HARMONIC ANALYSIS OF SIMPLE PENDULUM TO DYNAMICS STUDY AND MONITORING OF A COLUMN OF STEMS BCP PUMP

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Abstract. *This paper presents preliminary results of the dynamical analysis of simple and multiple pendulums, where is taking into account the sinusoidal term of differential equation to be used as analogy to column of stems in Progressive Cavities Pump (BCP) installed downhole in petroleum well. One of the first studies in pumps vibration analysis which tried to monitor the equipment installed downhole from the surface noted that, due to noise interference, it was better measure the amplitude and shape of the pump's acoustic oscillation instead of making spectral investigation of its vibration. The analysis of obtained results only shows the possibility of monitoring a pump system by its vibration transmitted through the tubing. The main objective in the present research is to know the dynamics of the column of stems and to determine the frequencies that are developed in the bending vibrations against the tubing. Next, from the analogy of column stems in petroleum well with multiple pendulums, it is also verified the dynamics disturbance of multiple pendulums due to the sinusoidal term in differential equation versus to displacement angle of the pendulum (stems). The displacement of column stems causes beating in the inner production pipe wall and difficult the monitoring. This beating could damage the structure leading to production stops and an increase on well interventions. The analysis and investigation of the dynamic behavior of the column stem can help in the development of monitoring vibration systems for the reduction on both the maintenance costs and petroleum well intervention.*

Keywords*: Simple pendulum, pendulum multiple, harmonics, column stems of BCP pump.*

1. INTRODUCTION

In the acoustic and vibration analysis of operation oil wells, mainly in a column of stems BCP pump for onshore oil wells, should consider the different sources of noises, sometimes, indicate the responsible factors for breaks of those stems. The dynamics of the stems BCP pump when oscillate can be compared with multiple pendulums, that when oscillate causes beating in the inner production pipe wall. In this paper we present, initially, results preliminaries of dynamic analysis of multiple pendulums due occurrence of the sinusoidal term in the differential equation. Next, makes an analogy of the stems BCP pump with multiple pendulums and their disturbances at the oscillations due the sinusoidal terms of the pendulums differential equations. We will present the results with detailed discussion for changes in the pendulums (stems) frequency with initial angle and we will suggest that those disturbances (frequency variation) interferes in the oscillation of stems BCP.

1.1. Progressive Cavities Pump

Recently, Rodrigues (2008) developed a simulation and dynamic analysis in a column of stems BCP pump using analogy with multiple pendulums, where was observed that the column of stems caused shocks (beaten) in the inner wall of the production piping, then the simulation showed the column stems movements in the production column. Those beats can cause damages in the structure provoking the stop of oil well production, interventions in the well to change of stems and noises that cause interference in the data acquisition of vibration for monitoring. The Progressive Cavities Pump has a column of stems that is submitted a rotation generated by an electric motor at the surface. The column stems works in the production tubing. Figure 1, present the configuration of oil well and equipments.

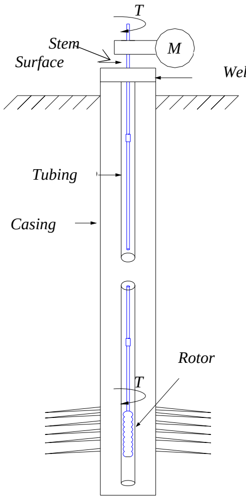

Figure 1. Configuration of oil well equipped with typical BCP installation (electric motor and headstock).

Each stems has 7.62 meters of length connected at the end by screw-glove. The rotation movements combined with the great length of column, which depends on the depth of the well, causing bending movements of column of stems inside the tubing (Thomas, 2001). For the monitoring of vibrations in this system is necessary to know the natural frequencies. Beyond the natural frequencies of vibration torsional, bending, longitudinal and flow of oil, this work comes to identify noises that disturbs the pump system. In the Rodrigues (2008), the use of analogy with pendulums becomes easy to make experimental tests in place of system mass-spring.

2. OSCILLATION OF THE SIMPLE PENDULUM

Suppose a pendulum of length *z*, fixed for a wire with a mass *m*. Let us yet that wire makes a vertical angle θ and that *g* is the acceleration of gravity (Craig, 1986). The differential equation representing the motion of simple pendulum is given by:

$$\ddot{\theta} + (g/z)\sin\theta = 0 \tag{1}$$

It is known that there will be an envelope curve in generation of pendulum oscillations. The envelope curve occurs due to sinusoidal term in the equation of motion. In this work, first we made the analysis of simple pendulum oscillations considering the initial angle. Next, analyze the equation that describes the variation frequency of oscillation versus initial angle. Using the Fourier Transform we analyzed the oscillations for $\theta_0=30^o$. After analyze of oscillation curve, have been defined an envelope curve and using a low pass filter, show the FFT for harmonic curve. Then compare the results obtained from the harmonic analysis of simple pendulum curve with curve of oscillation of a simple harmonic motion, and the relationship with the envelope curve. Finally, analysis of results will be obtained for oil well operation equipped with BCP pump.

The Figure 2 presents a simplified diagram of simple pendulum, with weight $m_0.g$ and length *z*, displaced a angle θ. We used $P_0= m_0.g$, knowing that for an arm length *z*, the angular displacement is given by $s=\theta z$, the speed by $v=\dot{\theta}z$, and the acceleration by $a=\ddot{\theta}z$ (Meirovitch, 1990).

The torque of simple pendulum is shown in Eq. (2). Using $T_0=P.(z/2)$ and $a=\ddot{\theta}z$, obtains the differential equation that describes the angular displacement of pendulum, Eq. (3).

$$T_0 = P_0(z/2)\sin\theta_0 \tag{2}$$

and

$$m_0.a.z = m_0.\ddot{\theta}.z^2 = m_0.(z/2).g.\sin(\theta_0) \Rightarrow \ddot{\theta} = (g/2z)\sin(\theta_0) \tag{3}$$

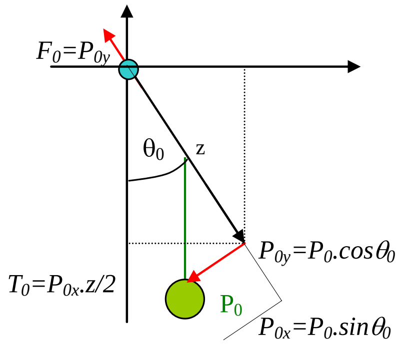

Figure 2. Diagram of simple pendulum.

The general solution of differential equation requires the use of elliptic functions (Meirovitch, 1990). The equation of period versus initial angle (θ_0) is shown in Eq. (4).

$$\tau = 4\sqrt{\frac{z}{g}\int_0^{\pi/2}(1-k^2\sin^2\beta)^{-1/2}d\beta} \tag{4}$$

where $k=\sin(\theta_0/2)$ and $\beta=\sin^{-1}(\sin(\theta_0/2)/k)$. The oscillations of pendulum, due sinusoidal term, present a beat that can be called by envelope. The Figure 3 shows the frequency of pendulum as a function of initial angle obtained from the equation (4) which considers the term sinusoidal, with $z = 0{,}25m$ and $g = 10\ m/s^2$. From the results was collected a frequency of $f = 1{,}066\ Hz$ to θ_0 near to zero.

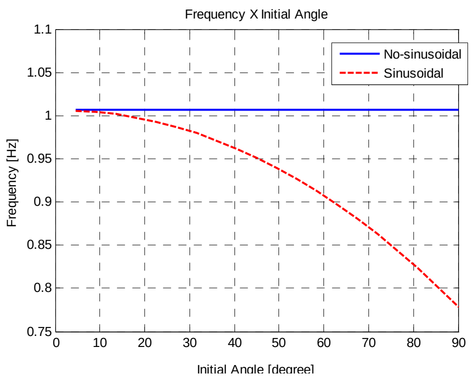

Figure 3. Frequency versus initial angle θ_0.

In the Figure 3, show can see that frequency of oscillation of simple pendulum (stem) decreases when initial angle increases. The bending movements of stems cause a disturbance in the system due the frequency change.

The blue curve represents the pendulum frequency from the approach given by *sen* $(\theta)\simeq\theta$ that is defined as

$$f = \left(\tfrac{1}{2\pi}\right)\sqrt{g/z} \qquad [\mathrm{Hz}] \tag{5}$$

where f is the pendulum frequency (stem), g is the gravity and z is the pendulum length. The red dotted curve represents the sinusoidal term that is defined as

$$f = \left(\tfrac{1}{2\pi}\right)1/\tau \qquad [\mathrm{Hz}] \tag{6}$$

where τ is the period defined by Eq. (4).

The Figure 4 presents the results of stem with length of 7.62 m. There is a natural frequency from the motion of a simple pendulum of *0,1823 Hz* and sinusoidal variation of the term depending on angle of initial displacement.

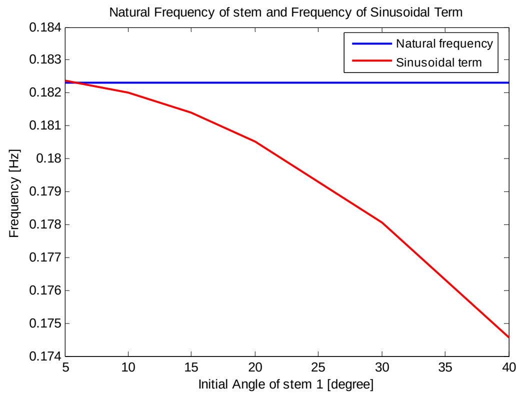

Figure 4. Natural frequency, Sinusoidal Term of stem versus Initial Angle.

3. MOVEMENT OF ONE SIMPLE PENDULUM (STEM)

3.1 Pendulum movements

Consider the data for pendulum where the Fig. 5 presents results curves obtained from simulation of simple pendulum with $g = 10\ m/s^2$ and $z = 0,25\ m$. In this case the pendulum was released from initial angle of $\theta = 30°$.The blue solid curve at the top of Fig. 5 represents the waveform generated by pendulum displacement. In the same position of the Fig. 5, the red dotted curve shows the envelope curve. The Figure 6 shows the envelope curve that was obtained from low-pass filter and the curve representing the pendulum frequency ($f = 1,0066\ Hz$) obtained from standard Fourier Transform (FFT) of pendulum displacement.

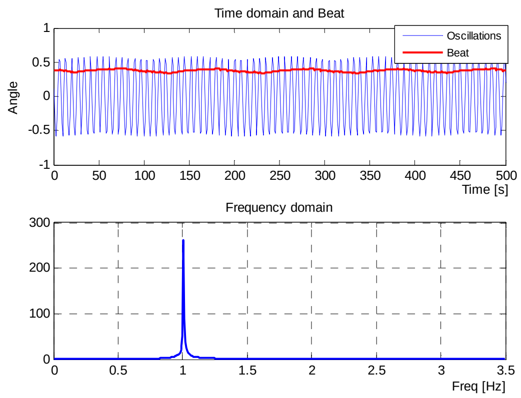

Figure 5. Result of time domain and Beat, and standard Fourier transform.

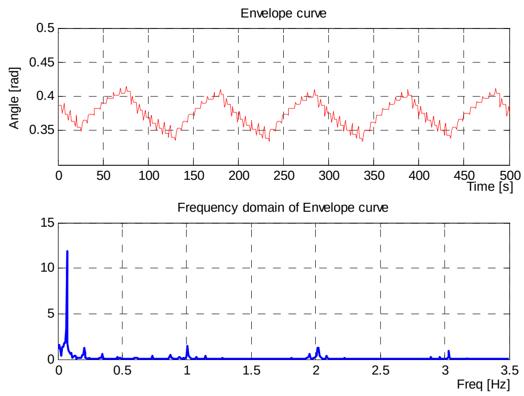

Figure 6. Envelope curve and response in frequency of Envelope curve

The Figure 6 show the Envelope curve obtained after filtering curve of pendulum oscillation and frequency domain after standard Fourier transforms processing which the harmonic envelope has a frequency about $0,07\ Hz$.

3.2 Analogy with simple pendulum for pump stems

Next in the Fig. 7 and Fig. 8, show the influences that sinusoidal term cause at the one stem vibrations where of stem length is 7,62 m.

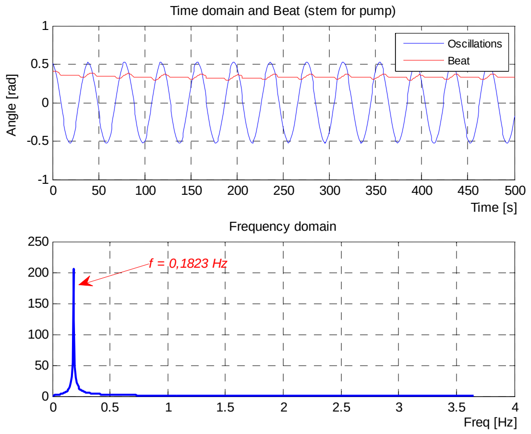

Figure 7. Oscillations of stem with response in time domain and frequency domain.

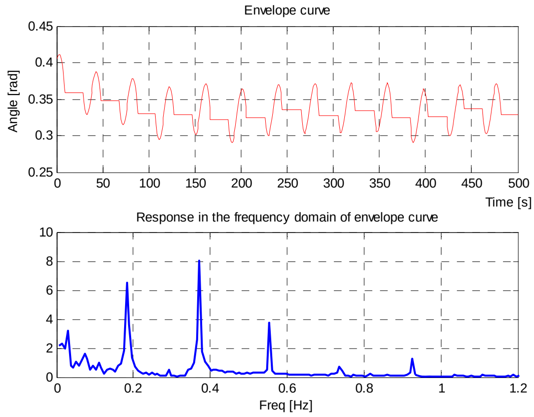

Figure 8. Envelope Curve of sinusoidal term and response in frequency.

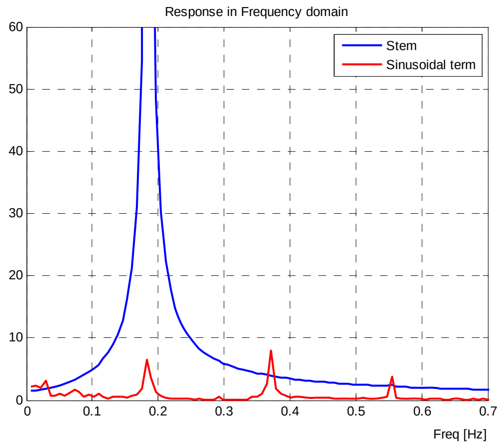

Figure 9. Response in frequency domain for stem and sinusoidal term.

The response in frequency domain for stem (sinusoidal term) has frequencies peak behind the stem natural frequency (*f= 0.1823 Hz*), shown in Fig. 7, Fig. 8 and Fig. 9, and others, showing that the frequency variation depending on the angle of displacement. These components of frequency act like noise that can cause damage in the column of stems and provoke production stop at the oil well.

3.3 Analysis for envelope curve of pendulum

For the pendulum, shown in Fig. 10, presents the Fourier transform of Envelope curve versus initial angle for oscillation of the pendulum, which varies around 5% of natural frequency (*1,0067 Hz*). The initial angle range is 0-60 degree. Fig. 10 shows the values that can disturb the structure and indicate damage.

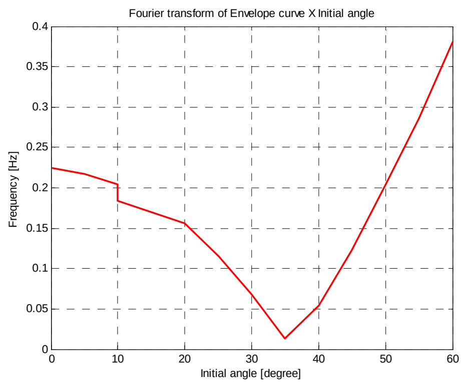

Figure 10. Response in frequency of envelope curve versus initial angle θ_0.

The initial angle changes frequency of the pendulum and the harmonics when angle variation. The Figure 11 shows is the frequency of oscillation of the pendulum and harmonics calculated versus initial angle of oscillation. In red curve the value calculated using the Eq. (4) and black dotted curve presents the values calculated using the harmonics of the Fourier transform in this equation.

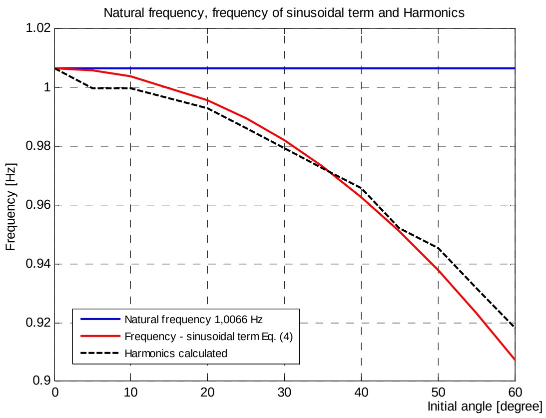

Figure 11. Natural frequency, frequency of sinusoidal term and harmonics.

4. SIMULATION FOR TWO PENDULUMS (TWO STEMS)

From Spong (1989),was studied the dynamics of two pendulums (stems) coupled as shown in Fig. 12. In Equation (7) shows the torque due to pendulum 2, and Eq. (8) present a differential equation of pendulum 2 oscillation. In Equation (9) shows the torque due to pendulum 1, and Eq. (10) present a differential equation of pendulum 1 oscillation.

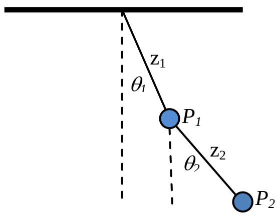

Figure 12. Configuration of Two pendulums

$$T_2 = (P_2 / 2)\sin(\theta_1 + \theta_2) \tag{7}$$

$$\ddot{\theta}_2 = \frac{P_2 / 2}{m_2 z}\sin(\theta_1 + \theta_2) \tag{8}$$

$$T_1 = \left(\frac{P_1}{2} + P_2\right)\sin(\theta_1) + T_2 \tag{9}$$

$$\ddot{\theta}_1 = \left\{\left(\frac{P_1}{2} + P_2\right)\sin(\theta_1) + T_2\right\} / m_1 z_1 \tag{10}$$

Comparing with differential equation of a single pendulum given below,

$$\ddot{\theta}_0 = \left\{\left(\frac{P_0}{2}\right)\sin(\theta_0)\right\} / m_0 z \tag{11}$$

with equation (10), where differ in torque term T_2 .

Now, can see in the Fig. 13 the stem simulator that was used for data collect (Rodrigues, 2008). Was used a length for two stems of *0,25 m*.

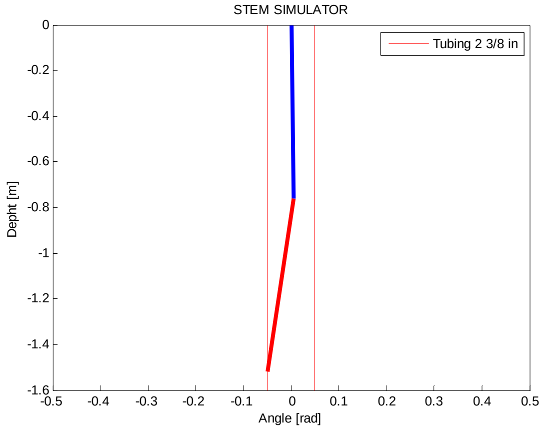

Figure 13. Stems simulator.

We simulate the movements of the double pendulum which were considering two situations:

a) In the first situation, the pendulum 2 is fixed to pendulum 1. This way, the double pendulum works like simple pendulum, and.

b) In the second situation, the pendulum 2 is free.

The simulations were made in positioning the stem 1 in $\theta_1=5^o$ and stem 2 in $\theta_2=0$. The Figure 14 shows the curve of time domain with stem 1 free and stem 2 fixed. Shows at the bottom of Fig. 16 the response in frequency domain for stem 1 ($f_1 = 17\ Hz$).

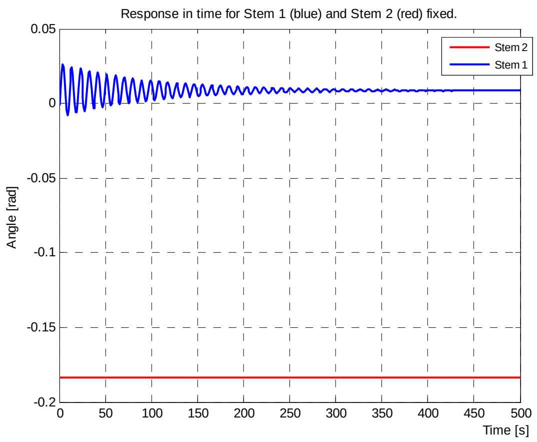

Figure 14. Response in time domain for stem 1 with stem 2 fixed.

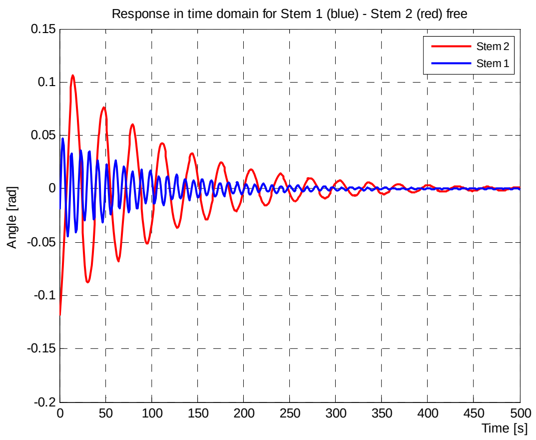

Figure15. Response in time domain for stem 1 and stem 2 free,

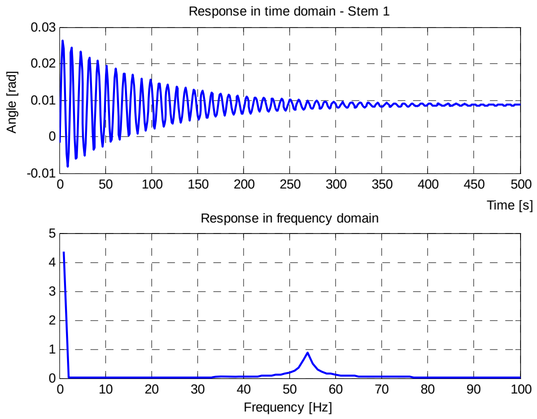

Figure 16. Response in time and frequency domain for stem 1 with stem 2 fixed.

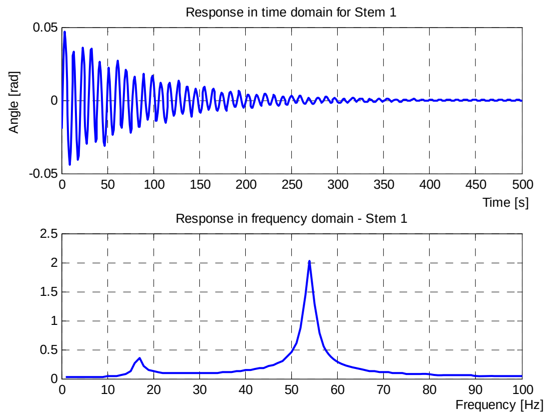

Figure 17. Response in time and frequency domain for stem 1 with stem 2 free.

The Figure 15 shows the response in time domain for stem 1 and stem 2 free when $\theta_1=5^o$ and $\theta_2=0$. At the bottom of Figure 17 shows the response in frequency domain for stem 2 where was observed the two frequencies of stem 1 (f_1 = $54Hz$) and stem 2 (f_2 = $17Hz$).

In the differential equation for θ_2 that the shape of envelope curve for stem 2 depends on the initial angles of stem 1 and 2, which presents a different shape of envelope curve for simple pendulum. Next, were simulated the double pendulum with different initial angle θ_1= (5^o, 10^o, 15^o, 20^o, 30^o, 40^o) when θ_2= (5^o, 10^o, 15^o, 20^o, 40^o).

The Figure 18 shows the frequency variation for stem 2 versus initial angle for stem 1 (θ_1 range of 5 to 40^o). The green curve represents the value calculating using the Eq. (4). The blue curve represents the maximum frequency for stem 2 and the others curves are the positions of stem 1. Can see that stem 2 in $\theta_2=5^o$, $\theta_2=10^o$ and $\theta_2=15^o$ presents frequencies higher than the frequency obtained with Eq. (4).

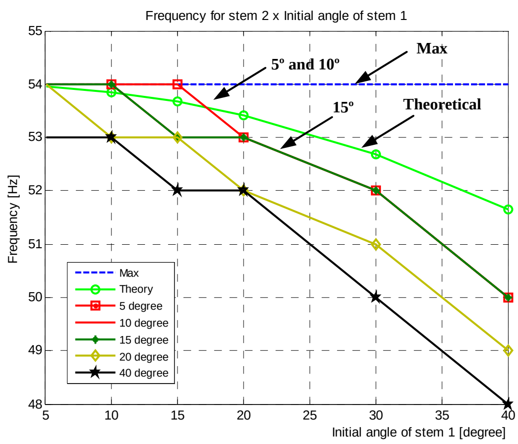

Figure 18. Frequency variation for stem 2 versus initial angle for stem 1.

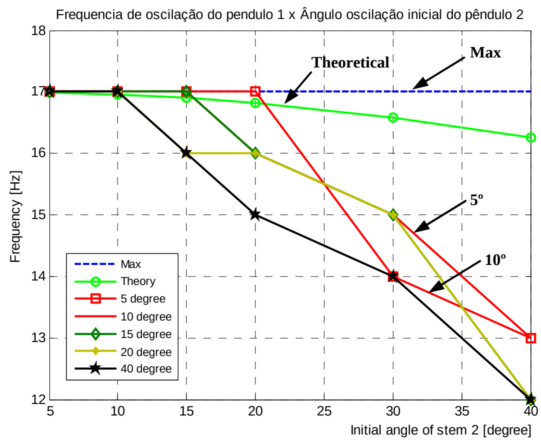

Figure 19. Frequency variation for stem 1 versus initial angle for stem 2.

The Figure 19 presents the frequency variation for stem1 with variation initial angle for stem 2. Can see that the frequencies of stems for $\theta_1=5^o$ present values lower than frequency obtained with Eq. (4). Then, that varying the positions of each stem on other the variation frequency is observed, that can cause damage in the structure of column stems.

5. DISCUSSION AND CONCLUSIONS

With the advance in research and development of systems for monitoring structural health, know the frequencies and the disturbances of the system is very important. For a monitoring system and collect data (vibrations) for oil well, should consider the torsional and bending vibrations that cause damage in the stems and provoke breaks and stops in the production oil and avoid high costs of maintenance.

To verify the proposed experiences and the future system monitoring, was calculated the harmonics of BCP column of stem pump for onshore oil well when consider the sinusoidal term in the differential equation for pendulum. Was observed that the disturbance in pendulum trajectory creates an envelope curve, which is visible and is a function of the initial angle.

The preliminary results showed that the frequencies of pendulum varies with initial angle, and the simulation of stems column for BCP pump presented the variation of frequencies with values lower the analytic curve.

These results help the correct choice about the diameter of stem and tubing and increase the data for system of monitoring structural health that will be developed.

In the future works, will analyze the differential equations of multiple pendulums to verify dependence on the initial angle and apply in the simulator. Knowing of the envelope curve of columns of stems BCP pump, the Supervisor and Petroleum Engineer can choice the correct equipment to equip the oil well and reduce the number of maintenance.

6. REFERENCES

Cavalcanti, J. H. F. and Ferneda, E, 1995," Intelligent Control of an Inverted Pendulum - Training and Evolution", *SEMISH'95*, Canelas, RS, Brazil, pp 627-637.

Craig, John J., 1986, "Introduction to Robotic Mechanics & Control", Addison-Wesley Publishing Company.

Inman, D. J, 2001,"Engineering Vibration", Prentice Hall, 2nd edition, Englewood Cliffs, NJ, 650p.

Meirovitch, L., 1990, "Dynamics and Control of Structures", John Wiley & Sons, New York, 434p.

Rodrigues, M. C. and Cavalcanti, J. H. F, 2008," Simulação e Análise Dinâmica de uma Coluna de Hastes de Bombeio BCP Usando Analogia com Movimentos de Pêndulos Múltiplos", V Congresso Nacional de Engenharia Mecânica, Salvador, BA, Brazil.

Spong, M. W. and Vidyasagar, M., 1989, "Robot Dynamics and Control", John Wiley and Sons, New York.

Thomas, J. E., 2001," Fundamentos de Engenharia de Petróleo", 1ª ed., Interciência: Petrobras, Rio de Janeiro, Brazil.

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