A SENSITIVITY ANALYSIS FOR ESTUARINE HYDRODYNAMICS REGARDING MEASUREMENTS OF FREE SURFACE LEVEL AND LONGITUDINAL VELOCITY

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Abstract *This article presents a sensitivity analysis applied to the Macaé estuary that is located at the north coast of the Rio de Janeiro state in Brazil. The two-dimensional model was built upon the MOHID® simulator considering the integration over the depth. A sensitivity analysis of the model was performed regarding measurements of longitudinal velocities, the free surface level taken at two different stations, with the final objective of estimating two hydrodynamic parameters, the turbulent viscosity and the roughness height.*
The basic idea is to evaluate the best approach to be used to solve the inverse problem and determinate the best location to take measurements, if one should use measurements of the free surface level or the velocity field at a certain point and if it would be possible to estimate simultaneously the roughness height and the turbulent viscosity. The first parameter interferes on the friction coefficient that influences the sink term present in the set of equations that constitute the hydrodynamic model, usually being used at its calibration.
The modeled domain included an extension of approximately 20 km, from the head to the outer region of the estuary, at the coast. It was adopted a spatial discretization of 40 m, being created quadratic cells. The bathymetry data of the coast region was taken from the nautical chart 1507, edited by the Brazilian Navy in 1974, while the upper region bathymetry was obtained from Amaral (2003).
As a general guideline, the sensitivity of the state observable variable with respect to the parameters we want to estimate must be high enough in order to allow the estimation of such unknowns within reasonable confidence bounds. Besides, when two or more unknown parameters are sought to be estimated simultaneously they must be uncorrelated, and such behavior can be deduced from the observation of the sensitivity coefficients.

Keywords*: Sensitivity Analysis, Inverse Problems, Water resources, Water Quality, Dynamic Model.*

1. INTRODUCTION

The use of models with different complexity is gaining importance in applications related to environmental and water resources management. Although mathematical and computational models are an abstraction and simplification of the simulated real systems, such models, if properly calibrated and validated, are extremely useful, mainly for their capacity to create different scenarios and to simultaneously manipulate a great number of variables.

The deterministic mathematical and computational models allow the simulation and provide numerical solutions for the hydrodynamics and mass transport phenomena of a given substance in a water body. These models obey to a certain sequence of development and calibration. First, it must be developed a hydrodynamic model (HDM), which can solve the space-time distribution of velocities and water level. This model is superposed with a transport model of conservative substance (TMCS), and finally, a constituent non conservative substance transport model (NCSTM) is developed. The level of empiricism of such models also grows obeying the same sequence. So, the HDM is based on the well established equations from fluid mechanics, and some simplifications are assumed. The TMCS is expressed by an equation where the main uncertainty is the definition of the dispersion coefficients, while in the NCSTM much of the uncertainty involves the functional dependencies of the reactions that represent the removal and production of the substance in the aquatic environment. These last two categories of models are generically called Water Quality Models (WQM).

The set of equations that represents the deterministic model usually is solved numerically, creating what is called numerical simulation. The basic idea of the numerical simulation is the discretization process, which reduces the physical continuum domain with an infinity number of variables to a discretized domain problem, with a finite number of variables, in a way that it can be computationally solved.

In this work the results of a two-dimensional (depth integrated) model, developed using the MOHID® platforms (Water Modelling System, Instituto Superior Técnico – Universidade de Lisboa) are presented. The developed model was applied to the Macaé river estuary, located in the southeast Brazilian coast, in order to perform a sensitivity analysis to study the possibility of estimating both the roughness height and the turbulent viscosity using an inverse problem approach. The basic idea is to evaluate the best approach to be used to solve the inverse problem and answer the following questions:

- Where is the best location to take measurements?
- One should use measurements of the free surface level or the velocity field at a certain point?
- Is it possible to estimate simultaneously the roughness height and the turbulent viscosity?

2. MATHEMATICAL FORMULATION AND SOLUTION OF THE DIRECT PROBLEM

The mathematical formulation adopted to simulate the mass transport in estuaries and rivers usually takes into account the temporal variable. On space, such formulation can be three, two or one-dimensional, depending on the physical and morphological characteristics of the system. The estuarine flow is mainly three-dimensional, as a function of the stratification that takes place in the water column, with the salt water (denser) flowing under the freshwater. However, estuaries subjected to large tidal amplitude (more than 4 meters) and with a relatively small river flow, can be simulated with two-dimensional hydrodynamic equations (Rosman, 1989, Dyer, 1997, Miranda et al., 2002), where it is considered that the flow variables have negligible changes in the depth direction. These equations are

$$\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}+\frac{\partial (vh)}{\partial y}=0 \tag{1}$$

$$\frac{\partial (uh)}{\partial t}+\frac{\partial (hu^2)}{\partial x}+gh\frac{\partial h}{\partial x}-f_0 v=-gh\frac{\partial b}{\partial x}+D_x\frac{\partial^2 u}{\partial x^2}+F \tag{2}$$

$$\frac{\partial (vh)}{\partial t}+\frac{\partial (hv^2)}{\partial y}+gh\frac{\partial h}{\partial y}+f_0 u=-gh\frac{\partial b}{\partial y}+D_y\frac{\partial^2 u}{\partial y^2}+F \tag{3}$$

where h is the free surface level, u and v are respectively the longitudinal and transversal velocity components, g is the gravity acceleration, f_0 is the Coriolis parameter, b is the estuarine depth and F is the source/sink term, that can include parameterization of the momentum generation/dissipation due to wind force or the friction in the bed.

The transport of solute and suspended constituent can be described by a 2D-horizontal advection-dispersion equation, expressed by

$$\frac{\partial c}{\partial t}+\frac{\partial (uc)}{\partial x}+\frac{\partial (vc)}{\partial y}=D_x\frac{\partial^2 c}{\partial x^2}+D_y\frac{\partial^2 c}{\partial y^2}+R \tag{4}$$

where c is the solute or suspended constituent concentration , D_x and D_y are turbulent dispersion coefficients and R takes into account all internal reactions and external sources/sinks that can change the concentration of the simulated constituent.

There are several numerical approaches available for the solution of this set of equations. The MOHID simulator solves these equations with the Finite Volume Method (Versteeg e Malalasekera, 1995), using a mixed upwind-central differences for computing the advective terms, central differences for the dispersive terms and Crank-Nicolson in time. Detailed expositions of the equations, numerical methods and order of accuracy, and algorithms used in MOHID are available at the software official home-page (www.mohid.com), and documents such as Martins (1999) and MOHID (2006).

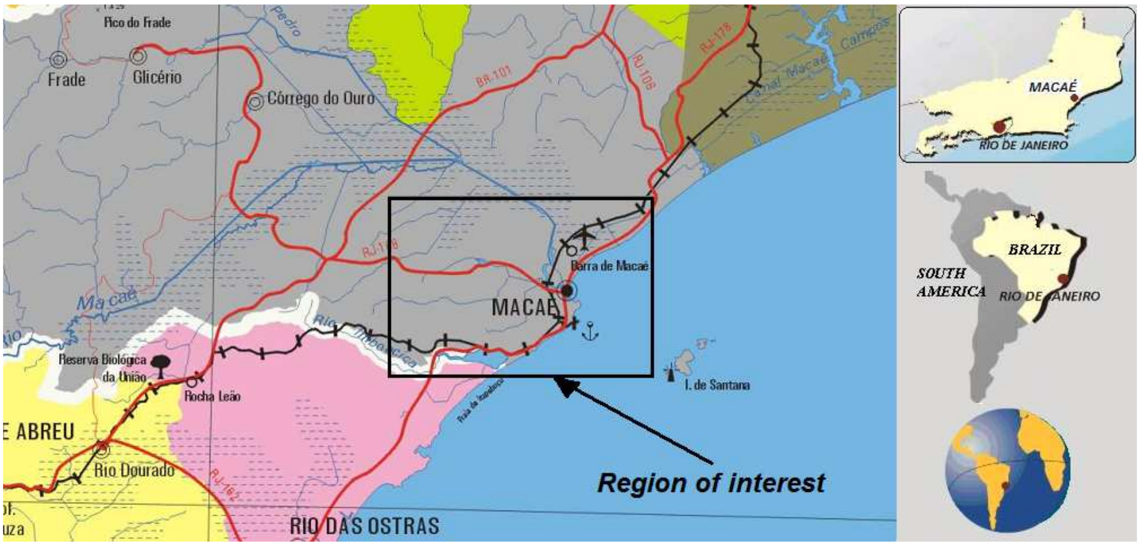

Figure 1 - Estuary localization and detail of simulated region in this study (adapted from Fundação CIDE, http://www.cide.rj.gov.br)

The modeled domain included an extension of approximately 20 km, from the head to the outer region of the estuary, at the coast. It was adopted a spatial discretization of 40 m, being created quadratic cells. The bathymetry data of the coast region was taken from the nautical chart 1507, edited by the Brazilian Navy in 1974, while the upper region bathymetry was obtained from Amaral (2003). Figure 2 shows the discretized domain, and the adopted bathymetry.

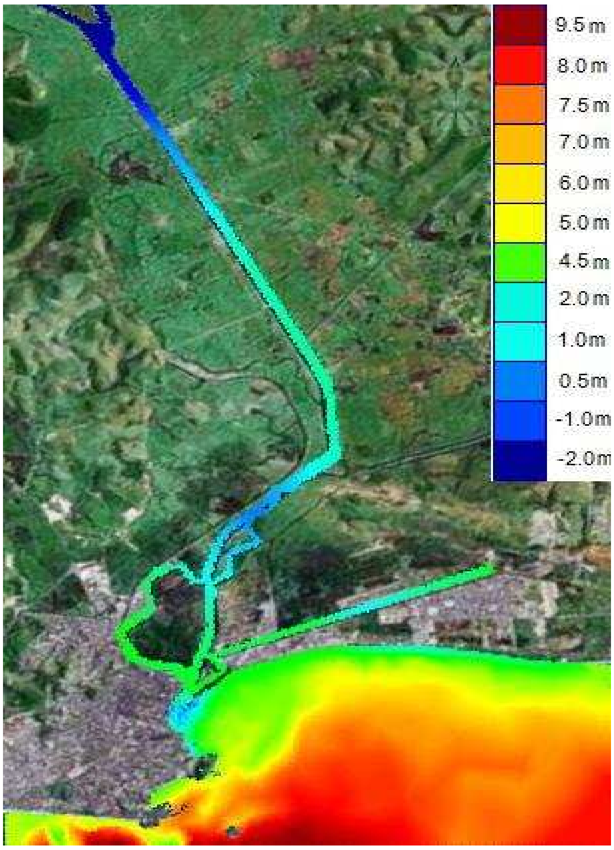

Figure 2 - Simulated domain and model bathymetry used.

Two boundary conditions were informed to the hydrodynamic model: in the riverine boundary it was set a river discharge of 7.8 $m^3\ s^{-1}$, typical of the dry season for the Macaé river close to estuarine region (Amaral, 2004); in the marine boundary it was simulated an astronomic tide with 20 components, of which the amplitude and phase are shown in Table 1.

Table 1 - Period and amplitude of the astronomic sea tide components created by the model

Variable	Period (s)	Amplitude (m)
M2	44714.16	0.369
S2	43200.00	0.191
O1	92949.63	0.100
K1	86164.09	0.059
K2	43082.05	0.054
N2	45570.05	0.046
Q1	96726.08	0.026
M4	22357.08	0.024
P1	86637.21	0.021
MN4	22569.03	0.012
MS4	21972.02	0.011
2N2	46459.35	0.010
M1	89399.69	0.009
M3	29809.44	0.004
MO3	30190.69	0.003
MK3	29437.70	0.002
SN4	22176.69	0.002

2.1 Solution of the Direct Problem

The solution of Eq. 1 to 3, i.e. the hydrodynamic model, gives the temporal variation of the velocity field (u and v) and the free surface (h). The knowledge of the distribution of these variables is important due to many aspects. For example, the erosion/deposition zones can be defined, as well as the potential areas of flooding. Furthermore, the advective transport of any substance is governed by the field of velocities and, finally, the estuary volume itself can be dynamically calculated, with obvious implications on the dilution of substances that eventually reach the water body. The hydrodynamic model is usually calibrated and validated from free level data, which unfortunately is not available for the Macaé estuary.

In order to evaluate the synoptic behavior of hydrodynamic variables, two virtual monitoring stations were positioned alongside the estuary, being referred to as stations 1 and 2, located respectively at 0.5 and 9.51 km from the estuary head, as shown in Fig. 3.

Figure 3 - Simulated domain and the virtual monitoring stations.

Although the model results information of the variables u and v for the velocity field, in this work we considered the resultant of them according to

$$vel = \sqrt{u^2 + v^2} \tag{5}$$

where vel represents the resultant of both horizontal velocities.

Results obtained with this model are presented at Lima, et al (2008) and Rodrigues et al (2009).

Figures 4 and 5 show the synoptic variation of the surface level, the velocity field, calculated by the model, respectively at stations 1 and 2. The simulation period was 28 hours (10^5 s), including a spring tide, which represent the higher tidal amplitude within a given 28 days period of time. The dynamic limit of influence of the sea level oscillation is near station 1, located at 0.5 km from the estuary head.

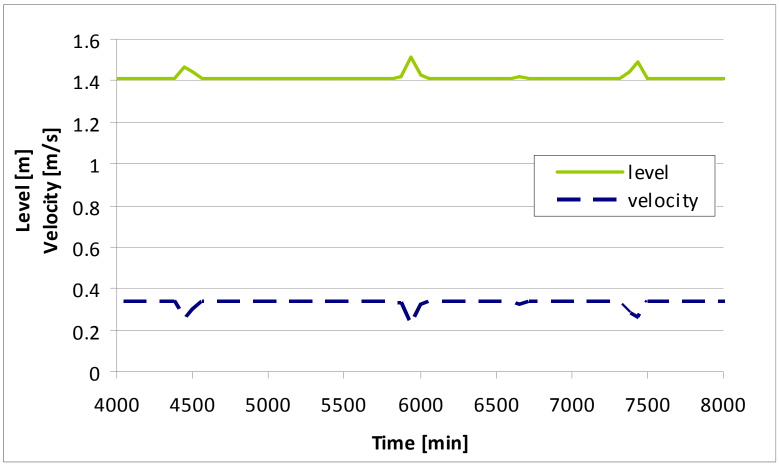

Figure 4 - Hydrodynamic variables behavior at monitoring station 1.

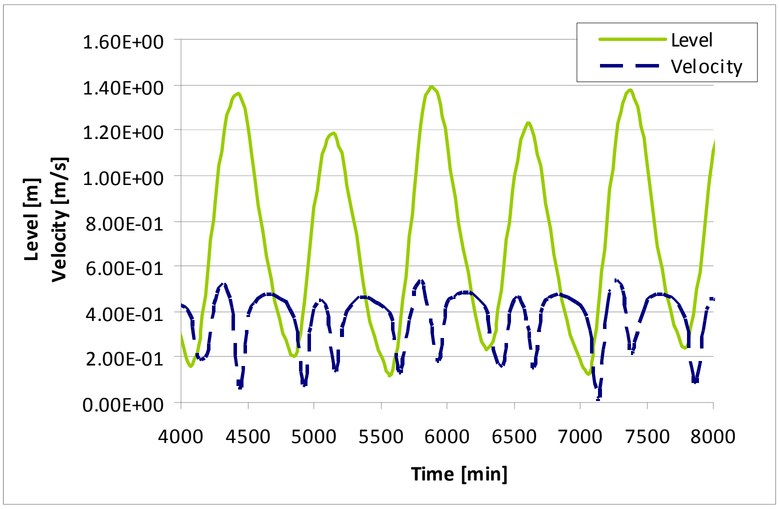

Figure 5 - Hydrodynamic behavior at monitoring station 2.

3. SENSITIVITY ANALISYS

At this point a sensitivity analysis of the model is presented regarding the hydrodynamic parameters “roughness height” and turbulent viscosity. The first parameter interferes on the friction coefficient that influences the sink term present in the set of equations that constitute the hydrodynamic model, usually being used at its calibration.

The sensitivity analysis plays a major role in several aspects related to the formulation and solution of an inverse problem (Lugon Jr. et al., 2008). The basic idea here is to evaluate the possibility of estimating the parameters of interest using measured values of longitudinal velocity or free level surface at station 1 or 2. Such analysis may be performed with the study of the sensitivity coefficients. Here we use the modified, or scaled, sensitivity coefficients defined as

$$X_P^{V_i}(t_i) = P\frac{\partial V_i(x,y,t_i)}{\partial P}, \qquad i = 1, 2, \cdots, M \tag{6}$$

where V_i with $i = 1,2,\ldots,M$, is the observable variable, that is, a particular measurement of longitudinal velocity or free surface level, M is the total number of measurements, and P, is a particular unknown of the problem, which in the situations of interest in the present work is the roughness height or the turbulent viscosity.

As a general guideline, the sensitivity of the state observable variable with respect to the parameters we want to estimate must be high enough in order to allow the estimation of such unknowns within reasonable confidence bounds. Besides, when two or more unknown parameters are sought to be estimated simultaneously they must be uncorrelated, and such behavior can be deduced from the observation of the sensitivity coefficients (Dowding, et al., 1999).

In Figs. 6 and 7 are represented the sensitivity coefficients for the estimation of the roughness height, r, considering measurements of the longitudinal velocity or free surface level at stations 1 and 2. From this analysis it was decided to use the measurements of the free surface level, which is easier to obtain in the field, at the same time being as much sensitive as the velocity measurements. Regarding the sampling site, station 2 showed to be quite more sensitive than station 1 to variables fluctuations.

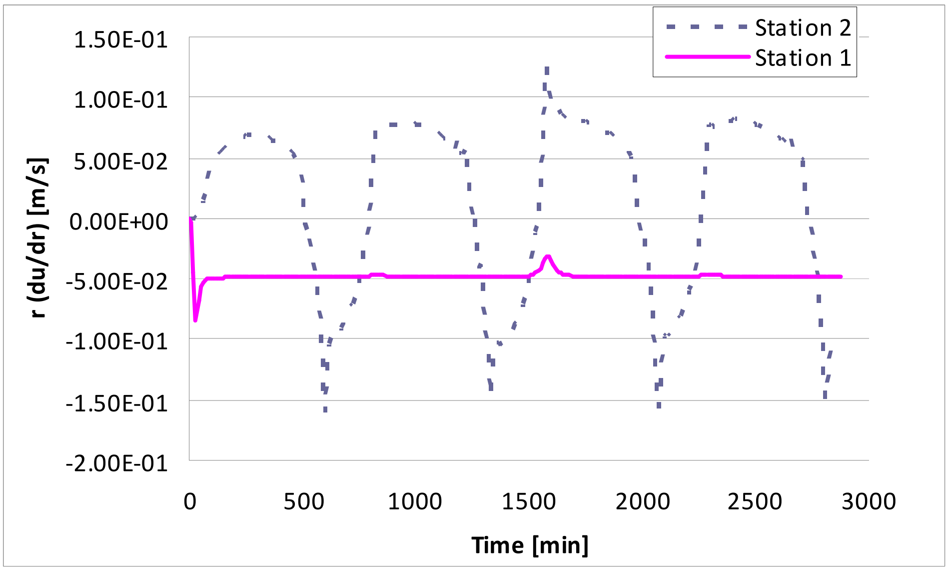

Figure 6 - Sensitivity coefficients for the estimation of the roughness height using velocity measurements.

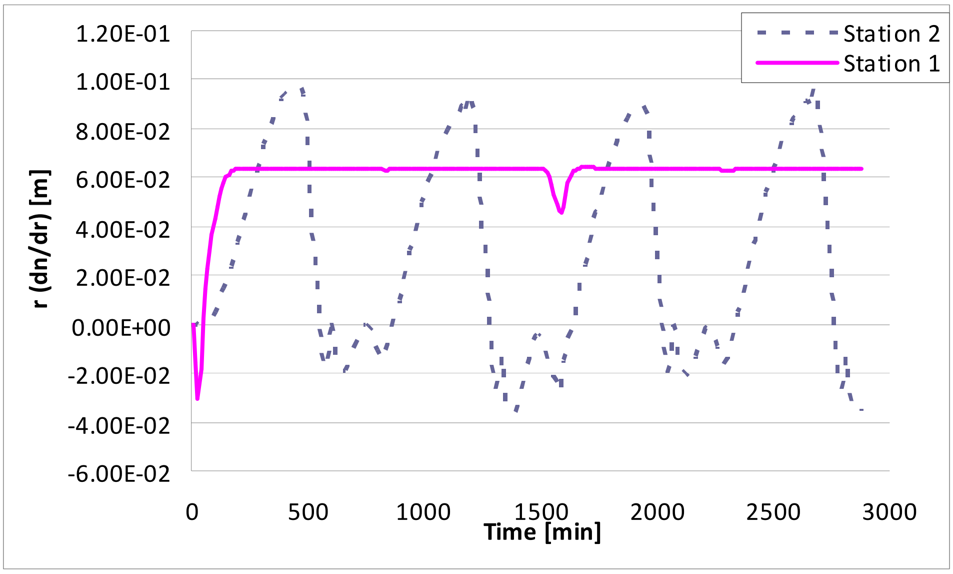

Figure 7 - Sensitivity coefficients for the estimation of the roughness height using free surface level measurements.

As expected the sensitivity coefficients calculated for Station 1 are much lower than those for Station 2. Therefore, we concentrate our analysis on the information provided by the latter.

In Fig. 8 are represented the sensitivity coefficients for the estimation of the turbulent viscosity, T, considering measurements of velocity and free surface level at Station 2. Again it is better to use the free surface level measurements because of the same reasons, that is, the sensitivity is almost the same and it is much easier to measure the free surface level.

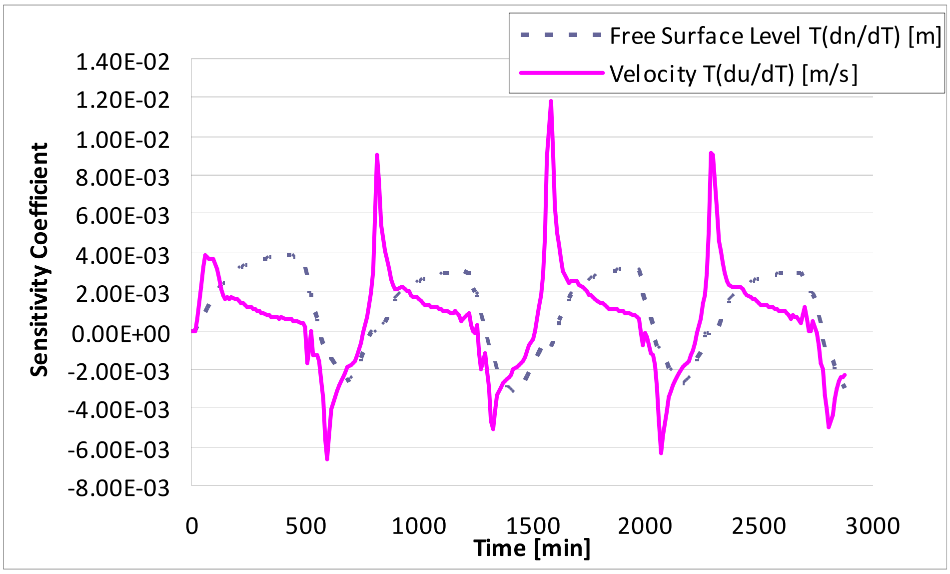

Figure 8 - Sensitivity coefficient for estimating the turbulent viscosity considering measurements taken at Station 2.

In Fig. 9 are represented the sensitivity coefficients for the simultaneous estimation of both the turbulent viscosity (T) and the roughness height (r) considering measurements of velocity and free surface level at Station 2. Then, it is concluded that it is not possible to perform the simultaneous estimation of both parameters, mainly because the sensitivity for the turbulence coefficient is too low and from the graphics analysis it seems to be correlated to the sensitivity for the roughness height.

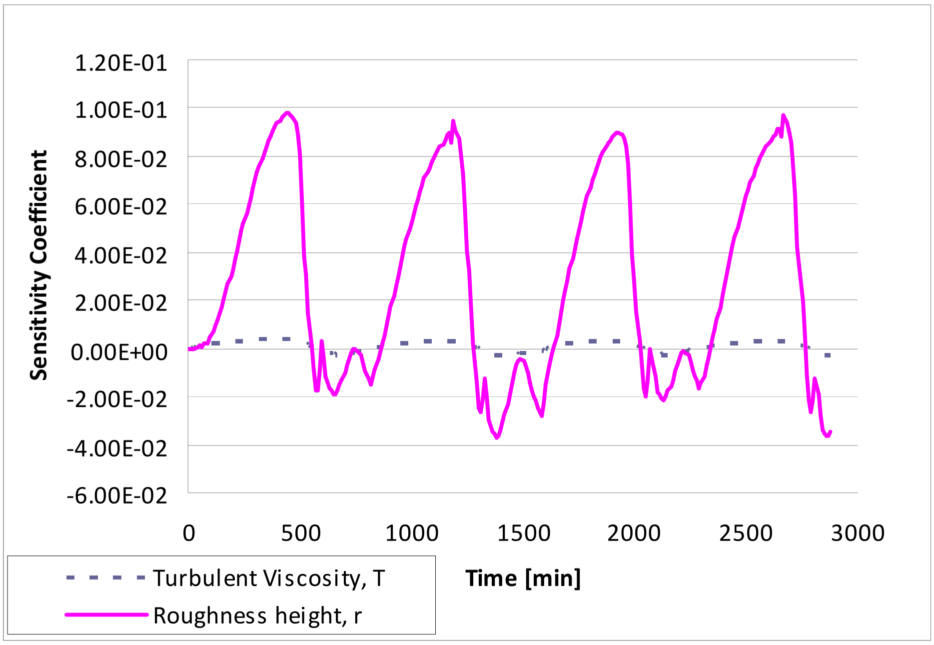

Figure 9 - Sensitivity coefficients considering measurements of free surface level taken at Station 2.

4. CONCLUSIONS AND FUTURE WORKS

From the results presented for the sensibility coefficients, we are able to conclude that it may be possible to perform the estimation of the roughness height that interferes on the friction coefficient using measurements of the free surface level taken at a certain point of the domain. On the other hand, the simultaneous estimation of the turbulence coefficient is not an easy task because the sensitivity to that parameter is too low and its effect seems to be correlated to the roughness height.

This is an on going study and we are going to investigate other possibilities, such as using salinity measurements to improve sensitivity. The next step will be to perform the estimation of the parameters of interest considered unknowns using simulated experimental data, that is, to use the solution of the direct problem corrupted with a certain level of noise. After those studies, real experimental data are going to be used.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

Amaral, K., 2003, Macaé Estuary: Modelling as a Tool for an Integrated Management of Water Resources, M.Sc. Thesis, COPPE/UFRJ, Rio de Janeiro, RJ, Brazil, 150 pp. (in Portuguese).

Baird, J.I. and Whitelaw, K., 1992, Water Quality Aspects of Estuary Modelling. In Roger A. Falconer (Ed.) Water Quality Modelling. Ashgate Publishing Limited, Aldershot, 139 pp.

CONSÓRCIO INTERMUNICIPAL MRA-5. 2004, The Design of Macaé River (RJ) Water Basin Plan, Rio de Janeiro, 52 p, (in Portuguese).

Dyer, K., 1997, Estuaries. A Physical Introduction. John Wiley & Sons Ltd., London, 195 pp.

Dowding, K. J., Blackwell, B. F. and Cochran, R. J., 1999, “Applications of Sensitivity Coefficients for Heat Conduction Problems”, Numerical Heat Transfer, Part B, v. 36, pp. 33-55.

Lima, Edgar B., Silva Neto, Antônio J., Rodrigues, Pedro P. G. W., 2008, “Simulação bidimensional do escoamento e transporte no estuário do rio macaé”, Encontro de Modelagem computacional, Volta Redonda, Brazil.

Lugon Jr., J. Silva Neto, A.J. and Rodrigues, P.P.G.W., 2008, Assessment of Dispersion Mechanisms in Rivers by Means of an Inverse Problem Approach, Inverse Problems in Science and engineering, Vol. 16, No. 8, pp. 967-979.

Martins, F.A.B.C., 1999, Modelação matemática tridimensional de escoamentos costeiros e estuarinos usando uma abordagem de coordenada vertical genérica, Ph. D, Thesis, Universidade Técnica de Lisboa, Instituto Superior Técnico.

Miranda, L. B., de Castro, B.M. and Kjerfve, B., 2002, Estuaries Physical Oceanography Principles, Editora Universidade de São Paulo, São Paulo, 414 pp. (in Portuguese).

MOHID, 2006, Mohid Description, pp. 1-122, MARETEC – Marine and Environmental Technology Research Center, Instituto Superior Técnico – IST, Technical University of Lisbon, Portugal, pp. 1-122. Disponível em <http://maretec.mohid.com/PublicData/Products/Manuals/Mohid_Description.pdf>. Acesso em 05/08/2009.

Rodrigues, P.P.G.W., Lima, E.B., Silva Neto, A.J., Lugon Junior, J., 2008, “Modelling and sensitivity analysis for estuarine hydrodynamics”, IX Simposio Internacional de Automatización, Havana, Cuba.

Rosman, P.C., 1989, Circulation Models in Shallow Water Bodies, In R.C. Vieira da Silva (Ed.), Numerical Methods in Water Resources, ABRH, Rio de Janeiro, 381 pp. (in Portuguese).

Versteeg, H.K. and Malalasekera.,W., 1995, Computational Fluid Dynamics, An Introduction to Finite Volume Method. Logman Group, London, 257 pp.

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