MINIMUM-TIME ANTI-SWING CONTROL OF GANTRY CRANES WITH HOISTING CONTROL USING LINEAR PROGRAMMING

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Abstract. *The problem of minimum-time anti-swing transfer of a load in a ship-to-pier gantry crane is investigated in this paper. The load is assumed to be initially at rest at the vertical position at the loading point above the ship and equally at rest at the unloading point above the hopper. The trolley is also assumed to be at rest at both points. A complete model is presented for the crane system where the nonlinear dynamic equations are linearized for sufficiently small swing angles and then rewritten in dimensionless form. The minimum-time solution is sought by considering as control variables both the force applied on the trolley that produces its horizontal motion and the hoisting speed of the load as functions of time. A predictor-corrector iterative method using Linear Programming (LP) is proposed based on a discrete-time model of the system where the control variables are taken as stepwise constants. At the corrector step, the hoisting motion is assumed given and a minimum-time solution is obtained by solving a sequence of LP problems representing fixed-time maximum-range problems. At the predictor step, a linearized model is employed to obtain an optimal correction of the hoisting motion using LP. The minimum-time control problem is formulated by taking into account practical constraints on the maximum speeds of both the trolley and the load hoisting, as well as on the maximum force that can be applied to the trolley. Numerical results are presented and show the effectiveness of the method.*

Keywords*: Anti-swing control, minimum-time control, optimal control, linear programming, gantry crane control*

1. INTRODUCTION

An efficient operation of gantry cranes requires the unloading of cargo from ships (and the loading) to be as fast as possible. The objective of this paper is to propose a numerical procedure to compute a solution to the minimum-time anti-swing control of gantry crane systems. The control variables are the force applied to the trolley and the speed of hoisting. Practical constraints on the speed of the trolley, velocity of hoisting and force applied to the trolley are included. Furthermore, both the load and the trolley must be at rest at the end point of the motion. The originality of the present approach resides in the application of Linear Programming (LP) for solving the problem. The optimal control law is obtained by a predictor-corrector iterative scheme. At the corrector step, an LP problem is formulated in order to find the force to be applied to the trolley for a given hoisting profile. At the predictor step, the hoisting is corrected by another LP problem. The iterative process stops when the hoisting correction becomes sufficiently small.

Several papers deal with the subject in the literature. Sakawa and Shindo (1982) minimize the load swing but not the time. Auernig and Troger (1987) apply the Pontryagin maximum principle to solve the minimum-time anti-swing control, but they consider that either the hoisting motion must be given, or the initial and final cable lengths must be equal. Khan (1993) and Al-Garni et al. (1995) apply the Feasible Sequential Quadratic Programming (FSQP) to minimize a hybrid performance measure index composed of weighting matrices where the time is just one of the goals and therefore the solution is sub-optimal. Golafshani and Aplevich (1995) apply the Sequential Quadratic Programming algorithm to minimize the transfer time in tower cranes. Corriga et al. (1998) propose an implicit gain-scheduling controller for cranes only to minimize the swing of the load but not the transfer time. Singhose et al. (2000) investigate the effects of hoisting on the input shaping control of gantry cranes, but only the non-swinging of the load is considered. Cho and Lee (2000) propose an anti-swing control for a three-dimensional overhead crane using position servo controller and fuzzy logic, the minimum time being of no concern. Lee and Choi (2001) propose a model-based anti-swing control for high hoisting speed based on the Lyapunov stability theorem to obtain a sub-optimal minimum-time solution. Scardua, Cruz and Costa (2002) describe the use of the reinforcement learning (RL) technique for anti-swing minimum-time control for a given hoisting. Lee (2004) proposes a high speed hoisting anti-swing control for bi-dimensional systems using Lyapunov stability theorem and extending the results to an adaptive scheme, but the minimization of time is out of the scope of the paper. In another paper (2005), Lee extends the results from bi-dimensional to three-dimensional systems for a given hoisting but yet the minimization of time is not approached. Wang et al. (2006) employ the parallel differential (PD) eingenvalue assignment methodology for an anti-swing control of overhead cranes with constant cable length. Solihin and Wahyudi (2007) propose a sensorless model-based anti-swing control but the solution is not time-optimal and there is no hoisting. Finally Cruz and Leonardi (2007) propose an anti-swing time-optimal control using LP for a given hoisting.

2. MODELING

The modeling is based on a gantry crane as illustrated in Fig. 1.

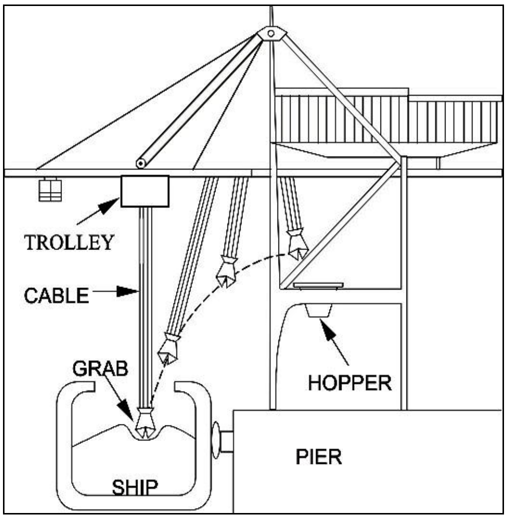

Fig. 1 – Scheme of the crane.

The gantry crane system consists of a trolley that holds a cable with a grab to grasp the material to be transferred. In this configuration there is a motor for moving the trolley on the bridge and a motor for the load hoisting, where both can work independently. The automatic unloading cycle begins at a given point above the ship deck, where the grab is already loaded, and ends at another given point above the hopper, where the grab unloads the material. At both points the trolley and the grab must be at rest.

All motions are assumed to occur in the plane of the figure. The dissipative effects are negligible in face of the forces involved in the system (Auernig and Troguer, 1987). The load is considered a material point. The mass of the cable is negligible and cable is inextensible. Schematically, the variables involved in the system can be illustrated as shown in Fig. 2.

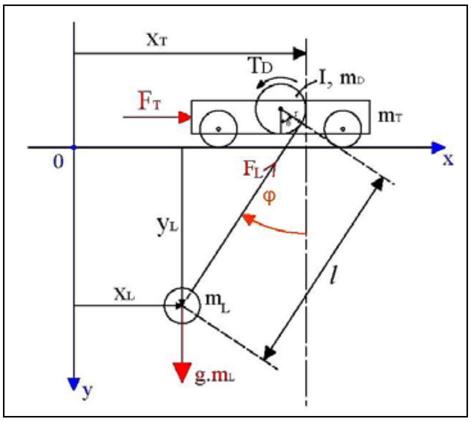

Fig. 2 – Schematic representation of the variables involved in the gantry crane dynamics

Using a dot over a variable to denote its derivative with respect to dimensional time, the following set of dynamical equations represents the system dynamics (Auernig and Troger, 1987):

$$\ddot{x}_T + \frac{m_L}{m_T}\left[\ddot{x}_T - \ddot{l}\varphi - 2\dot{l}\dot{\varphi} - l\ddot{\varphi}\right] = \frac{F_T}{m_T} \tag{1}$$

$$2\dot{l}\dot{\varphi} + l\ddot{\varphi} + g\varphi = \ddot{x}_T \tag{2}$$

$$-\left(1 + \frac{m_L}{m_D}\right)\ddot{l} + \frac{m_L}{m_D}\left(\ddot{x}_T\varphi + g\right) = \frac{F_L}{m_D} \tag{3}$$

where m_T is the trolley mass, including the equivalent contribution of the rotating parts of the trolley, x_T is the horizontal position of the car, F_T is the force applied to the trolley by the traversing motion motor, m_L is the load mass, m_D is the mass of hoisting drum, I is the moment of inertia of hoisting drum, T_D is the torque of hoisting motor, r is the radius of the hoisting drum, θ is the angular displacement of the hoisting drum, l is the length of the cable, φ is the load angle deviation with respect to vertical, F_L is the force applied to the cable that holds the load, g is the acceleration of gravity (9,81 m/s^2), $m_D = I/r^2$ and $F_L = T_D/r$.

3. VARIABLES TRANSFORMATION TO DIMENSIONLESS FORM – (ADIMENSIONALIZATION)

In order to improve the numerical stability of the system model a dimensionless form is adopted for its variables (Auernig and Troger, 1987):

$$u_T = \frac{F_T(t)}{F_{T\max}} \tag{4}$$

$$\tau = t\sqrt{\frac{g}{l_{\min}}} \tag{5}$$

$$\sigma = \frac{g}{a_{T\max}.l_{\min}} x_T \tag{6}$$

$$\lambda = \frac{l}{l_{\min}} \tag{7}$$

$$\phi = \frac{g}{a_{T\max}} \varphi \tag{8}$$

where $F_{T\max}$ is the maximum force that can be provided by the trolley traverse motor, $a_{T\max} = F_{T\max}/m_T$, t is time in seconds, l_{min} is the minimum cable length in meters and φ is the angle in radians. Derivatives with respect to dimensionless time are denoted by an apostrophe. Substitution of the dimensionless variables into Eqs. (1-3) gives:

$$\sigma'' + \alpha\phi = u_T \tag{9}$$

$$2\lambda'\phi' + \lambda\phi'' + (1+\alpha)\phi = u_T \tag{10}$$

where $\alpha = m_L/m_T$ is the ratio of the mass of the load grab plus the carried material and the mass of the trolley. According to Auernig and Troger (1987), $\lambda'' << 1$. Therefore, and taking into account that the load hoisting acceleration time is usually a small fraction of the whole traversing time, it is considered that $\lambda'' \cong 0$. For systems controlled by acceleration of the car, α can be set to zero.

Defining $x_1 = \sigma$, $x_2 = \dot{x}_1 = \dot{\sigma}$, $x_3 = \phi$ and $x_4 = x_3' = \phi'$, Eqs. (9) and (10) can be rewritten in state space form as:

$$\mathbf{X}'(\tau) = \mathbf{A}(\tau).\mathbf{X}(\tau) + \mathbf{B}(\tau).u(\tau) \tag{11}$$

where

$$\mathbf{A}(\tau) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\dfrac{(1+\alpha)}{\lambda(\tau)} & -\dfrac{2\lambda'(\tau)}{\lambda(\tau)} \end{bmatrix} \tag{12}$$

and

$$\mathbf{B}(\tau) = \begin{bmatrix} 0 & 1 & 0 & 1/\lambda(\tau) \end{bmatrix}^T \tag{13}$$

4. MINIMUM-TIME SOLUTION FOR A GIVEN HOISTING – THE CORRECTOR STEP

The general solution of Eq. (11) is well known:

$$\mathbf{X}(\tau) = \mathbf{\Phi}(\tau,0).\mathbf{X}(0) + \int_0^\tau \mathbf{\Phi}(\tau,\xi).\mathbf{B}(\xi).u_T(\xi)d\xi \tag{14}$$

where $\mathbf{\Phi}(\tau,\xi)$, the state-transition matrix of the system, is the solution of the differential equation:

$$\frac{\partial \mathbf{\Phi}(\tau,\xi)}{\partial \tau} = \mathbf{A}(\tau)\mathbf{\Phi}(\tau,\xi) \tag{15}$$

with initial condition $\mathbf{\Phi}(0,0) = \mathbf{I}$ where **I** is the identity matrix

4.1 Discretization of the model

Considering $\mathbf{u_T}$ piecewise constant and denoting by $u_T(i)$ the value of $u_T(\tau)$ in the interval $(i-1)\Delta\tau \le \tau < i\Delta\tau$, where $\Delta\tau$ is the discretization time step, it follows that:

$$\mathbf{X}(k) = \sum_{i=1}^{k} \mathbf{\Gamma_k}(i).u_T(i) \tag{16}$$

where:

$$\mathbf{\Gamma_k}(i) = \int_{\tau_{i-1}}^{\tau_i} \mathbf{\Phi}(\tau_k,\xi).\mathbf{B}(\xi).d\xi \tag{17}$$

Notice that Eq. (16) takes into account that $\mathbf{X}(0) = \mathbf{0}$, which means that both the trolley and the grab are initially at rest. $\mathbf{X}(k)$ denotes the value of $\mathbf{X}(\tau_k)$ where $\tau_k = k\Delta\tau$. Denoting by n the number of discretization steps in the interval $[0,\tau_f]$, the final time is given by $\tau_f = \tau_n = n\Delta\tau$.

4.2 LP formulation of the minimum-time problem for a given hoisting profile

It is discussed in the following how to obtain the minimum-time solution by solving a sequence of maximum-range fixed-time problems. An iterative search process is defined for which the duration time is fixed at each step and the goal is to maximize the distance traversed by the trolley. In this way the maximum range attained for a fixed time corresponds to the maximum average speed of the trolley. Since the system dynamics is linear, all constraints of the problem are also linear. Furthermore the traversed distance in a fixed time is a linear function of the control variables too. This is the obvious advantage of the present approach. See (Cruz and Leonardi, 2007) for more details.

Assume thus that τ_f is given. The objective function can then be written as:

$$\max_{\mathbf{u_T}}\left(x_1(n)\right) = \max_{\mathbf{u_T}}\left[\sum_{i=1}^{n}\left(\Gamma_{n,1}(i)\cdot u_T(i)\right)\right] \tag{18}$$

where vector $\mathbf{\Gamma_{n,1}}$ is the first row of matrix $\mathbf{\Gamma_n}$. The following constraints must be satisfied:

$$x_2(\tau_f) = 0 \tag{19}$$

$$x_3(\tau_f) = 0 \tag{20}$$

$$x_4(\tau_f) = 0 \tag{21}$$

which mean that both the trolley and the grab must bet at rest at the final time. The following additional constraints are included in order to take into account some practical limitations of the crane system:

$$|u_T(\tau)| \leq 1 \tag{22}$$

$$|x_2(\tau)| \leq \sigma'_{\max} \,. \tag{23}$$

The above equations represent limitations respectively on the control effort and on the maximum speed of the trolley for every τ in the interval $0 \leq \tau \leq \tau_f$.

The distance to be traversed by the trolley is actually given *(σ_f)* The value of τ_f is then iteratively sought until

$$x_1(\tau_f{}^*) = \sigma_f \tag{24}$$

where $\tau_f{}^*$ denotes de optimal time. This adjustment is performed here by linear interpolation:

$$\tau_f^{(j+1)} = \frac{\left(\sigma_f - x_1^{(j-1)}(n)\right)\cdot\left(\tau_f^{(j)} - \tau_f^{(j-1)}\right)}{x_1^{(j)}(n) - x_1^{(j-1)}(n)} + \tau_f^{(j-1)} \tag{25}$$

where *j = 1, 2, ...* denotes the index for the sequence of time adjustment and $x_1(n)$ is the state representing the distance at final time. For *j=1*, it can be set $\tau_f^{(0)} \equiv 0$ and $x_1^{(0)}(n) \equiv 0$.

5. OPTIMIZATION OF HOISTING – PREDICTOR STEP

In this section a method for evaluating an optimal increment on the hoisting profile is proposed. It is assumed that the hoisting profile increments are small and that the value of the time duration of the trolley motion is $\tau_f{}^*$, the one resultant from the previous corrector step. An LP problem is formulated in order to maximize the increment of the trolley traversed distance. The idea behind this proposal is the same as that from the corrector step, namely, to maximize the mean trolley speed.

In order to show explicitly the dependence of matrix $\mathbf{A}$ on λ and λ' Eq. (11) is rewritten in the form:

$$\mathbf{X}'(\tau) = \mathbf{A}(\lambda(\tau), \lambda'(\tau)).\mathbf{X}(\tau) + \mathbf{B}(\lambda(\tau)).u(\tau) \,. \tag{26}$$

The hoisting variables λ and λ' can be written as

$$\lambda(\tau) = \overline{\lambda}(\tau) + \Delta\lambda(\tau) \tag{27}$$

$$\lambda'(\tau) = \overline{\lambda}'(\tau) + \Delta\lambda'(\tau) \tag{28}$$

where $\overline{\lambda}$ and $\overline{\lambda}'$ represent the given hoisting profile of the previous section, whereas $\Delta\lambda$ and $\Delta\lambda'$ are, respectively, the increments of the hoisting variables. Furthermore if the state and control vectors are denoted correspondingly as

$$\mathbf{X}(\tau) = \overline{\mathbf{X}}(\tau) + \mathbf{\Delta X}(\tau) \tag{29}$$

and

$$u(\tau) = \overline{u}(\tau) + \Delta u(\tau) \tag{30}$$

then the following linear approximation is valid:

$$\mathbf{\Delta X'}(\tau) = \mathbf{A}(\overline{\lambda}(\tau), \overline{\lambda}'(\tau)) \cdot \mathbf{\Delta X}(\tau) + \mathbf{\Delta A}(\overline{\lambda}(\tau), \overline{\lambda}'(\tau)) \cdot \overline{\mathbf{X}}(\tau) + \mathbf{B}(\overline{\lambda}(\tau)) \cdot \Delta u(\tau) + \mathbf{\Delta B}(\overline{\lambda}(\tau)) \cdot \overline{u}(\tau) \tag{31}$$

Assuming that $\Delta\lambda$ and $\Delta\lambda'$ are sufficiently small, then

$$\mathbf{\Delta A}(\bar{\lambda}(\tau),\bar{\lambda}'(\tau)) \cong \left.\frac{\partial \mathbf{A}}{\partial \lambda}\right|_{\bar{\lambda}(\tau),\bar{\lambda}'(\tau)} \cdot \Delta\lambda(\tau) + \left.\frac{\partial \mathbf{A}}{\partial \lambda'}\right|_{\bar{\lambda}(\tau),\bar{\lambda}'(\tau)} \cdot \Delta\lambda'(\tau) \tag{32}$$

$$\mathbf{\Delta B}(\bar{\lambda}(\tau)) \cong \left.\frac{\partial \mathbf{B}}{\partial \lambda}\right|_{\bar{\lambda}(\tau)} \cdot \Delta\lambda(\tau) \tag{33}$$

Defining:

$$v(\tau) = \Delta\lambda'(\tau)\,, \tag{34}$$

$$\mathbf{E_{1,1}}(\tau) = \mathbf{A}(\bar{\lambda}(\tau),\bar{\lambda}'(\tau)) \tag{35}$$

$$\mathbf{E_{1,2}}(\tau) = \left.\frac{\partial \mathbf{A}}{\partial \lambda}\right|_{\bar{\lambda}(\tau),\bar{\lambda}'(\tau)} \cdot \overline{\mathbf{X}}(\tau) + \left.\frac{\partial \mathbf{B}}{\partial \lambda}\right|_{\bar{\lambda}(\tau)} \cdot \bar{u}(\tau) \tag{36}$$

$$\mathbf{G_{1,1}}(\tau) = \mathbf{B}(\bar{\lambda}(\tau)) \tag{37}$$

$$\mathbf{G_{1,2}}(\tau) = \left.\frac{\partial \mathbf{A}}{\partial \lambda'}\right|_{\bar{\lambda}(\tau),\bar{\lambda}'(\tau)} \cdot \overline{\mathbf{X}}(\tau) \tag{38}$$

$$\mathbf{E}(\tau) = \begin{bmatrix} \mathbf{E_{1,1}}(\tau) & \mathbf{E_{1,2}}(\tau) \\ \mathbf{0} & 0 \end{bmatrix} \tag{39}$$

$$\mathbf{G}(\tau) = \begin{bmatrix} \mathbf{G_{1,1}}(\tau) & \mathbf{G_{1,2}}(\tau) \\ 0 & 1 \end{bmatrix} \tag{40}$$

$$\boldsymbol{\chi}(\tau) = \begin{bmatrix} \mathbf{\Delta X}(\tau) \\ \Delta\lambda(\tau) \end{bmatrix} \tag{41}$$

$$\boldsymbol{\mu}(\tau) = \begin{bmatrix} \Delta u(\tau) \\ v(\tau) \end{bmatrix} \tag{42}$$

from Eqs. (31) to (33) it follows that

$$\boldsymbol{\chi}'(\tau) = \mathbf{E}(\tau) \cdot \boldsymbol{\chi}(\tau) + \mathbf{G}(\tau) \cdot \boldsymbol{\mu}(\tau) \tag{43}$$

5.1 Discretization of the incremental model

Proceeding analogously as in section 4 and considering $\boldsymbol{\mu}(\tau)$ piecewise constant, and taking into account that both the trolley and the grab must be initially at rest, i.e,

$$\boldsymbol{\chi}(0) = \mathbf{0}\,, \tag{44}$$

it follows that

$$\boldsymbol{\chi}(k) = \sum_{i=1}^{k} \mathbf{\Omega_k}(i) \cdot \boldsymbol{\mu}(i) \tag{45}$$

where

$$\mathbf{\Omega_k}(i) = \int_{\tau_{i-1}}^{\tau_i} \mathbf{\Phi}_\Delta(\tau_k, \xi).\mathbf{G}(\xi).d\xi \,, \tag{46}$$

$\mathbf{\Phi}_\Delta(\tau_k, \xi)$, the state-transition matrix of the system, is the solution of the differential equation:

$$\frac{\partial \mathbf{\Phi}_\Delta(\tau, \xi)}{\partial \tau} = \mathbf{E}(\tau) \cdot \mathbf{\Phi}_\Delta(\tau, \xi) \tag{47}$$

with initial condition $\mathbf{\Phi}_\Delta(0,0) = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix.

5.2 LP formulation of the optimal incremental policy for the hoisting profile

In what follows, an LP problem is formulated in order to maximize the increment of the trolley traversed distance. Hence, the objective function can be written as

$$\max_{\boldsymbol{\mu}}(\Delta x_1(n)) = \max_{\boldsymbol{\mu}}\left(\sum_{i=1}^{n} \mathbf{\Omega_{n,1}}(i) \cdot \boldsymbol{\mu}(i)\right) \tag{48}$$

where the vector $\mathbf{\Omega_{n,1}}$ is the first row of the matrix $\mathbf{\Omega_n}$.

Since both the trolley and the grab must be at rest at the final time, the following constraint must be satisfied:

$$\mathbf{P} \cdot \boldsymbol{\chi}(n) = \mathbf{0} \tag{49}$$

where $\mathbf{P}$ is the (4x5) matrix

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{50}$$

Additionally the following inequality constraints must be satisfied at any time:

$$\left|\bar{x}_2(\tau) + \Delta x_2(\tau)\right| \le \sigma'_{\max} \,, \forall \tau \in [0, \tau_f] \tag{51}$$

which means that the trolley speed must not be higher than the maximum permissible value;

$$\left|u_T(\tau) + \Delta u(\tau)\right| \le 1 \,, \forall \tau \in [0, \tau_f] \tag{52}$$

since the control effort cannot be higher than its maximum value in any instant;

$$\left|\lambda'(\tau) + \Delta\lambda'(\tau)\right| \le \lambda'_{\max} \,, \ \forall \tau \in [0, \tau_f]. \tag{53}$$

because the hoisting speed must not be higher than its maximum value.

In order to avoid the shock of the grab with the ship or with the crane structure a constraint on the cable length along the trajectory is included:

$$\lambda_{\min}(\tau) < \left(\lambda(\tau) + \Delta\lambda(\tau)\right) < \lambda_{\max}(\tau) \,, \ \forall \tau \in [0, \tau_f], \tag{54}$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are given. This is an approximation since the bounds are in fact known as functions of σ.

To guarantee the validity of linear approximations made in Eqs. (32) - (33), the following constraints are added in:

$$\Delta x_5(\tau) \le \varepsilon \cdot \bar{\lambda}(\tau) \tag{55}$$

$$v(\tau) \le \gamma \cdot \lambda'_{\max} \tag{56}$$

where $0 < \varepsilon, \gamma << 1$, since the LP problem above has been solved, the new hoisting variables λ and λ' can be updated as:

$$\lambda(k) = \bar{\lambda}(k) + \Delta\lambda(k) \tag{57}$$

$$\lambda'(k) = \bar{\lambda}'(k) + \Delta\lambda'(k) \tag{58}$$

for $k = 1,2,\ldots,n$.

The solution values $\Delta u(k)$, $k = 1,2,\ldots,n$, are discarded since they are just approximated linear predictions of the "true" ones, which are then evaluated in the next corrector step. At this point the reason of giving the names *predictor* and *corrector* to the steps of the proposed method should be evident.

6. SUMMARY OF THE PREDICTOR – CORRECTOR ITERATIVE METHOD

In summary, the computation of the minimum-time anti-swing control of gantry cranes must proceed according the following steps:

a) Define an initial feasible hoisting profile.
b) Solve the minimum-time problem set in Section 4 for the given hoisting profile (*corrector step*).
c) Solve the LP problem set in Section 5 to update the hoisting profile (*predictor step*).
d) Compute again the optimization as described in section (4), the corrector step, on the nominal model, however using now the new corrected hoisting vector. Solve the minimum-time problem set in Section 4 for the updated hoisting profile (*corrector step*).
e) Repeat steps (c) and (d) until convergence has been attained.

7. RESULTS

Actual data from Sepetiba Port, RJ, Brazil (Cruz and Leonardi, 2007), were used in order to illustrate the application of the proposed approach to a real gantry crane operation. The following data were used: $x_{Tf} = 50m$, $l_{\min} = 16m$, $l_{\max}(\sigma_h) = 30m$ (for the hoisting region), $l_{\max}(\sigma_r) = 18m$ (for the retracted-cable region), $\dot{x}_{T\max} = 2{,}4m/s$, $m_T = 19.500kg$, $F_{T\max} = 19.500N$, $\alpha = 2.1$, $\dot{l}_{\max} = 2m/s$, $\varepsilon = 0{,}02$, $\gamma = 0{,}02$, $n = 100$.

The admissible region for the cable length, defined by the dash-dotted lines labeled as "maximum hoisting" and "minimum hoisting", together with the resulting hoisting profile are given in Fig. 3 and Figs. 4 – 11 show the remaining results obtained. As it can be seen, all the constraints were satisfied. The value of the minimum-time obtained was 27,638 seconds.

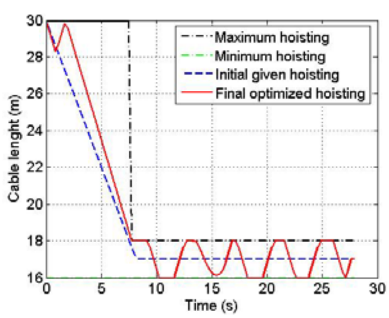

Fig. 3 – Hoisting profile.

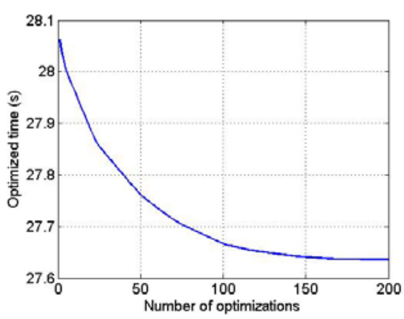

Fig. 4 – Evolution of minimum-time along the predictor-corrector steps.

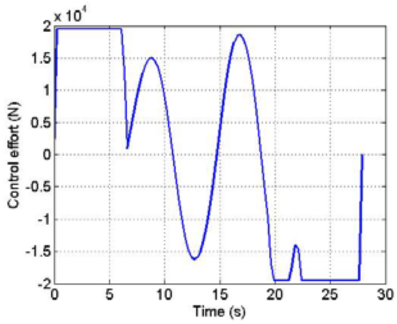

Fig. 5 – Control effort on trolley.

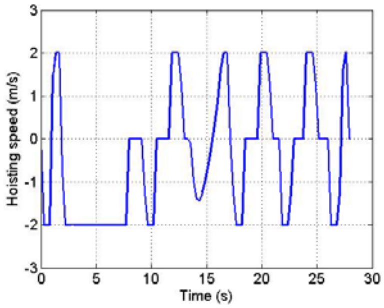

Fig. 6 – Hoisting speed.

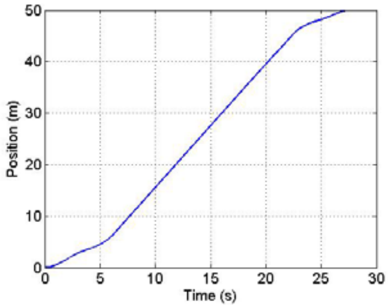

Fig. 7 – Distance traversed by the trolley.

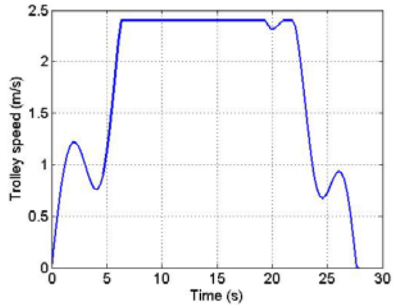

Fig. 8 – Trolley speed.

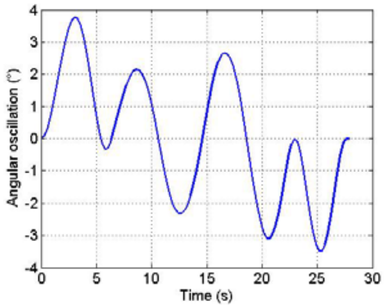

Fig. 9 – Angular oscillation.

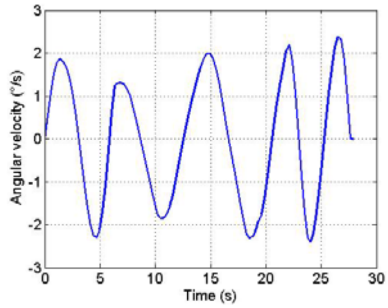

Fig. 10 – Angular velocity.

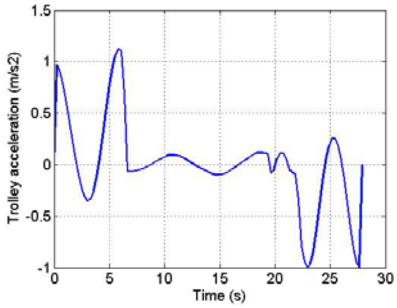

Fig. 11 – Trolley acceleration.

In this case it can be seen that the hoisting profile has small influence on the optimization of transfer time of the load. As Fig. 4 shows, the transfer time gain with hoisting optimization was only 0,4234 seconds, which represents an improvement of just about 1,5%. Notice that this occurs despite the optimal hoisting profile being significantly different from the initially given hoisting profile. Another interesting point to be noticed is that the optimal solution observed has an oscillatory form, which may reflect a nonlinear profile built by the method along the predictor-corrector steps. In the results presented by Auernig and Troger (1987), with the use of Pontryagin Maximum Principle, it can also be seen that the hoisting profiles have little influence on the minimum times obtained. Furthermore it can be seen in that paper that the best transfer time is the one corresponding to the most oscillatory hoisting profile, which agrees with the present study.

From a practical point of view, the oscillating hoisting solution may be difficult to implement. This problem can be circumvented by a narrower region admissible for the cable length as shown in Figs. 12 – 13. Obviously the price to be paid is an increase in the minimum-time which, in this case, was 27,9584 seconds, much closer to the value corresponding to the given initial hoisting profile.

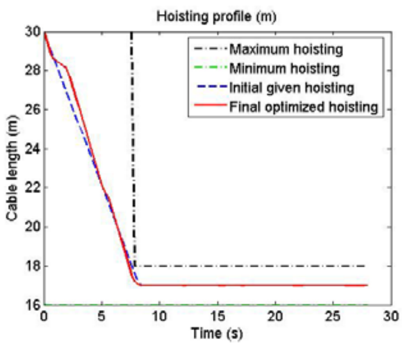

Fig. 12 – Hoisting profile

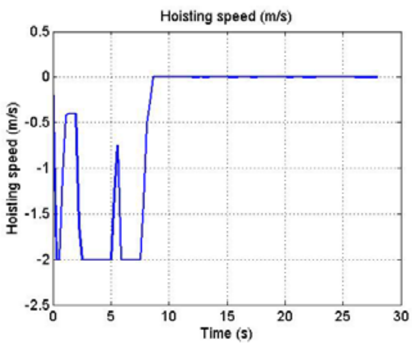

Fig.13 – Hoisting speed

8. CONCLUSIONS

An LP-based predictor-corrector algorithm to solve the minimum-time anti-swing control of grab gantry cranes with hoisting control has been proposed in this paper. The inclusion of a variety of important practical constraints in the LP problems has been quite simple. The examples presented have shown the numerical effectiveness of this method.

It should be emphasized that, since the optimal control solution is an open-loop one, robustness issues were out of the scope of the paper. In a practical implementation, a command-following control loop should be designed to cope with model uncertainties.

An interesting point of the present investigation is the observation of the oscillatory character of the hoisting profile shown by the examples run so far. This fact may reflect a nonlinear profile built by the method along the predictor-corrector steps.

Another important conclusion of this study is the small influence of the hoisting policy on the minimum-time transfer solution observed in the examples run so far. Therefore it may be possible to work with a more convenient smooth sub-optimal solution with little increase on total transfer time.

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11. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.