GVT).

This paper presents a numerical model of pylon/stores to be integrated on the fighter aircraft using results from the free-free boundary condition GVT for update the airframe structural dynamic model for aeroelastic analysis. The pylon launcher and store are considered as rigid body. The pylon and launcher are modelled as a single body. All dynamic stiffness are incorporated in three torsional springs. The equation of motion is based on Lagrange formalism. Small displacement hypotheses simplify the cinematic model. Only three degree of freedom (DOF) are considered, as in Marto and Alonso (2005).

In order to correlate with experimental model, the wing/pylon link is replaced by a simple beam/pylon link. This procedure allows adjustment of spring constants using results from free-free boundary condition GVT results. It allows the incorporation of the appropriated model, with clamped boundary condition, on the aircraft numerical model.

Table 1. Inertial properties and CG position of bodies

	Pylon			**Launcher**			**Missile**		
	x	**y**	**z**	**x**	**y**	**z**	**x**	**y**	**z**
CG Position (m)	-0.15	0	-0.12	-0.46	0	-0.28	-0.92	0	-0.44
Weight (kg)	27			37.5			120.4		
Iyy, Izz (kg m^2)	2.2			16.8			107		
Ixx (kg m^2)	0.075			0.3			0.54		

2. NUMERICAL MODELING

The model shown in Fig. 1 is a rigid body representation of the wing-pylon-store set. Inertial properties and center of gravities of each body are presented in Tab. 1. The beam to which the pylon is fixed takes no part of the model and, initially, shall be neglected.

Considering small displacement approximations, it should be possible to represent the kinematic of the model in function of only three degree of freedom, the generalized coordinate **q**.

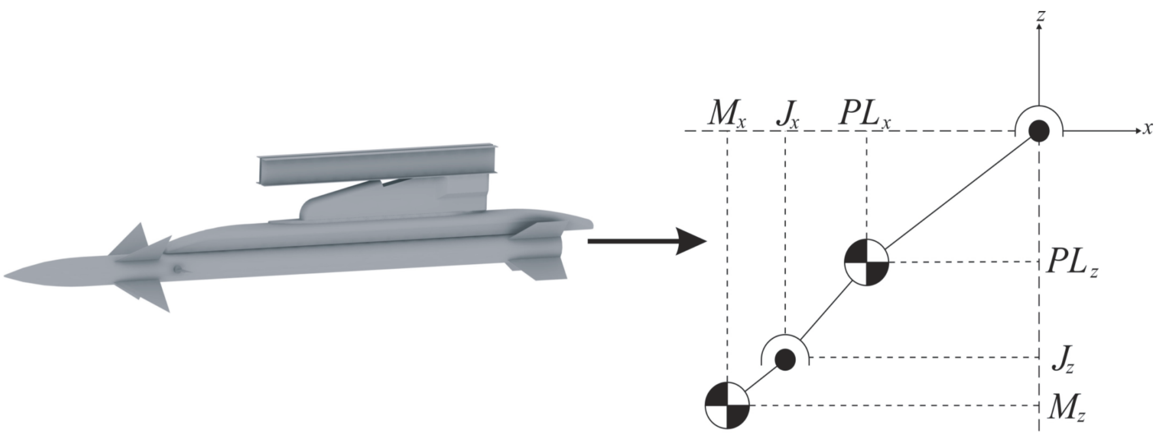
2.1 Pylon-launcher-stores kinematic description

The real model, schematically presented in the Fig. 1a, can be considered as two rigid body linked as in Fig. 1b. The pylon and launcher P is supposed to be a single body rigidly connected, and the other body is the external store S. The connection with the wing shall be represented as a roll torsional spring K_{θ_x} linked to the wing, including also a roll spring link. Although some stores has a flexible behavior, for global aeroelastic model these bodies might be considered as a rigid body. All distributed stores stiffness are incorporated in two connection springs (yaw torsional spring K_{θ_y} and pitch torsional spring K_{θ_z} which connects the launcher with the pylon in a joint J.

Using small displacement approximations, i.e.

$$sin(\alpha) \approx \alpha \text{ and } cos(\alpha) \approx 1 \,, \tag{1}$$

the translational Pylon-Launcher displacements, d_{x_P}, d_{y_P} and d_{z_P}, the translational store displacements, d_{x_P}, d_{y_S} and d_{z_S} and rotational displacements of each link θ_x, θ_y and θ_z can be expressed in function of assumed generalized coordinates



(a) (b)

Figure 1. Schematic of CG position considering pylon and launcher as multibody and rigid body, respectively in the left and in the right

q as:

$$\mathbf{d} = \left\{ \begin{array}{c} d_{x_P} \\ d_{y_P} \\ d_{z_P} \\ d_{x_S} \\ d_{y_S} \\ d_{z_S} \\ \theta_x \\ \theta_y \\ \theta_z \end{array} \right\} = \left[\begin{array}{ccc} 0 & -z_P & 0 \\ +z_P & 0 & 0 \\ 0 & +z_P & 0 \\ 0 & -z_S & 0 \\ z_S & 0 & z_S - x_J \\ 0 & x_S & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] \left\{ \begin{array}{c} \theta_x \\ \theta_y \\ \theta_z \end{array} \right\} = \mathbf{A}\,\mathbf{q}\,, \tag{2}$$

where **A** is a transformation matrix, between two physical spaces of coordinates:

$$\mathbf{A} = \left[\begin{array}{ccc} 0 & -z_P & 0 \\ +z_P & 0 & 0 \\ 0 & +z_P & 0 \\ 0 & -z_S & 0 \\ z_S & 0 & z_S - x_J \\ 0 & x_S & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] , \tag{3}$$

The terms included in the above matrices and vectors refers to localizations of the pylon-to-launcher set center of mass P and Store S, well as of the joint J, in a cartesian coordinate system as described below:

- Pylon-Launcher x_P, z_P,
- Store x_S, z_S and,
- Joint x_J .

2.2 Equation of motion

Using Lagrangean formulation, as described for instance, in Shabana (1991), the equation of motion can be expressed in function of generalized coordinates **q**:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\mathbf{q}}}\right] + \frac{\partial L}{\partial \mathbf{q}} = \mathbf{F}\,, \tag{4}$$

where L is the Lagrangean operator which includes the kinematic energy T and potential energy U,

$$L = T - U\,. \tag{5}$$

Neglecting the gravitational potential energy, the potential energy U becomes only due to spring deformation, and can be represented as:

$$U = \frac{1}{2}\mathbf{q}^T\mathbf{K}\mathbf{q}\,, \tag{6}$$

where **K** represent a stiffness matrix that relates to the torsional spring properties of each link.

The Kinetic energy T written in function of **q** is

$$T = \frac{1}{2}\dot{\mathbf{d}}^T\mathbf{I}_n\dot{\mathbf{d}} = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{A}^T\mathbf{I}_n\mathbf{A}\dot{\mathbf{q}}\,. \tag{7}$$

Replacing the Eq. 7 and 6 on the Eq. 5, the equation of motion can be written as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}\,, \tag{8}$$

where **M** mass matrix that contains the inertia properties of each body through the matrix $\mathbf{I}_n$

$$\mathbf{M} = \mathbf{A}^T\mathbf{I}_n\mathbf{A}\,. \tag{9}$$

The $\mathbf{I}_n$ is a diagonal matrix that relates Pylon-Launcher mass m_P, Store mass m_S and moments of inertia I_{xx}, I_{yy} and I_{zz}:

$$\mathbf{I}_n = \mathtt{diag}\left[\begin{array}{ccccccccc} m_P & m_P & m_P & m_S & m_S & m_S & I_{xx} & I_{yy} & I_{zz}\end{array}\right]\,, \tag{10}$$

F is the generalized force vector.

The eigenvalue problem of Eq. 8 when F is null can be replaced in modal space as in Maia et al. (1997)

$$\mathbf{I}\ddot{\mathbf{q}} + \Lambda \mathbf{q} = 0\,, \tag{11}$$

where the modal matrix are defined as

$$\mathbf{I} = \Phi^T \mathbf{M} \Phi \;\text{ and }\; \Lambda = \Phi^T \mathbf{K} \Phi\,, \tag{12}$$

when Φ is matrix that contains the natural system eigenvectors, normalized by mass, **I** is the identity matrix and Λ is a diagonal matrix which contains eigenvalues, the quadratic angular natural frequency ω^2 of each r mode.

2.3 Adjusting the mode shapes and boundary condition

In most os the cases external stores can be considered as rigid bodies in the sense of aeroelastic modeling, as explained earlier. In fact, the Pylon-Launcher-Store is flexible system that might be approximated by lumped masses to estimate the corresponding system mode shapes and frequencies.

The pitch is associated to a rotational displacement θ_y, the yaw to a θ_z and roll to a θ_x. Analyzing, for each mode shape, the xy plane we can estimate the θ_z by linear regression of store translational displacements. We can also estimate the θ_y by linear regression of store translational displacements, also in the xz plane. The slope between the wing conection point and center of mass of store, considering all of other estimation, is obtained on the yz plane.

Obviously the rotational displacements vectors estimated are not orthogonal among each other. Therefore it is chosen Grand Schmidt method for estimating a orthogonal mode shapes set. Once the mode shape are identified, they are normalized by the model mass matrix **M** corresponding to the decoupled pylon-launcher-store lumped masses combination.

Another issue to be considered, necessary to extract the appropriated dynamic parameters for aeroelastic analysis, are the boundary condition. The difficulties to build the sufficient rigid support to reproduce the clamped boundary condition, lead to the choice of a soft suspension simulating free-free boundary condition.

There is also the need for considering the beam mode shapes coupling to the experimental modal analysis incorporate to the simple model presented before. These beam is considerated as rigid body too. All modal rotational displacements are compensated with modal angles identified for the beam. Therefore, in order to adjust the model for modal analysis the matrix A is:

$$\mathbf{A} = \begin{bmatrix} 0 & -z_P & 0 \\ +z_P & 0 & 0 \\ 0 & +x_P & 0 \\ 0 & -z_S & 0 \\ z_S & 0 & z_S - x_J \\ 0 & x_S & 0 \\ 0 & -z_B & 0 \\ z_B & 0 & 0 \\ 0 & x_B & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{13}$$

where z_B and x_B are the localization of center of mass of beam in cartesian coordinate. Another modification on the model is the Inertia matrix $\mathbf{I}_n$, where it is necessary to consider the inertia of the beam: mass m_B and moments of inertia.

$$\mathbf{I}_n = \mathtt{diag}\left[\begin{array}{cccccccccccc} m_P & m_P & m_P & m_S & m_S & m_S & m_B & m_B & m_B & I_{xx} & I_{yy} & I_{zz} \end{array}\right]. \tag{14}$$

2.4 Extracting the Stiffness matrix

The equation of motion in modal base is expressed as Eq. 12. We suppose the matrix mass **M** has the form as in Eq. 9. The mode shapes are estimated as described in section 2.3 before. The mode shapes are orthogonalized and normalized by the assumed mass matrix **M**. Therefore, the stiffness matrix **K** can be calculated through the inverted modal transformation:

$$\mathbf{K} = \mathbf{M}\Phi\Lambda\Phi^T\mathbf{M}\,. \tag{15}$$

3. EXPERIMENTAL MODAL ANALYSIS

In order to extract the modal parameters from free-free boundary condition GVT, a flexible support was developed as proposed by Fernandes and Marto (2008). The pylon-launcher-store set is fixed to the beam as shown in Fig. 2. The pylon

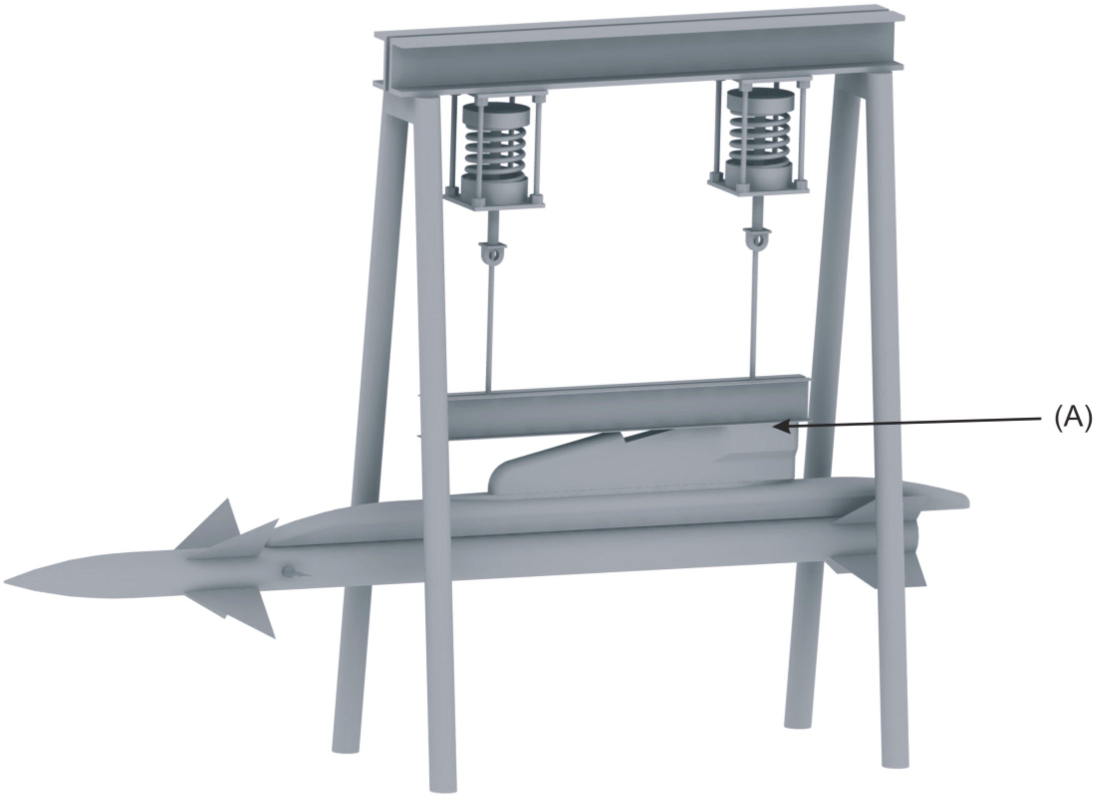


Figure 2. Device to the ground vibration test

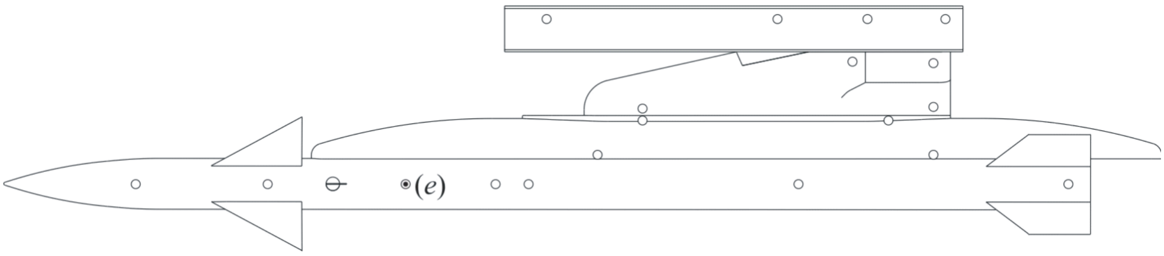


Figure 3. Accelerometers and excitation point location

was clamped to the supporting beam and incorporated to the model in order to maintain the set in a static equilibrium. The beam is the boundary condition incorporated to the model as already explained before. Whole conjunct is hanged through steel cables to a spring mechanism attached in the support.

The pylon-launcher-weapon set was mapped as shown in Fig 3 using 19 accelerometers. A shaker positioned at the indicated points e excited the structure with random input forces.

The accelerations were captured simultaneously with 1024 Hz sampling frequency by means of a Data Acquisition System. In order to avoid the aliasing problem, the signals passed through the 400 Hz low-band pass filter. The frequency response function FRF, $H(\omega)$, is estimated from spectrum analysis of 50 samples as in Eq. 16 below

$$H(\omega) = \frac{S_{fx}(\omega)}{S_{xx}(\omega)} \ , \tag{16}$$

where S_{xx} are the output autospectrum that correspond to accelerations output autospectrum, and S_{fx} are the cross spectrum among input and outputs. The modal shapes are estimated using a dedicated modal analysis software TestLab$^{\text{TM}}$, developed by LMS$^{\text{TM}}$, using the Polimax$^{\text{TM}}$method.

This method adjust the experimental $H(\omega)$ in $z-$domain, as described in (Peeters et al., 2004), through the

$$[H(\omega)] = \sum_{r=0}^{P} z^r [\beta_r] \left[\sum_{r=0}^{P} z^r [\alpha_r] \right]^{-1} \tag{17}$$

where $\left[\beta_r \in \Re^{l\times m}\right]$ are the numerator matrix polynomial coefficients; , $\left[\alpha_r \in \Re^{l\times m}\right]$ are the denominator matrix polynomial coefficients, p is the model order and

$$z = e^{-j\omega\Delta t} \quad . \tag{18}$$

Basically, the unknown polynomial coefficients α_r, β_r are found as the least-squares solution of these equations (after linearization). they are related to the eigenfrequencies ω_r(rad/s) and damping ratios ξ.

The interpretation of the stabilization diagram yields a set of poles ω_r and corresponding participation factors. The orthogonality is verified in order the appropriated modes.

4. RESULTS AND DISCUSSION

4.1 Modal shapes identified

The three first natural frequency identified from the experimental analysis are presented on Tab. 2 and the mode shapes associated can be seen on the Fig. 4.

Table 2. Natural frequencies identified on the experimental analysis

Mode Shape	Frequency [Hz]
first	10.8
second	23.7
third	28.9

4.2 Slopes of Modal Shapes estimated

Applying the linear regression we estimate slopes on the stores for each mode as illustrated in the Fig. 5 and on the beam as in the Fig. 6. After the orthogonalization by Grand-Schmidt procedure, these estimated mode shapes $\Phi_{estimated}$ can be represented as

$$\Phi_{estimated} = \begin{bmatrix} -0.2343 & -0.9722 & 0.0000 \\ -0.0000 & -0.0000 & -1.0000 \\ -0.9722 & 0.2343 & 0 \end{bmatrix} \tag{19}$$

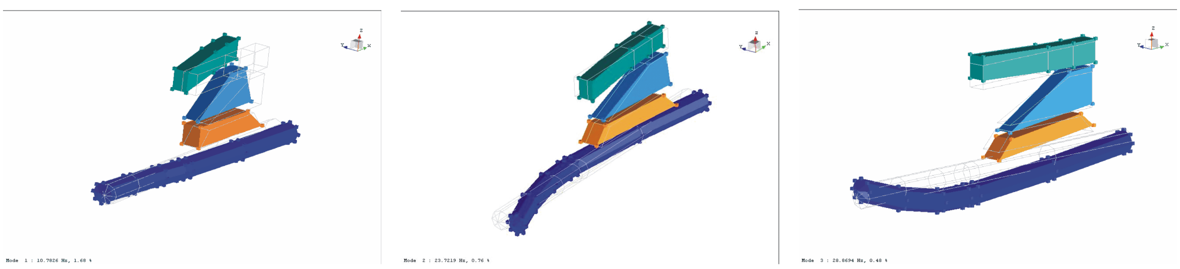
Normalizating these mode shape by mass we obtain:

$$\Phi_{normalized} = \begin{bmatrix} -0.0168 & -0.0393 & 0.0000 \\ -0.0000 & -0.0000 & -0.0704 \\ -0.0698 & 0.0095 & 0 \end{bmatrix} \tag{20}$$

4.3 Stiffness Matrix calculated

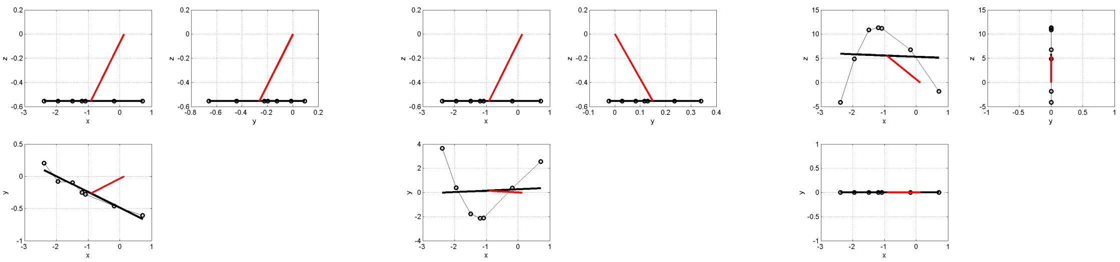
Using the mode shapes estimated from experimental modal analysis and the considered matrix mass from the numeric model, stiffness matrix is calculated as in Eq. 15

$$K = 10^6 \times \begin{bmatrix} 14.9 & 0 & 0.3 \\ 0 & 6.7 & 0 \\ 0.3 & 0 & 0.6 \end{bmatrix} \frac{N\,m}{rad} \tag{21}$$



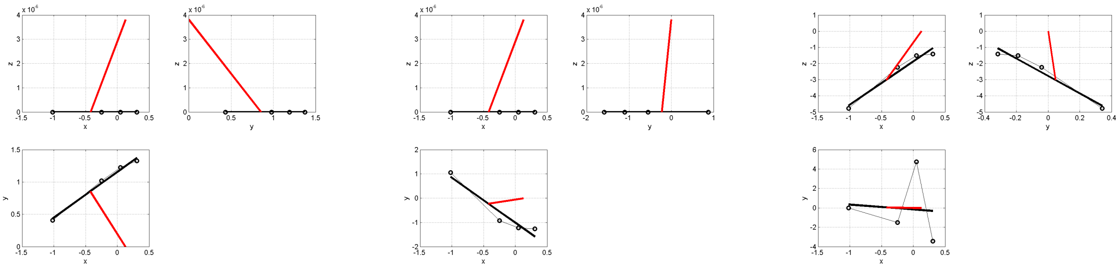
(a) First mode shape 10.8 Hz (b) Second mode shape 23.7 Hz (c) Third mode shape 28.9 Hz

Figure 4. The mode shapes extracted from experimental modal analysis



(a) First mode shape (b) Second mode shape (c) Third mode shape

Figure 5. The store slopes estimated from modal shapes



(a) First mode shape (b) Second mode shape (c) Third mode shape 28.9 Hz

Figure 6. The beam slopes estimated from modal shapes

5. Conclusion

This methodology allows to incorporate the dynamic of flexible Stores in aircraft aeroelastic analysis processes. The estimation of generalized coordinate is possible by a linear regression of translational displacements. The difficulties in order to extract dynamic parameters from a clamped support can be replaced by a free-free boundary condition incorporating a simple and representative part on the numeric model. This part added to the model can be easily subtracted from the model in order to extract the appropriated model to be coupling on the aircraft.

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7. Responsibility notice