

ASSESSMENT OF THE FATIGUE STRENGTH OF NOTCHED SPECIMENS BY THE STAIRCASE METHOD

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Abstract. The presence of notches and other stress concentration in turbine blades and other notch hydraulic components is a current problem in engineering. It causes a reduction of endurance limit of material. In that sense, specimens of the ASTM A743 CA6NM alloy steel using in several hydrogenator turbine components was tested. The specimens were tested under uniaxial fatigue loading with a load ratio equal to -1, and the considered stress concentration factors, K_t , values, calculated with respect to net area, were 1.55, 2.04 and 2.42. In order to determine the fatigue limit for such notch type, a reduction data method by Dixon and Mood, Staircase method was used. This approach is based on the assumed target distribution of the fatigue limit. For such geometry at least 8 specimens were tested. In addition, the Peterson and Neuber's notch fatigue factor were compared through fatigue notch reduction factor, K_f , obtained from experimental data. According to results obtained it was possible to conclude that the tested material is less sensitive to notches than the prediction of the Peterson and Neuber's empirical models.

Keywords: Notch, fatigue strength, staircase method, up and down method, ASTM A743 CA6NM

1. INTRODUCTION

The fatigue analysis in turbine blades and other hydraulic notch components is a very important problem because these components are designed to operate below its endurance limit and because fatigue failures in service invariably occur at stress concentrations. In the cases that the loading is dynamic, the ductile materials behave as brittle materials due to the fatigue. Therefore, the stress concentration factor should be modified according to material sensitivity to geometric discontinuities. In that sense, appears the fatigue notch reduction factor, K_f . It establishes an important transformation factor that relates the fatigue strength of notch specimen, S_f , with the fatigue strength of unnotched specimen, S_{fe} . This relation can be expressed by Eq. (1), where q is the notch sensibility factor and $0 \leq q \leq 1$. However, the most definition accepts is the relation showed by Eq. (2). Experimental investigations indicate that the K_f value is practically unaffected for 10^6 to 10^8 cycles. In addition, when the fatigue life is less than 10^6 cycles, the K_f value decrease quickly with respect the cycle number (Buch, 1988).

$$K_f = 1 + q(K_t - 1) \quad (1)$$

$$K_f = \frac{S_f}{S_{fe}} \quad (2)$$

In the last 40 years many expressions have been developed to the notch fatigue notch reduction factor, K_f . According to its consideration, they can be classified in three models: (i) Average stress models, (ii) Fracture mechanic models and (iii) Stress field intensity models (Yao *et al.*, 1994). In this work, we work only average stress models: Neuber and Peterson's relation. The first model shows by Kuhn and Hardraht (1952) became base for average stress models. This model assumes the fatigue failure occur when the average stress about characteristic length from root notch equals the endurance limit of a smooth specimen S_f . The Eq. (3) presents an obtained expression by Kuhn and Hardraht, where ρ is root notch radius, w is notch opening angle, and A is a material constant that is a function of the tensile strength.

$$K_f = 1 + \frac{K_t - 1}{1 + \frac{\pi}{\pi - w} \sqrt{\frac{A}{\rho}}} \quad (3)$$

Neuber (1961) formulated the Eq. (3) as the Eq. (4), where $a_p = f(\sigma_n)$ is a constant material that can be quantified in function of tensile strength, S_{rt} , for steels with $S_{rt} > 550$ MPa, according to Eq. (5)

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{a_N}{\rho}}} \quad (4)$$

$$a_p = \begin{cases} 0,185 \cdot \left(\frac{700}{S_{rt}}\right), & S_{rt} < 700MPa \\ 0,0254 \cdot \left(\frac{2079}{S_{rt}}\right)^{1,8}, & S_{rt} > 700MPa \end{cases} \quad (5)$$

Peterson (1959) assumed that the fatigue failure occur when the stress at distance d_0 from root notch is equal or more than limit fatigue strength of a smooth specimen. Obviously, the Peterson's model is a particular case of average stress model. However, Peterson proposed that the stress near to notch decrease linearly. The Eq. (6) express this relation, where a_N is a constant material and it can be estimate in function of tensile strength according to Eq. (7).

$$K_f = 1 + \frac{K_t - 1}{1 + \left(\frac{a_p}{\rho}\right)} \quad (6)$$

$$a_N = 10^{\frac{134 - S_{rt}}{586}} \quad (7)$$

2. EXPERIMENTAL PROCEDURE

2.1. Material

The test material was a steel alloy, ASTM A743 CA6NM, which has been used in the fabrication of hydraulic turbine components and it requests high mechanical strength and that it resist the corrosion. Its chemical properties according to ASTM A 743/A 743M (2006) are showed in Tab. (1). The mechanical (Young modulus, E , tensile strength, S_{rt} , and yield strength, S_y) and fatigue properties based on Parallel-projected method according to and DaSilva *et al.* (2009a) are showed in Tab. (2) and Tab. (3), respectively.

Table 1. Chemical properties of ASTM A743 CA6NM alloy steel (ASTM A 743/A 743M, 2006)

Composition (%)							
C	Mn	Si	Cr	Ni	Mo	P	S
≤0.06	≤1.00	≤1.00	11.5-14	3.5-4.5	0.4-1.0	≤0.04	≤0.03

Table2. Mechanical properties of ASTM A743 CA6NM alloy steel

E (GPa)	S_y (MPa)	S_{rt} (MPa)	Hardness (HB)
198 ± 4	575 ± 35	918 ± 1	273.0 ± 7.0

Table 3. Basquin's fatigue parameter obtained of Parallel-projected method

Parameter	Estimate value		Confidence intervals	
	Estimate	Deviation	Lower	Upper
A	1659.1	116.4	1416.3	1901.9
b	-0.108	0.006	-0.120	-0.097

According to Parallel-projected method for fatigue life of $2 \cdot 10^6$ cycles, the endurance limit is 344.00 ± 24.13 MPa (DaSilva *et al.*, 2009a). However, DaSilva *et al.* (2009b) predict 360.1 ± 14.0 MPa according to Staircase method. Therefore, this is the fatigue limit used in this work. The specimens were produced in accordance with ASTM E 466-96 (ASTM, 2002) and forged into flat plates as shows the Fig. (1) and its geometric parameters are shown in Tab. (4). In addition, Tab. (4) presents the analytic stress concentration factor calculated by Eq. (8), (Young and Budynas, 2002).

$$K_t = 3.065 + 3.370 \left(\frac{2a}{c} \right) + 0.647 \left(\frac{2a}{c} \right)^2 + 0.658 \left(\frac{2a}{c} \right)^3 \quad (8)$$

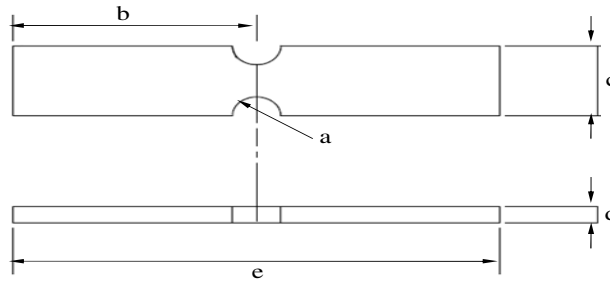


Figure 1. Flat plate specimen

Table 4. Specimen geometric data

K_t	a (mm)	b (mm)	c (mm)	d (mm)	e (mm)	Net area (mm ²)
2.42	3.00	80.00	30.00	7.50	160.00	105.00
2.04	5.00	80.00	30.00	7.50	160.00	150.00
1.55	8.00	80.00	30.00	7.50	160.00	180.00

2.2. Staircase method

The Staircase method has been recommended by many standards (British Standard Institution, 1966; Japanese Society of Mechanical Engineers, 1981, L'association Francaise de Normalisation, 1991) to evaluate the fatigue limit statistical properties. Historically, its origin is associated to biological assay. Biological assay is a set of techniques used in comparison of alternative but similar biological stimuli. In other words, it is basically the measurement of the potency of any stimulus by observing the reaction that is produces in a living organism (Finney, 1971). In both cases, one want to determine the level stimulus, stress or dose, for that an acceptable proportion of specimens survives.

Two methods of data reduction for statistical properties of fatigue strength at a specific fatigue life are most used: Dixon-Mood (1948) and Zhang-Kececioglu (1998). Both approaches are based on maximum likelihood estimation and assume target distribution of the fatigue limit, Normal and Weibull, respectively. According to Lin *et al.* (2001), the Dixon-Mood method (DM) provides better and more conservative predictions than Zhang-Kececioglu method (ZK). Therefore it will be used in this work.

The DM method was popularized by Little (1975). It uses a simple systematic methodology where the specimen is tested at initial stress for a specific fatigue life. Initially, the fatigue limit and its standard deviation are estimate, for example, through of Parallel-projected or S-N curve. If the specimen fails before to infinite life (say $2 \cdot 10^6$ cycles), the next specimen will be tested at a lower stress level. Otherwise, a new test will be conducted at upper stress level. In that way, each test depends on the previous test results and the experiment continues in this manner in sequence with the stress level being increased or decreased (Lin *et al.*, 2001).The stress increments are usually constant and are in the range of half to twice the standard deviation of the fatigue limit. Lee *et al.* (2005) recommends a value 5% less than fatigue limit initially estimated. Collin (1993) recommends running the test at least 15 specimens.

2.3. Statistical analysis

The DM method provides approximate formulas to calculate the mean, μ_{DM} , and standard deviation, σ_{DM} , of the fatigue limit. It requires that the two statistical properties be determined by using the data of the less frequent event, either only the survivals or only the failures.

The stress levels S spaced equally with a chosen increment d are numbered i where $i=0$ for the lowest stress level S_0 . The equations proposed by Dixon and Mood (1948) respect three assumptions: (i) the fatigue strength should be normally distributed; (ii) the sample size should be big, around 40 to 50 specimens or more and (iii) the standard deviation of normal distribution should be grossly estimated previously in order to specify the step of stress increments. However, Brownlee *et al.* (1953) assures that the samples from 5 to 10 specimens are reliable to determine the mean fatigue limit.

Denoting by n_i the number of the less frequent event at the stress level i , two quantities can be calculated: A and B , Eqs. (8) and (9), respectively.

$$A_i = \sum in_i \quad (8)$$

$$B_i = \sum i^2 n_i \quad (9)$$

The Equation (10) shows the estimate of the mean, where the plus sign is used if the more frequent event is survival and otherwise, it is used minus sign. The Eqs. (11) and (12) show the estimate of standard deviation.

$$\mu_{DM} = S_0 + d \left(\frac{A}{\sum n_i} \pm \frac{1}{2} \right) \quad (10)$$

$$\sigma_{DM} = 1.62d \left[\frac{B \sum n_i - A^2}{(\sum n_i)^2} + 0.029 \right] \text{ if } \frac{B \sum n_i - A^2}{(\sum n_i)^2} \geq 0.3 \quad (11)$$

or

$$\sigma_{DM} = 0.53d \text{ if } \frac{B \sum n_i - A^2}{(\sum n_i)^2} < 0.3 \quad (12)$$

The Staircase methods are notably accurate and efficient in terms to quantify the mean fatigue strength but very difficult to predict estimate accurate of fatigue limit standard deviation using these methods with small samples at high cycle fatigue (Pollak *et al.*, 2006). This method concentrates the most experimental points near the mean therefore is more difficult to obtain an accurate standard deviation. Braam and van der Zwaag (1998), Svensson e de Maré (1999), Lin *et al.* (2001) e Rabb (2003) worked in order to evaluate and to improve the reliability of standard deviation and proposed a linear correction factor and found to be an improvement in all maximum-likelihood evaluation procedures.

The Eq. (13) shows the estimate of standard deviation corrected by Svensson-Lóren, σ_{SL} , where σ_{DM} is the standard deviation by Dixon-Mood and N is the total specimen number. This correction is a strictly function of sample size and tend increase the deviation estimate by Dixon-Mood.

$$\sigma_{SL} = \sigma_{DM} \left(\frac{N}{N-3} \right) \quad (13)$$

A modified correction was developed which attempted to allow a greater range of unbiased estimation than the Svensson-Lóren correction. The form of the proposed standard deviation estimate, σ_{PC} , is shown in Eq. (14), where A , B , and m are constants dependent on the number of samples, see Tab. (5).

$$\sigma_{PC} = A \sigma_{DM} \left(\frac{N}{N-3} \right) \left(B \frac{\sigma_{DM}}{d} \right)^m \quad (14)$$

In this work the largest deviation will be used, σ_{SL} or σ_{PC} .

Table 5. Constants used in proposed standard deviation correction (Pollak et al., 2006)

Specimens # (<i>N</i>)	<i>A</i>	<i>B</i>	<i>m</i>
8	1.30	1.2	1.72
10	1.08	1.2	1.10
12	1.04	1.2	0.78
15	0.97	1.2	0.55
20	1.00	1.2	0.45

2.4. Fatigue testing

In order to determine experimentally the fatigue limit of notched materials for 2.10^6 cycles, all testing was performed at a stress ratio, *R*, of -1 at a frequency of 15 Hz with a universal servo-hydraulic MTS machine. The Tab. (6) shows the standard deviation number used to determine the class of Staircase method; the quantity of class used, *S*; the step size, *d*; the percentile between step size and material endurance limit for 2.10^6 cycles. In addition, the upper and lower staircase limit intervals are presented too in the same table. It can be observed from Tab. (6) that the stress increments are less than 5% of fatigue limit initially estimated by Staircase method in DaSilva *et al.*, (2009b) as Lee *et al.*, (2005) recommends.

Table (6). Experimental parameters of Staircase method

Notch radius (mm)	Deviation	Class	<i>d</i> (mm)	<i>d</i> / σ_f (%)	Upper lim. (MPa)	Lower lim. (MPa)
3	1.6	10	3.99	1.1	190.2	154.3
5 ₁	1.6	10	5.47	1.5	223.0	173.8
5 ₂	2.2	10	1.82	0.5	228.5	212.1
8	1.6	10	11.18	3.0	303.2	202.6

¹ first testing stage and ² second testing stage

3. RESULTS AND DISCUSSION

3.1. Endurance limit for notch radius of 8 mm

Three experimental geometric conditions were tested in this study and the results are shown in Tab. (9). Figure (2) plots the Staircase testing results for notch radius of 8 mm. The fatigue limit for this case is 355.1 ± 19.1 MPa.

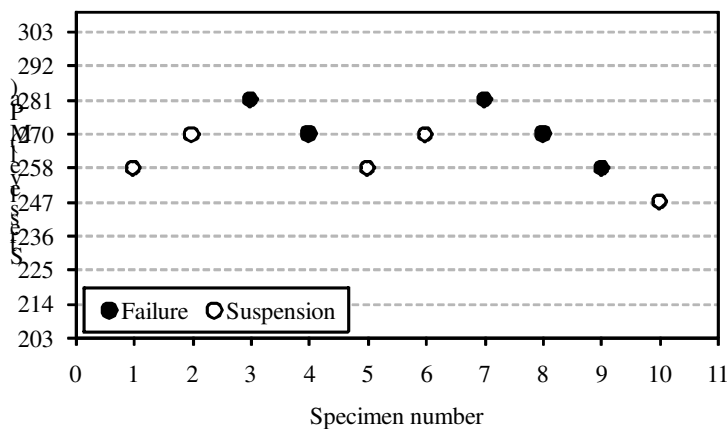


Figure 2. Plot of Staircase testing results for notch radius of 8 mm for 2.10^6 cycles

3.2. Endurance limit for notch radius of 3 mm

Figure (3) plots the Staircase testing results for notch radius of 3 mm. The average fatigue limit and its scatter determined by Dixon-Mood and Svensson-Lören equations, respectively, for this case are 184.2 ± 2.6 MPa. In order to reduce experimental time, the tests started from at bigger class. It can be observed that this class corresponds 1.6

standard deviation above average fatigue strength estimate, the probability of experimental limit fatigue is under is more than 89% for a normal distribution. In addition, from second specimen to eighth specimen the experimental behavior is regular and constant. It can be verified that experimental results for the specimens from 1 to 5 is statistically similar to results obtained with specimens 6 to 10. Therefore, in this case five specimens were sufficiently to determine the average fatigue limit but with less accuracy in the standard deviation.

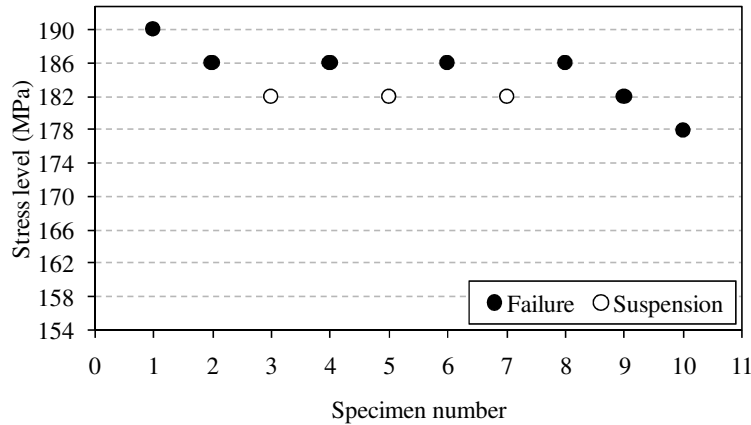


Figure 3. Plot of Staircase testing results for notch radius of 3 mm

3.3. Endurance limit for notch radius of 5 mm

Starting from the obtained results previously the Staircase method for notch radius was executed at two stages. Firstly, five specimens were used and the endurance limit determined was 220.3 ± 3.6 MPa. Figure (4) shows the testing results. Following, the result was refined and more five specimens were tested as it can be seen in Fig. (5). The fatigue limit take account second stage is 214.6 ± 1.2 MPa.

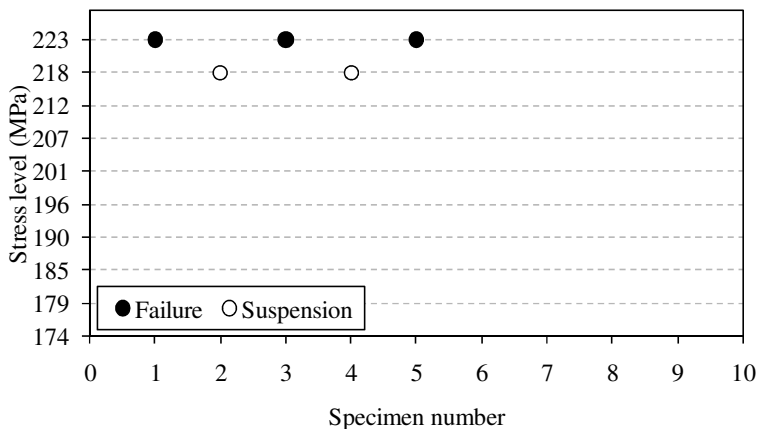


Figure 4. Plot of Staircase testing results for notch radius of 5 mm (stage 1)

The results obtained in second stage presented a reduction at average endurance limit because the refinement in the stress increment of the Staircase method. The standard deviation decreased three times in relation to first stage. The decreased of step size is responsible for this fact.

3.4. Notch strength and notch fatigue reduction strength. K_f behavior

According to obtained results starting from experimental data and shown in Tab. (6) and Fig. (5) we can to observe a decrease in endurance limit when notch radius decrease for ASTM A743 CA6NM alloy steel.

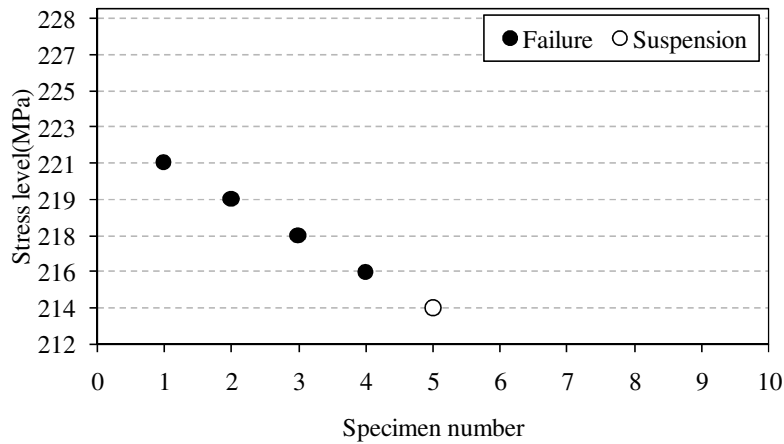


Figure 5. Plot of Staircase testing results for notch radius of 5 mm (stage 2)

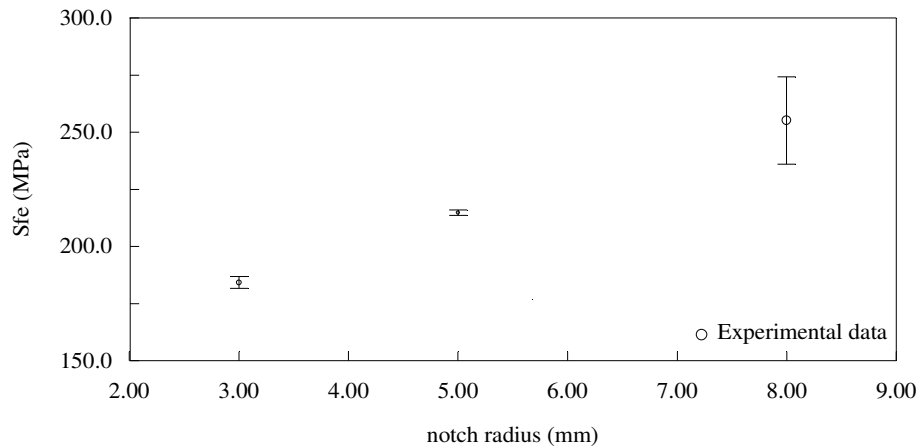


Figure 5. Comparison between endurance limits of notched specimens

Table 6. Endurance limit for each notch based on Staircase method

Notch radius (mm)	Endurance limit. S_{fe} (MPa)	
	Mean	Deviation
3	184.2	2.6
5	214.6	1.2
8	255.1	19.1

Table (8) and Fig. (6) show an increase of notch fatigue reduction strength with decrease of notch radius. In addition, the Peterson and Neuber's empirical models are statistically very similar when notch radius is big. However, when decreases notch size the notch factor are more different. The comparison between experimental notch fatigue factor and the empirical models shows that ASTM A743 CA6NM alloy steel is less sensitive than predictions.

Table 8. Experimental and theory fatigue notch reduction factor estimate for each geometry

Notch radius (mm)	Fatigue notch reduction factor. K_f		
	Experimental Eq. (2)	Neuber Eq. (4)	Peterson Eq. (6)
3	1.96 ± 0.08	2.27 ± 0.03	2.37 ± 0.03
5	2.04 ± 0.07	1.95 ± 0.05	2.02 ± 0.05
8	1.55 ± 0.12	1.51 ± 0.06	1.54 ± 0.06

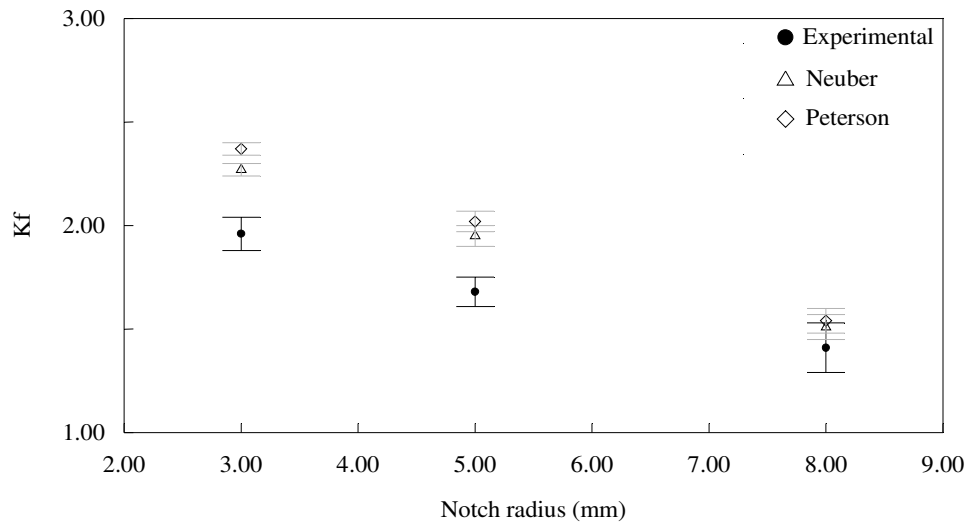


Figure 6. Comparison between experimental and theory models for notch fatigue reduction factor.

Table 9. Staircase testing results for 2.10^6 cycles ($R=-1$)

Notch radius (3 mm)			Notch radius (5 mm)			Notch radius (8 mm)		
Specimen #	S (MPa)	Cycles	Specimen #	S (MPa)	Cycles	Specimen #	S (MPa)	Cycles
1	190.19	$1.2 \cdot 10^6$	1	223.03	$8.2 \cdot 10^5$	1	258.49	run out
2	186.19	$1.4 \cdot 10^6$	2	217.56	run out	2	269.67	run out
3	182.20	run out	3	223.03	$5.2 \cdot 10^5$	3	280.86	$1.1 \cdot 10^6$
4	186.19	$8.1 \cdot 10^5$	4	217.56	run out	4	269.67	$9.6 \cdot 10^5$
5	182.20	run out	5	223.03	$4.0 \cdot 10^5$	5	258.49	run out
6	186.19	$1.2 \cdot 10^6$	1	221.20	$7.4 \cdot 10^5$	6	269.67	run out
7	182.20	run out	2	219.38	$7.6 \cdot 10^5$	7	280.86	$15 \cdot 10^5$
8	186.19	$9.1 \cdot 10^5$	3	217.56	$4.5 \cdot 10^5$	8	269.67	$4.1 \cdot 10^5$
9	182.20	$1.7 \cdot 10^6$	4	215.73	$9.4 \cdot 10^5$	9	258.49	$2.2 \cdot 10^5$
10	178.21	$7.5 \cdot 10^5$	5	213.91	run out	10	247.30	run out
S_f (MPa)	184.2 ± 2.6		S_f (MPa)	214.8 ± 1.2		S_f (MPa)	255.1 ± 19.1	

4. CONCLUSIONS

The aim of this work was to evaluate the effect of notch fatigue behavior of ASTM A743 CA6NM alloy steel. The Staircase method was used to determine the endurance limit and its standard deviation for three different notch sizes. A reduction data method by Dixon and Mood was very efficiently to determine fatigue limit. The statistical properties are easily determined and the mean value is has very accuracy because the technique concentrates points around average. However, the standard deviation does not reliable. The fatigue limit decreases when notch radius decreases as it was waited. Therefore, the notch fatigue reduction factor decreases when notch radius increases. For big radius, the predictions are statistically similar and can to predict the fatigue behavior, otherwise Peterson and Neuber's empirical models does not predict correctly. In addition, the alloy steel tested in this research is less sensitive than theory models used.

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