A ROBUST NON-LINEAR DESIGN FOR AUTOPILOT CONTROL LAW USING DYNAMIC INVERSION AND H-INFINITY LOOP-SHAPING

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Abstract. *This paper presents a robust control law design for a commercial aircraft pitch autopilot dedicated to perform the approach maneuver. The design involves only the longitudinal axis. Traditionally, in the aerospace industry, the design of autopilot functions follows a process that involves the partitioning of the flight operational envelope into separate flight conditions, the design of a compensator, for each of the flight conditions, to satisfy closed-loop specifications and, finally, a gain schedule is designed to make the final control law cover the full flight envelope. However, the practice shows that the resulting control law is not able to deal satisfactorily with plant variations and the intrinsic non-linear behavior of the aircraft. The process outlined in this paper combines two modern control law design techniques, namely Dynamic Inversion (DI) and H-infinity loop-shaping, in order to provide a robust control law. The design is divided into two stages: (a) Dynamic Inversion is used in the inner-loop design to generalize the dynamic of the aircraft. It uses feedback linearization in order to deal with the aircraft non-linear behavior and replaces it by a linear time invariant system defined by the inner-loop compensator, witch eliminate the design of a gain schedule. (b) Robust H-infinity loop-shaping technique is used on the outer-loop design. The objective is to adjust the open-loop frequency response of the system to a desirable reference system with specified robust stability and performance requirements. The designed compensator adjusts both the open-loop singular value frequency response to the reference system and a coprime factor that is directly related to the stability and performance margins. The final result is evaluated by means of frequency domain singular value response and time domain simulation, using Dryden wind disturbance mode.*

Keywords: *Robust Control; Dynamic Inversion; Automatic Control*

1. INTRODUCTION

The control law designed in this project has a structure with two loops. The inner-loop comprises the application of the Dynamic Inversion technique (Enns *et al.*, 1994) to adjust the longitudinal movement, pitch and pitch rate dynamics. The outer-loop is responsible to control the speed and the trajectory of the aircraft; it uses H_∞ loop-Shaping (Skogestad *et al.*, 2001) technique to adjust the frequency response of the system in order to satisfy performance and stability requirements.

The main objective with this type of structure is to combine the benefits of both techniques. The Dynamic Inversion tries to solve the problem of making control law design that works for a plant with considerable variation. The aircraft plant is an example of that, the control must keep the performance and stability for different flight conditions (Altitude, temperature and speed) and different configurations (flaps, landing gear, weight, etc). This technique tries to eliminate the gain scheduling process using the knowledge of the aircraft (model) in the structure of the control law.

The technique used for the outer-loop, H_∞ loop-shaping, is used to assure that the controlled system has robust stability and performance with respect to gain bounds for the open-loop frequency response of the system.

2. APPROACH DYNAMIC

The instrumented approach means that the approach maneuver is done with the support of ILS (“Instrumented Landing System”) (Stevens *et al.*, 1997). This radio system of the airport provides orientation to the approaching aircraft in terms of deviation of the reference path that would allow the aircraft to reach the runaway threshold. The aircraft must follow a ramp with a -3º inclination ramp, for example, and the radio signal from the ILS will provide the information of lateral and longitudinal deviation from the reference path, Glide-slope and Localizer signals.

The approach maneuver can be divided in three phases, Capture, Tracking and flare.

The Capture represents the transition from an altitude hold condition to start to follow the path defined with the support of the ILS of the runaway. During the Tracking phase the aircraft must follow the reference path. The Flare maneuver is the transition from the reference path to a reduced inclination path that will reduce the energy of the aircraft for the touchdown.

The objective of the Auto Pilot function of this project is to execute the Tracking maneuver; it should control deviation from a reference path and the reference speed. Figure 1 illustrates the deviation from the reference path that the automatic function has to control.

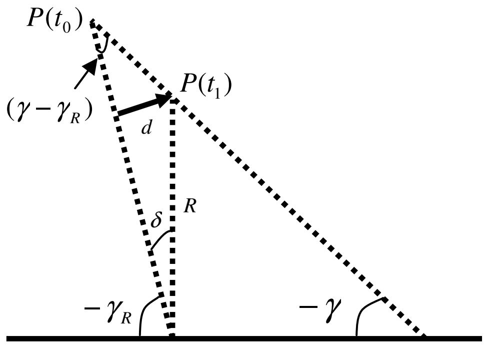

Figure 1. Definition of the deviation from the reference path

Equation (1) formulates the dynamic of the deviation.

$$\dot{d} = V_T \sin(\gamma - \gamma_R) \tag{1}$$

3. AIRCRAFT MODEL

The aircraft model used it this project was extracted from Roskan (2000), It has all the stability and controls derivatives (stability axis) from a commercial aircraft in an approach condition.

The aircraft is trimmed in the following condition, Table 1.

Table 2. Parameters of the trimmed condition

Parameter	Value	Description	Parameter	Value	Description
δ_h	-4.77	Horizontal Stabilizer (deg)	γ	-3	Flight path angle (deg)
δ_m	55	Throttle position (%)	V_T	65	Air Speed (m/s)
α	10.68	Angle of attack (deg)	H	305	Altitude (m)

The variables used to reach the trimmed condition were α , δ_m and δ_h . It was decided to trim the aircraft using the horizontal stabilizer not the elevator, the elevator position is null for the trimmed condition, and therefore the control law has the full nose up and nose down range of the elevator to control the aircraft.

The matrices A and B of the linear model of the aircraft for the condition described above are the following:

$$A = \begin{bmatrix} -0.0441 & 3.9395 & 0 & -9.7932 \\ -0.0045 & -0.4599 & 0.9721 & 0.0079 \\ -0.0001 & -0.3495 & -0.4070 & -0.0004 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1.6880 & 0 & 0.0267 & 0.0967 & 0 \\ -0.0051 & -0.0278 & 0.0055 & 0.0081 & 0.0279 \\ 0.0100 & -0.3602 & -0.0012 & 0.0053 & 0.3524 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Where the states are: $\begin{bmatrix} V_T & \alpha & q & \theta \end{bmatrix}'$; the inputs: $\begin{bmatrix} \delta_m & \delta_p & u_g & w_g & q_g \end{bmatrix}'$; witch are: throttle, elevator, horizontal gust, vertical gust and pitch rate gust.

4. CONTROLLER DESIGN

The robust controller for the tracking of the Glide-slope signal design was executed in two steps in order to take credit from two control design techniques with distinctive objectives; they are Dynamic Inversion and H_∞ loop-shaping. The main design objective is to obtain a controller robust to disturbance effects and high-frequency non-modeled dynamics.

The structure of the control law can be divided in two feedback loops. The inner-loop is used to generalize the longitudinal response. It generates command to the elevator in order to control the pitch and pitch-rate movement.

The outer-loop design uses H_∞ loop-shaping to elaborate a robust controller. The outer-loop generates command to the inner-loop and also controls the speed in order to keep the aircraft on the reference trajectory and to maintain the target speed.

4.1. Inner-Loop

The dynamic inversion technique in this project is used to adjust the dynamic of two states of the plant, the pitch angle (θ), that represents the phugoidal longitudinal movement, and the pitch rate angle (q), that represents the short period movement of the aircraft. The control of the short period allows that flying qualities criterion, like MIL-1787 (1997), can be used to define the closed loop performance. The control of pitch angle is used to make the interface with the outer-loop, witch is responsible to control the trajectory of the aircraft.

The Dynamic Inversion procedure comprises the choice of the controlled variable, the feedback linearization, that cancels the dynamic of the controlled variable, and the definition of the compensator, that will define the dynamic of the controlled variable.

4.1.1 Controlled variable and zero dynamics

In order to control both variables (q and θ), it was user the "Multiple Time Scale" technique (Ito *at al.*, 2002), witch is a procedure that allow the control the fast and slow dynamic separately.

For the linear case, the dynamic of both variables can be simplified like the equation (2) and (3)

$$[\dot{\theta}] = [A_{44}][\theta] + [A_{43}][q] \tag{2}$$

$$[\dot{q}] = [A_{31} \quad A_{32} \quad A_{33} \quad A_{34}]\begin{bmatrix} V \\ \alpha \\ q \\ \theta \end{bmatrix} + [B_{31}][\delta_p] \tag{3}$$

Applying the feedback linearization (4), (5):

$$[q_{cmd}] = [A_{43}]^{-1}\{[\dot{\theta}_{des}] - [A_{44}][\theta]\} \tag{4}$$

$$[\delta_p] = [B_{31}]^{-1}\left\{[\dot{q}_{des}] - [A_{31} \quad A_{32} \quad A_{33} \quad A_{34}]\begin{bmatrix} V \\ \alpha \\ q \\ \theta \end{bmatrix}\right\} \tag{5}$$

The Dynamic Inversion structure can be easily understood by a diagram. Figure 2 shows the "Multiple Time Scale" structure, the internal loop controls the pitch rate and the external loop control the pitch angle. Also, the A(4,4) term is zero and the A(4,3) term is one in the equation (4), then the input signal to the pitch rate compensator is the pitch angle error (6).

$$q_{cmd} = \theta_{cmd} - \theta_{sensor} \tag{6}$$

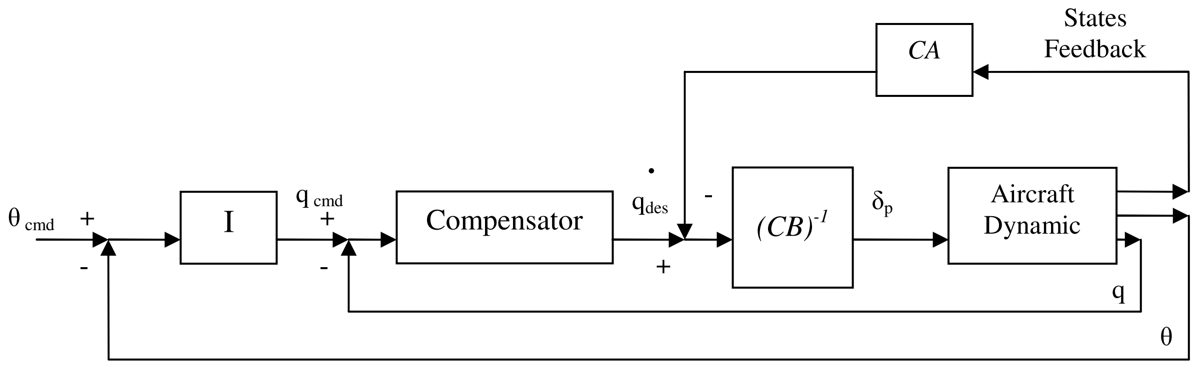

Figure 2. Definition of the deviation from the reference path

It is important to notice that it is necessary two compensators, one for each control. The main objective of the inner-loop is the control of the short period movement, the compensator for this control was designed to achieve some performance requirements. The compensator of the pitch angle is properly chosen equal to 1.

4.1.2 Compensator definition

The compensator structure has proportional and integral terms (7), and it is design in such a way that the closed loop transfer function becomes a first order dynamic with a desirable time constant.

$$\dot{q}_{cmd} = \left[K_B \left(\frac{1}{2} q_{cmd} - q \right) + \frac{{K_B}^2}{4} \frac{1}{s} \left(q_{cmd} - q \right) \right] \tag{7}$$

For this project it is desirable to place a pole at 0,5 rad/s, then the KB value must be equal to 1. The equation (10) is the close loop transfer function of the pitch rate response, considering that the feedback linearization was able to reduce the controlled variable dynamic to an integrator (8).

$$\frac{q}{\dot{q}_{cmd}} = \frac{1}{s} \tag{8}$$

$$q = \frac{1}{s} \left[K_B \left(\frac{1}{2} q_{cmd} - q \right) + \frac{{K_B}^2}{4} \frac{1}{s} \left(q_{cmd} - q \right) \right] \tag{9}$$

$$\frac{q}{q_{cmd}} = \frac{2 K_B s + {K_B}^2}{4 s^2 + 4 K_B s + {K_B}^2} = \frac{\frac{K_B}{2}}{s + \frac{K_B}{2}} \tag{10}$$

4.1.3 Non-linear case

The inner-loop compensator design is done using the linear model of the aircraft, this make possible the use of all the control theory in the design and in analysis of the controlled plant. However, the dynamic inversion can be applied using the non-linear model as well. The use of non-linear model on the control law structure can provide a better cancellation of the controller variable dynamic.

The non-linear system is given by (11) and (12), where F(x) and G(x) are non-linear function dependent of the states values:

$$\dot{x} = f(x) + g(x)u \tag{11}$$

$$\dot{y} = \frac{\partial h}{\partial x} \dot{x} = \frac{\partial h}{\partial x} f(x) + \frac{\partial h}{\partial x} g(x)u = F(x) + G(x)u \tag{12}$$

Applying the dynamic inversion theory to the non-linear model, the result is the equation (13).

$$[\delta_P] = G^{-1}(x)[[\dot{q}_{des}] - F(x)] \tag{13}$$

Where:

$$F(\alpha, q, V) = \frac{M_a}{I_{yy}} \tag{14}$$

$$G(V) = \frac{I_{yy}}{qScC_{m_{\delta_P}}} \tag{15}$$

$$M_a = qScC_m \tag{16}$$

$$C_m = C_{m\alpha}\alpha + \frac{C_{mq} qc}{2V} \tag{17}$$

4.2. Outter-Loop

The classic robust control design, the robust requirements are defined in terms of gain limit for the frequency response of the system. Basically it is defined a minimum gain limit for the low frequency region on the singular value graph of the open-loop and a maximum limit for the high frequency region (Skogestad *et al.*, 2001).

Figure 3 shows the generalized feedback configuration for the plant and the exogenous inputs, reference (r), disturbance (d) and noise (n). It helps the robustness analysis. G_p represents the plant (aircraft model after the output redefinition), K represents the controller.

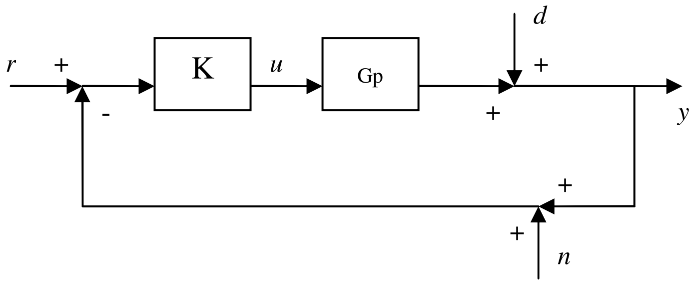

Figure 3. Generalized Feedback Configuration

From the figure above it is possible to analyze the effects of the reference, noise, and disturbance to the output of the system. Speaking in terms of closed loop behavior the transfer function of reference and noise to the output is T (co-sensitivity function) and the transfer function from disturbance to the output is S (sensitivity function).

$$S = (I + GK)^{-1} \tag{18}$$

$$T = GK(I + GK)^{-1} = I - S \tag{19}$$

To reach an satisfactory behavior of the closed loop system it is definite some criterion in terms for the sensitivity and co-sensitivity functions. This is described in terms of maximum and minimal singular values (Lewis, 2000):

1. $\underline{\sigma}(S)$ value must be small for disturbance rejection.
2. $\overline{\sigma}(T)$ value must be small for noise rejection.
3. $\overline{\sigma}(T) \approx \underline{\sigma}(T) \approx 1$ for reference tracking.
4. $\overline{\sigma}(T)$ value must be small for high frequency non-modeled dynamic rejection

Even if these requirements seems conflicting, they aren't because the characteristic of each exogenous signal. The criteria 1 and 3 influence the low frequency response while the 2 and 4 influence the high frequency response.

The above criteria can be approximated to open-loop criteria, from the fact that for low frequencies $\underline{\sigma}(GK) \approx \frac{1}{\overline{\sigma}(S)}$ and for high frequency $\overline{\sigma}(GK) \approx \overline{\sigma}(T)$, witch make simple the use of the loop-shaping technique. The criteria for open-loop frequency response can be summarized below:

1. $\underline{\sigma}(L)$ value must be large for disturbance rejection and reference tracking, low frequency.
2. $\overline{\sigma}(L)$ value must be small for noise rejection, high frequency.

4.2.1 H_∞ Control

The technique, propose by McFarlane and Glover (1990), is applied to system with one degree of freedom, and It is basically two steps design. The first step is to shape the singular value frequency response by selecting the pre and pos-compensator.

The second step is to obtain the sub-optimal controller that stabilize the shaped plant (G_S=W_2GW_1). The procedure for the second step is obtaining the minimum coprime factor; witch is a stability margin indication for the controlled system (20).

$$\gamma_{\min} = \frac{1}{\varepsilon_{\max}} = \left(1 + \rho(XZ)\right)^{\frac{1}{2}} \tag{20}$$

Where X and Z are positive defined unique solution of the two Riccati equation listed below:

$$(A - BS^{-1}D^T C)Z + Z(A - BS^{-1}D^T C)^T - ZC^T R^{-1}CZ + BS^{-1}B^T = 0 \tag{21}$$

$$(A - BS^{-1}D^T C)^T X + X(A - BS^{-1}D^T C) - XBS^{-1}B^T X + C^T R^{-1}C = 0 \tag{22}$$

$$R = I + DD^T \tag{23}$$

$$S = I + D^T D \tag{24}$$

The minimum coprime factor should have a value less than 4, this represents a stability margin of 0,25 ($\varepsilon_{\max} > 0{,}25$).

After the definition of the minimum coprime factor, it is selected another coprime factor to achieve the sub-optimal controller ($\gamma > \gamma_{\min}$). Usually a factor 10% higher than the minimum produces good results.

The sub-optimal controller must satisfy:

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} M^{-1} \right\|_{\infty} \leq \gamma \tag{25}$$

The solution for the controller is:

$$K = \begin{bmatrix} A + BF + \gamma^2 (L^T) ZC^T (C + DF) & \gamma^2 (L^T)^{-1} ZC^T \\ B^T X & -D^T \end{bmatrix} \tag{26}$$

Where,

$$F = -S^{-1}(D^T C + B^T X) \tag{27}$$

$$L = (1 - \gamma^2) I + XZ \tag{28}$$

4.2.2 Robusteness limits

The robustness limits are, basically a upper gain limit for the high frequency region of the singular value graph and a lower limit for the low frequency region.

The upper limit is defined by the first flexional mode of the aircraft, witch is represented by a second order dynamic with frequency of 40 rad/s and damping ratio 0.3. It is considered as a multiplicative uncertainty.

$$M(s) = F(s) - I = \frac{-s(s + 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^{\,2}} \tag{29}$$

Where: $\omega_n = 40 rad/s$, $\zeta = 0{,}3$

The upper bound is given by the magnitude de 1/M(s).

The lower bound, in this project, is defined by the influence of vertical wind gust. It was used the Dryden wind turbulence model, accord the norm MIL-1797 (1997).

For the vertical wind, it has the following transfer function (30).

$$H_w(s) = \sigma_w \sqrt{\frac{2L_w}{\pi V} \frac{1+\frac{2\sqrt{3}L_w}{V}}{\left(1+\frac{2L_w}{V}s\right)^2}} \tag{30}$$

When L_w represents a scale factor to the turbulence, V is the aircraft speed and σw the turbulence intensity. The parameters values are defined in Table 2.

Table 2. Parameters of the wind turbulence model

Parameter	Value	Description
h	1000 ft	Altitude
w_{20}	10 ft/s	Wind Speed at 20ft
$L_w = h/2$	500	Scale Factor
$\sigma_w = 0{,}1 * w_{20}$	1	Turbulence Intensity

4.2.3 Reference Model Definition

The desired reference model is defined in such a way that it respects the robustness bounds and gives the necessary performance for the system. Usually, and integrator with a desired crossover frequency attend the requirements. The definition of the reference model starts with a integrator, but it can be interactively modified to a higher order system in order to increase the low frequency gain or the roll off rate, if it is necessary. But it must keep the roll-off rate not more then -20 dB/dec at the crossover frequency.

After some iteration the desired reference model is (31).

$$G_{ref}(s) = 1.2 * \frac{(0.5s + 0.1)}{(s^2 + 0.1s)} \tag{31}$$

Figure 4 shows the result of the initial definition for the loop-shaping method, robustness bounds and the desired reference model.

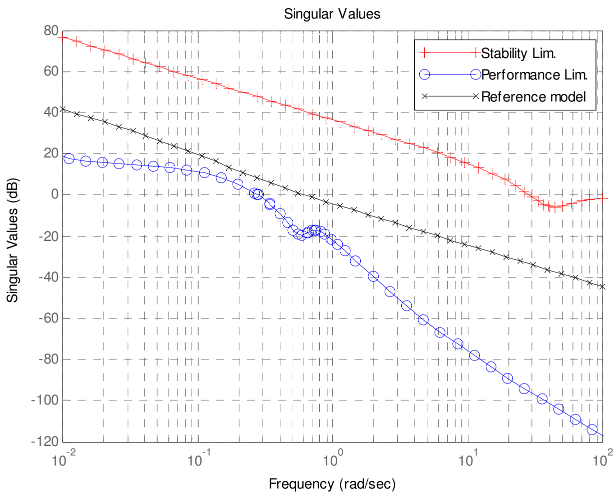

Figure 4. Preliminary definition of the H_∞ loop-shaping design

4.2.4 Robust stabilization

The H loop-shaping algorithm is already implement in standard available routines in commercial software (MATLAB) (Gu *et al.*, 2005), witch executes the two steps described in the item 4.2.1, based on the initial definition of the desired loop frequency response (bounds and reference model)

The shaped plant and the final controller are presented:

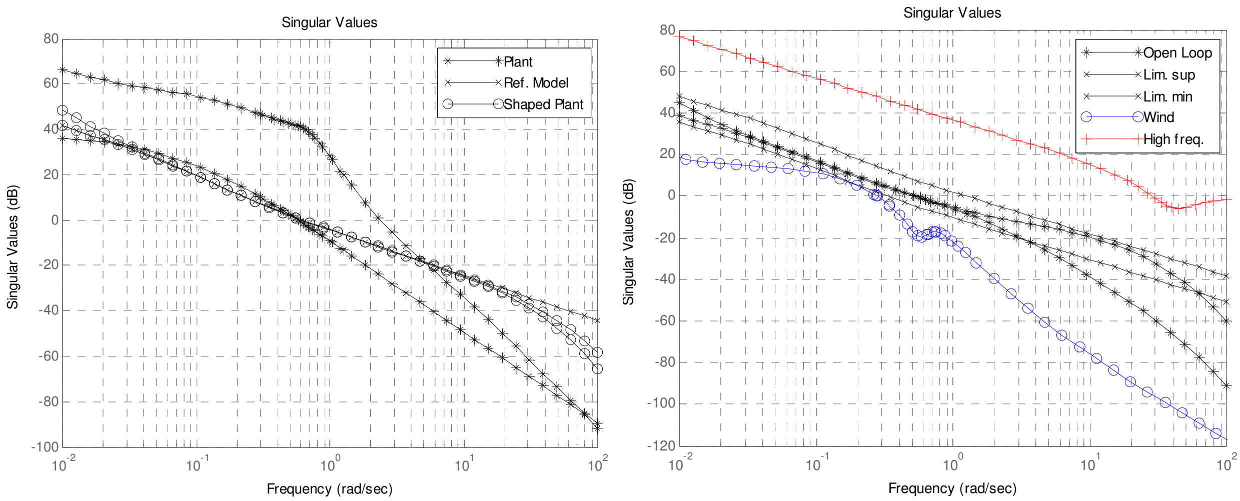

Figure 5. (a) Open-loop frequency response, (b) Open-loop controlled plant

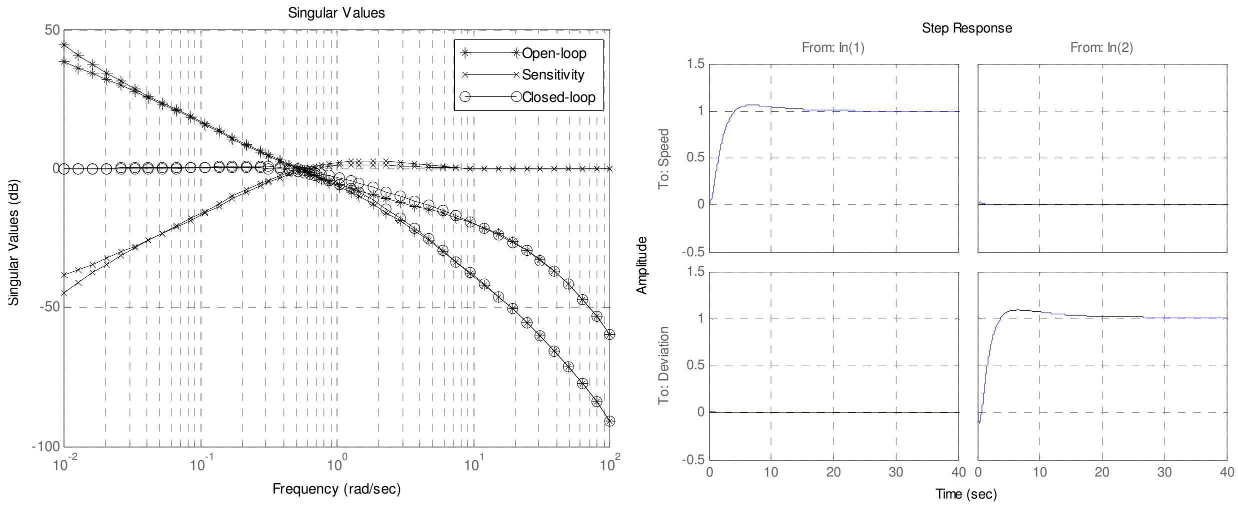

Figure 6. (a) Closed-loop frequency response, (b) Closed-loop step response

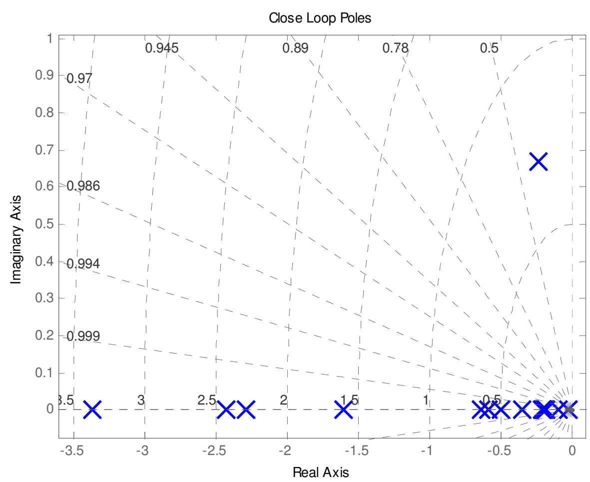

Figure 7. Closed-loop poles

Figure 5 shows the open-loop frequency response of the plant. (a) compares the maximum and minimum singular values of the original plant, the reference model and the shaped plant. Inside the frequency range, 0,01 to 100 rad/s the maximum and minimum singular values become very close each other and close to the reference model.

Figure 5 (b) shows the maximum and minimum singular values of the controlled plant (GK) and the robustness bounds. The two lines, above and below the controlled plant, are the limits of the coprime factor (Gd/γ e Gd*γ) obtained γ = 2.0380.

The coprime factor is a indication of the accuracy of the controller adjusted the original plant to the reference model and it is also a indication of the stability margin for plant variations.

Figure 6 shows the results for the closed-loop frequency response. (a) Sensitivity and co-Sensitivity functions. The Sensitivity function is related to the disturbance rejection and phase margin, usually with a Sensitivity peak less than 3 dB grantee the stability margin.

Figure 6 (b) shows the step response for both channels (speed and drift). The overshoot is less than 10% and no oscillatory behavior is observed. Also, it is important to notice that the channels are decoupled.

Finally, the designed controller usually has a large order; witch is 22 for this project. However, it is possible to use techniques to reduce the order of the controller but keep the system performance.

Again, standard available routines (MATLAB) are available, and provide good solution for order reduction. The technique applied to this project was Hankel singular values (Gu *et al.*, 2005), the theory is that the controller states with Hankel singular value too small (with respect to the other states) it can be discarded. Some trial and error is necessary. It was possible to reduce the controller order from 22 to 11 states, keeping the performance of the system very close to the original controller.

5. SIMULATION

The non-linear simulation is used to the final evaluation of the control law. The model has is its structure the non-linear feedback linearization, as described on item 4.1, and the robust controller, as presented on item 4.2.

The simulation starts with the condition presented on item 3, at 20 sec it is applied a wind gust of 10 m/s. The time history in Figure 8 show that the effect of the wind. The control law is able to reject both the speed and the deviation error.

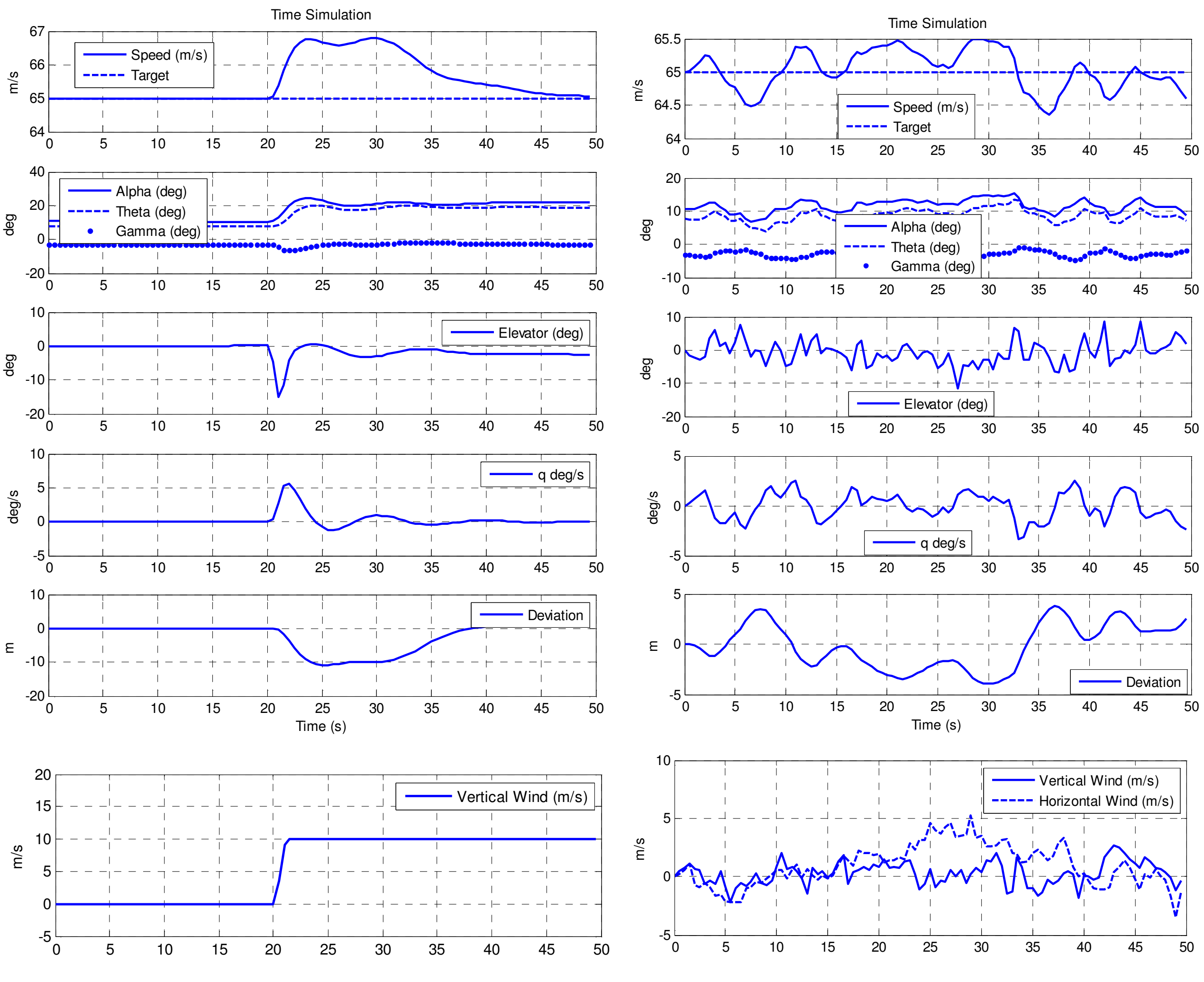

Figure 8. Non-linear simulation, (a) Wind gust, (b) Wind turbulence.

6. ACKNOWLEDGEMENTS

The Dynamic Inversion technique showed effective in the control of the state dynamic of θ and q, manly due to the use of the "multiple-time-scaling" method, which allowed the specification of a compensator for the fast dynamic and for the slow dynamic separately.

However, it is important to notice that the feedback linearization will not perfectly reduce the controlled variable dynamic to an integrator. This is because, the model is always a simplification of the real world and because of limitation of states feedback, and maybe not all the states are available for feedback for example. Besides that, the signal of the states feedback will always have some noise and delays.

In order to deal with this problem the compensator must satisfy not only the flying qualities criterion, it must provide satisfactory stability margins in order to deal with the plant uncertainty.

A relevant fact about the H∞ loop shaping result is the controller order. Even with a order reduction to half of the original order (22 states), keeping the performance, 11 states it is still considered a high order controller if it is compared with the result of other control law structure / technique.

However, nowadays it is not a barrier to the implementation, mostly to the fact that the computational capacity of today's computers allows the use of complex control law structures. Also, the use of the dynamic inversion shall eliminate the process of use gain tables to make the control law work for the entire operation envelope for this maneuver.

An important result of the H∞ "loop shaping" technique, in addition to the fact that the controller is stable, is the coprime factor (γ). The coprime factor represents the stability margin for plant variations. The final result of $\gamma = 2.0386$ allow that the control law keep the plant (with coprime factorization) stable even with a variation of $\varepsilon\text{max} = 1/\gamma = 49$ %.

7. REFERENCES

Enns, D., Bugaski, D., Hendrick, R., Stain, G., 1994, "Dynamic Inversion: an evolving methodology fro flight control design", International Journal of Control, Vol. 59, No. 1, pp. 71-91.

Gu, D., Petkov, P., Konstantinov, M., 2005, "Robust Control Design with MATLAB", Ed. Springer, New York, United States, 389 p.

Ito, D., Geoorgie, J., Valasek, J., 2002, "Reentry Vehicle Flight Controls Design Guidelines: Dynamic Inversion", National Aeronautics and Space Administration/TP, United States.

McFarlane, D. C., Glover, K., Sefton, J., 1992, "A tutorial on loop shaping using H-infinity robust stabilisation", Trans. Inst. M & C, Vol. 14, No. 3.

Roskan, J., 1998, "Airplane Flight Dynamics and Automatic Flight Control Systems – Part I and II", DAR Corporation.

Sandraey, M., Colgren, R., 2005, "Two DOF Robust Nonlinear Autopilot Design for small UAV using a combination of dynamic inversion and H-infinity loop-shaping", AIAA Guidance, Navigation and Control Conference and Exhibit, San Francisco, United States.

Skogestad, S., Postlethwaite, I., 2001, "Multivariable Feedback Control: Analysis and Design", Ed. John Wiley & Sons Inc., New York, United States, 595 p.

Stevens, B., Lewis, F., 1992, "Aircraft Control and Simulation", Ed. John Wiley & Sons Inc., New Jersey, United States, 664 p.

United States Department of Defense, 1997, "MIL-STD-1797: Flying Qualities of Piloted Aircraft", Washington, United Stated, 849 p.

8. RESPONSIBILITY NOTICE