MAXIMUM HEAT TRANSFER RATE DENSITY FROM A ROTATING MULTISCALE ARRAY OF CYLINDERS

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Abstract. *This paper studies the effect of laminar flow on multiscale heated rotating cylinders in cross-flow. The objective is to maximize the heat transfer rate density of the assembly, under a given pressure drop and subject to the constraint of materials and volume. This problem is solved numerically, with two main configurations: i) Both cylinders share a common centre-line, and ii) Both cylinders share a leading edge to the advancing coolant. Results are reported for two types of configurations; firstly, counter-rotating cylinders in cross-flow and then co-rotating cylinders in cross flow. The spacing between these two types of configuration is optimized. Results shows that the counter-rotating cylinders is more effective than the co-rotating cylinders at a higher pressure drop, while the difference is less noticeable at lower pressure drop.*

Keywords*: multiscale rotating cylinders, heat-transfer-rate density, co-rotating, counter-rotating*

1. INTRODUCTION

Due to the need for more efficient removal of heat from heat transfer generating equipment, research is currently being conducted in this area with the aim of accommodating more and more heat generating devices within a given volume. This will help in the design, manufacture and operation of such equipment. Modern electronic systems do produce high amounts of heat due to the power to volume or weight ratio employed in such systems. The heat produced, if not removed could lead to failure of parts of the system and in some cases failure of the whole system.

Effects of heat transfer from cylinders as well as the optimisation of design of such material have been conducted by a long list of researchers such as Badr and Dennis (1985), Mahfouz and Badr (1999), Gschwendtner (2004), Misirlioglu (2006), Paramane and Sharma (2009) amongst others. Optimisation of the spacing between cylinders have been conducted by Stanescu et al (1996) who found that with an increase in the Reynolds number the spacing between the cylinders consequently decreases. Mohanty et al (1995) compared the effect of rotation against pure cross flow with the same set of Reynolds numbers; they reported that at equal rotational and free stream Reynolds numbers, the localised heat transfer at the stagnation point of the rotational cylinder is lower than that of the pure cross flow. This view is somewhat supported by Tzeng et al (2007) where they found that at higher Reynolds' number the cooling efficiency is increased on high-velocity rotating machines.

According to Jones et al (1988), the heat transfer from a cylinder is governed by three mechanisms viz.: forced convection due to the bounding volume within which the cylinder is located forced convection due to rotation and natural convection. Part of the assumptions made in this work disregards the third mechanism mentioned above. It was further stated by Jones et al (1988) and confirmed by Joucaviel et al (2008) that rotation does enhance heat transfer, "viscous forces acting in the fluid due to rotation cause mixing of the fluid and augment heat transport in a way similar to turbulence"

In the words of Bello-Ochende and Bejan (2004), "Strategy and systematic search mean that architectural features that have been found beneficial in the past can be incorporated and compounded into more complex flow structures of the present". This work involves the addition of more length scales to those of Joucaviel et al (2008). The length scales are the number of cylinders, the difference in diameters of subsequent cylinders and the spacing between the cylinders.

This paper builds on prior research conducted by Bello-Ochende and Bejan (2004) and more recently added to by Joucaviel et al (2008), in which it is proved that rotational effects increase the heat-transfer-rate density for single scale cylinders. This paper focuses on the optimisation of the heat-transfer-rate density of multiscale cylinders cooled by cross flow fluid in the laminar regime. The flow is driven by a fixed pressure difference across the domain in consideration. Applications of heat transfer from rotating cylinders are found in rotating machineries, heat exchangers, viscous pumps, rotating electrodes, spinning projectiles as well as contact cylinder dryers in the paper industry.

2. MODEL

2.1. Cylinders aligned along the same centre-line

We start by considering a case when the cylinders are aligned along the same centre-line. Figure 1 depicts the model which represents a multi-scale array of cylinders set along the same centre-line, and due to the repetitive nature of the stack a domain containing two different size cylinders is chosen to represent the numerical region of interest. The figure further shows the flow across the domain is driven by a fixed pressure drop ΔP, the tip to tip distance between the cylinder is S and are assumed equal for a case with no eccentricity. The cylinders rotate with an angular velocity ω. The fluid inlet temperature T_∞ is fixed and lower than the temperature of the wall of the cylinders, T_w which are assumed constant.

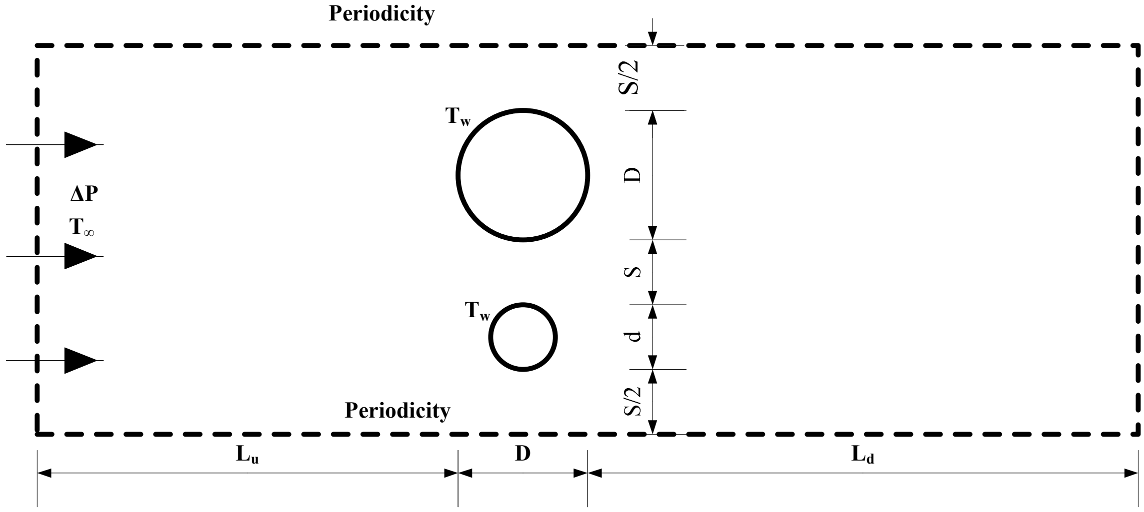

Figure 1 Both cylinders have a common centre-line.

Additional assumptions include steady, laminar, incompressible and two dimensional flow. All the thermo-physical properties are assumed constant. In this setup two configurations are investigated, that is, the cylinders rotating in the same direction (case I) and secondly both cylinders rotating in counter directions (case II) to each other. The domain upper and lower walls are chosen as periodic with the rotational direction reversed for cases of counter-rotation. The heat flux q' removed from the assembly per unit length perpendicular to the above figure can be written in the following form,

$$q' = D\int_0^{2\pi} q_w d\theta + d\int_0^{2\pi} q_w d\theta \tag{1}$$

The volume per unit height of interest occupied by the modeled assembly is $D(D + d + 2S)$, where D is the diameter of the bigger cylinder, d is the diameter of the small cylinder, and q_w is the heat transfer from the cylinder. The heat transfer rate density is given as

$$q''' = \frac{q'}{D(D+d+2S)} \tag{2}$$

And this represents the total heat per unit volume.

2.2.Mathematical Formulation

The governing equations of the fluid flow through the multi-scale rotating cylinders are the conservation of mass, momentum and energy equations. The computational domain is in two dimensions as shown in figure 2 and the assumptions made with respect to its solution have been given in the previous section.

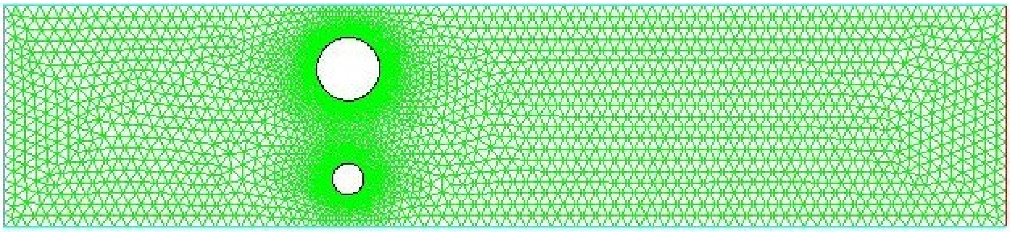

Figure 2 Discretized domain of cylinders with the same centre-line.

The equations, which represent the conservation of mass, momentum and energy equation in the primitive forms are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\mu\nabla^2 u \tag{4}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+\mu\nabla^2 v \tag{5}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\nabla^2 T \tag{6}$$

Where u and v represent the velocity components in the Cartesian coordinate directions, x and y. ρ is the density, μ is the viscosity, k is the thermal conductivity, c_P is the heat capacity, P is the pressure, and T is the temperature . Additional assumptions include constant solid and fluid properties and the heat transfer due to radiation is negligible. Equations (3) – (6) in dimensionless forms are,

$$\frac{\partial \tilde{u}}{\partial \tilde{x}}+\frac{\partial \tilde{v}}{\partial \tilde{y}}=0 \tag{7}$$

$$\frac{Be}{\Pr}\left(\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}}\right)=-\frac{\partial \tilde{P}}{\partial \tilde{x}}+\nabla^2\tilde{u} \tag{8}$$

$$\frac{Be}{\Pr}\left(\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}\right)=-\frac{\partial \tilde{P}}{\partial \tilde{y}}+\nabla^2\tilde{v} \tag{9}$$

$$Be\left(\tilde{u}\frac{\partial \tilde{T}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{T}}{\partial \tilde{y}}\right)=\nabla^2\tilde{T} \tag{10}$$

The non-dimensionalized variables used are:

$$\tilde{\mathrm{d}}=\frac{\mathrm{d}}{\mathrm{D}}\quad \tilde{x},\tilde{y},\tilde{S}=\frac{x,y,S}{D},\ \tilde{u},\tilde{v}=\frac{u,v}{\Delta PD/\mu}\quad \tilde{P}=\frac{P}{\Delta P}\qquad \tilde{T}=\frac{T-T_\infty}{T_W-T_\infty} \tag{11}$$

Where the Bejan and Prandtl numbers are $Be = D^2\Delta P/\mu\alpha$ and $\Pr = \nu/\alpha$.

The flow boundary conditions are: $\tilde{P}$ = 1 at the inlet plane and zero normal stress at the outlet plane. The thermal boundary conditions are $\tilde{T}$ = 0 at the inlet plane and $\tilde{T}$ = 1 on the cylinders surfaces. The horizontal surfaces of the domain correspond to periodic conditions due to the rotations of the multi-scale cylinders. The cylinders are rotating at ω and therefore an angular velocity is imposed as a boundary condition on the cylinder surfaces,

$$\tilde{\omega} = \frac{\omega\mu}{2\Delta P} \tag{12}$$

The objective function (i. e. heat transfer rate density), Eq. 2, can be written in dimensionless form as

$$\tilde{q} = \frac{q''' D^2}{k\left(T_w - T_\infty\right)} = \frac{1}{(1+\tilde{d}+2\tilde{S})}\int_0^{2\pi}\left[-\nabla\tilde{T}+\tilde{d}(-\nabla\tilde{T})\right]_n d\theta \tag{13}$$

2.3. Numerical solution

The finite volume code software FLUENT™ was employed to solve Eqs. (7) - (10). The domain was discretized using a second order discretization scheme. The pressure-velocity coupling was done with the SIMPLE algorithm. Numerical convergence was obtained in two ways, firstly convergence was obtained when the scaled residuals for mass and momentum equations were smaller than 10^{-4} and the energy residual was less than 10^{-7}. In the second option numerical convergence was obtained when there was no further change in the value of residuals for consecutive iterations in terms of the specified criteria such as conservation of mass flow rate in the domain.

An additional method used to ensure accuracy of the results was to perform grid refinement tests. The key quantity monitored in this regard was the overall heat transfer rate density. Figure 2 shows that the grid was refined by placing or concentrating the mesh in the region closest to the cylinders, where the thermal gradient was high. Virtual extensions, $\tilde{L}_u$ and $\tilde{L}_d$ had been added to the numerical domain downstream and upstream of the physical domain to adequately handle the pressure boundary conditions. The length of the virtual extension was chosen long enough so that with any further increase in length, the change in the heat transfer rate density between two iterations, i and i+1 is smaller than 1%.

Table 1a and 1b shows how the virtual extension was chosen. For Be = 10^3 and $\tilde{S}$ = 1, and $\tilde{\omega}$ = 0, the virtual extensions is $\tilde{L}_u = 5$, $\tilde{L}_d = 10$.

Table 1a. Keeping $\tilde{L}_d$ constant, $\tilde{\omega} = 0$, $\tilde{S} = 1$ and Be = 1000

$\tilde{L}_u$	$\tilde{q}$	$\left\lvert\frac{\tilde{q}_i - \tilde{q}_{i-1}}{\tilde{q}_i}\right\rvert$
3	17.5492	-
5	17.5788	0.0017
10	17.4778	0.0057
15	17.4218	0.0032

Table 1b. Keeping $\tilde{L}_u$ constant, $\tilde{\omega} = 0$, $\tilde{S} = 1$ and Be = 1000

$\tilde{L}_d$	$\tilde{q}$	$\left\lvert\frac{\tilde{q}_i - \tilde{q}_{i-1}}{\tilde{q}_i}\right\rvert$
5	17.4174	-
10	17.4778	0.0035
15	17.4587	0.0011
20	17.3673	0.0052

2.4.Optimal cylinder spacing

In order to obtain the optimized heat transfer rate density, the cylinder to cylinder distance is varied for a specified Bejan number, Prandtl number and dimensionless angular velocity. We proceeded with the optimisation by using the following criteria, the diameter of the bigger cylinder D was set at 1, while that of the smaller was fixed at $\tilde{d} = 0.25$, these values were obtained from previous work of Bello-Ochende and Bejan (2004) where it was found that for a multi-scale cylinder in cross-flow the optimal diameter of the second cylinder is robust and equal to 0.25. The range of parameter consider for this study is $10^3 \le \text{Be} \le 10^5$, $0 \le \tilde{\omega} \le 0.1$ and Pr = 0.71. The Reynolds number equivalence is in the laminar region of 10 – 10^3. Figures 3, 4 and 5 illustrate the heat transfer rate density obtained as a function of various rotational velocities and cylinder to cylinder spacing, $\tilde{S}$.

In the first case of cylinders on the same centre line with Be = 10^3 as seen in figure 3, it is observed that co-rotating cylinders provide better heat transfer rates with respect to the optimisation of the distance between the cylinders and the optimum distances between cylinders are smaller when compared to the optimal distance of the stationary cylinders.

The effect of counter rotating cylinders on the heat transfer rate densities is less when compared to stationary cylinders. For the co-rotating cylinders, an increase in the angular velocity translates to an increase in the heat transfer rate density.

Figure 4, shows the effect of rotation and cylinder to cylinder spacing on the maximum heat transfer rate density for Be = 10^4. The best configuration is that of the stationary flow, for both velocities used the co-rotating cylinders dissipate more heat than the counter rotating cylinders with a wide margin seen between 0.01 and 0.1, a possible explanation for

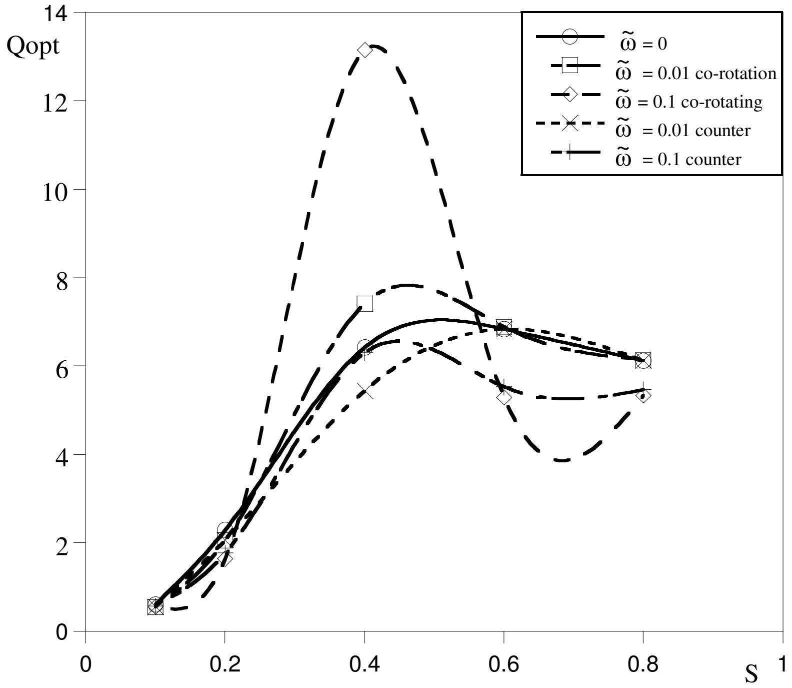

Figure 3 Optimal heat transfer rate density at Be = 10^3 for cylinders located inline.

this phenomenon is the entrainment of heated fluid due to the rotational direction of the cylinders.

Figure 5 shows the effect of rotation and the tip to tip spacing between the cylinders on the heat transfer rate density for Be = 10^5. At a lower dimensionless rotational velocity of 0.01 as shown in Figure 5, the effect of co-rotating cylinders on the heat transfer rate density is more noticeable when compare to the other two cases, that is there is heat transfer enhancement when the cylinders are co-rotating at that velocity. For other dimensionless rotational velocity their effect on the heat transfer rate density is insignificant when compared to that of a stationary cylinder.

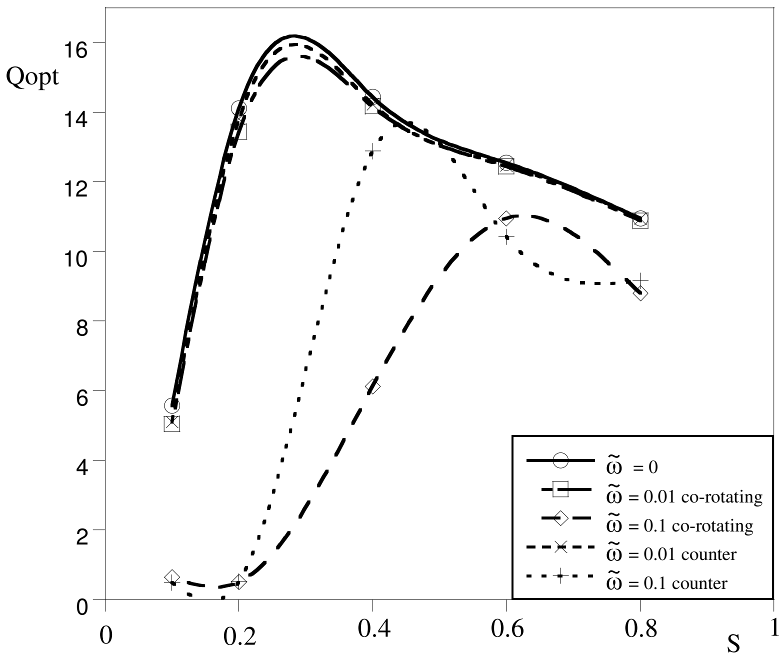

Figure 4 Heat Transfer Rate Density at Be = 10^4 for cylinders located inline.

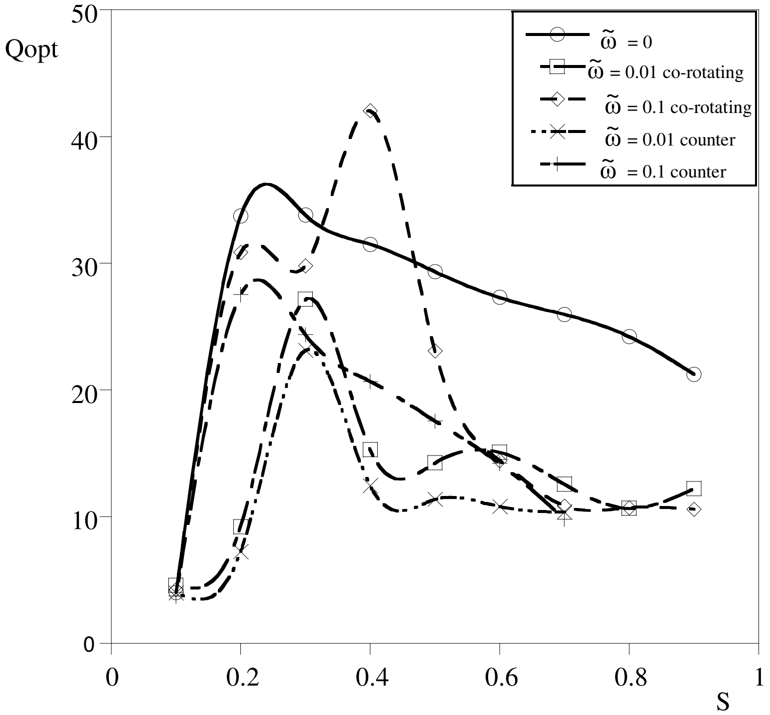

Figure 5 Heat transfer rate density for Be = 10^5 for cylinders located inline.

3. CYLINDERS ALIGNED ALONG THE LEADING EDGE

In the second type of configuration, we considered the case in which the cylinders share the same leading edge as is shown in figure 6 that is, both cylinders experience effect of the incoming coolant at the same time. The numerical procedure is the same as those of the case considered in Section 2 (when the cylinders are aligned along the same centre line) and described in Section 2.3. Figure 6 shows the computational domain while figure 7 shows the discretized domain for the second configuration.

We start the optimization procedure by setting the Bejan number to 10^2 and diameter of the smaller cylinder to 0.25 as obtained from Bello-Ochende and Bejan (2005) and shown in Figure 8. The result shows that there is no major improvement in heat transfer rate density by adding rotation to the array of cylinder, furthermore, the optimal distance is approximately equal to 1 for all the cases considered. The results also show for all the cases considered the heat transfer rate densities are identical and therefore all the curves in Figure 8 coalesce to a single curve at the optimal point.

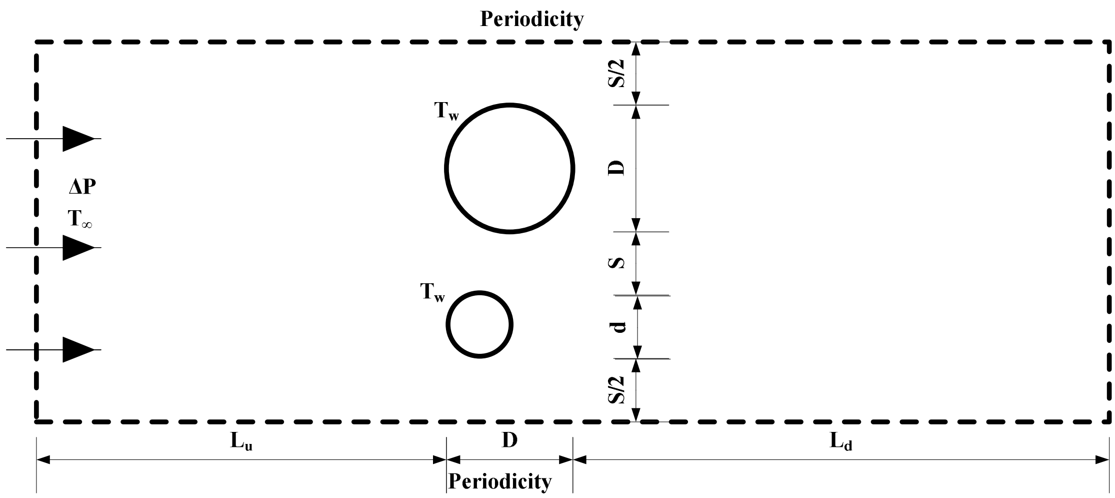

Figure 6 The cylinders with a common leading edge

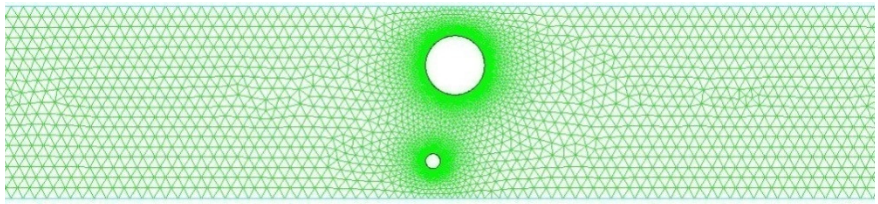

Figure 7 Discretized Domain of cylinders aligned along the leading edge

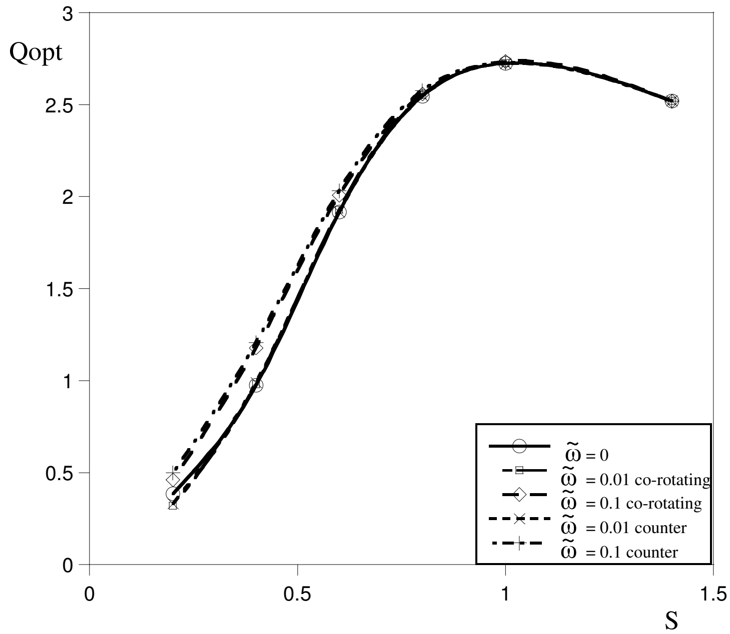

Figure 8 Heat transfer rate density at Be = 100 for cylinders sharing the same leading edge.

From Figure 9 we notice that the heat transfer rate density increases for both cases of co-rotating and counter-rotating cylinders when compared to the stationary cylinder for $\tilde{\omega} = 0.01$. Whereas, there is a significant difference in heat transfer rate density when compared to a stationary cylinder at a higher $\tilde{\omega}$ of 0.1. This might be due to the entrainment of heated fluids around the cylinder due to increase of rotation and the imposed pressure drop.

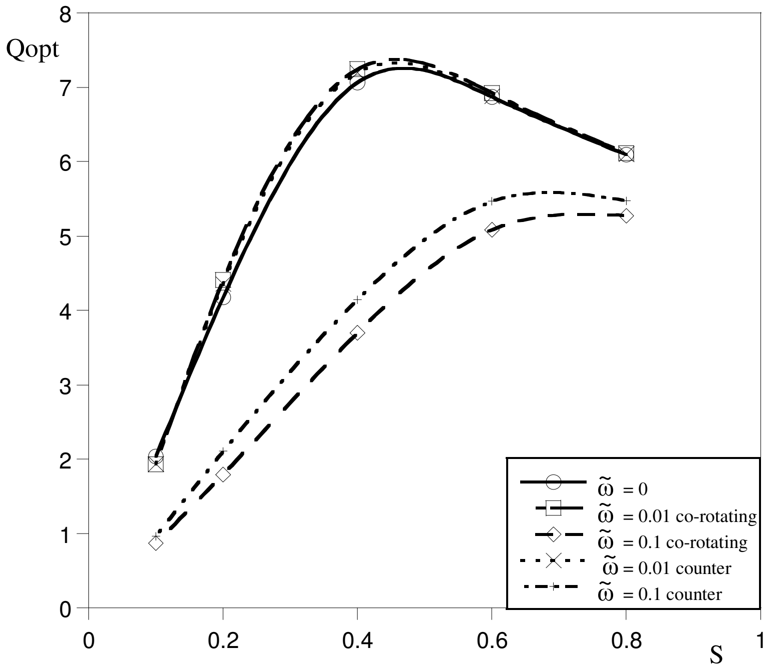

Figure 9 Heat transfer rate density at Be = 1000 for cylinders having the same leading edge.

Figure 10 shows that with rotation at a higher rotational speed there is a beneficial increase in the heat transfer and the optimal lower spacing is smaller as the Bejan number increases. From figure 10 it is seen that the heat transfer rate density at a higher angular velocity is higher than that of the lower angular velocity for both cases of co-rotating cylinders and counter-rotating cylinders. A further explanation for this is that the velocity of the incoming coolant fluid is more than that of the rotating cylinder hence a faster transfer of heat from the cylindrical surfaces.

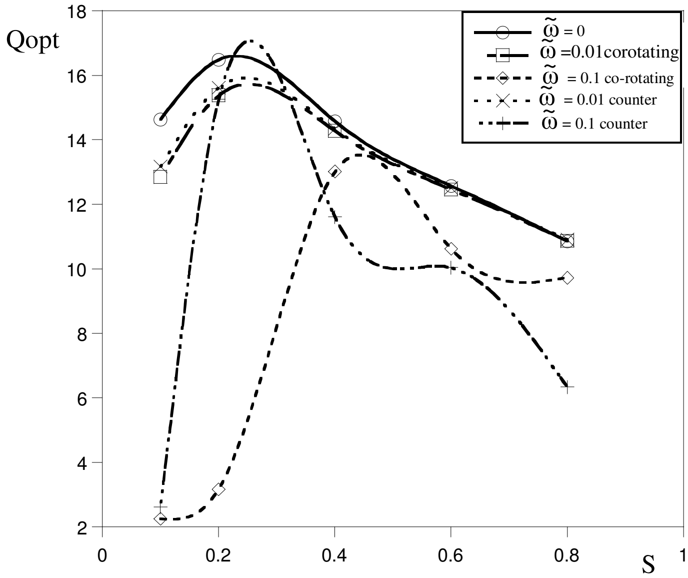

Figure 10 Heat transfer rate density at Be = 10^4 for cylinders having the same leading edge.

Figures 11 shows the velocity contours when the cylinders are aligned at the leading edge with the red region indicating the region of high velocity and blue indicates the region of low velocity and the colours between indicating the transition from the high velocity to the low velocity. In figure 12, the temperature contours of both stationary cylinders and co-rotating cylinders with the red region marking the points of high temperature, that is at the cylinder surface, $T_w = 1$ and the blue region marking places of low temperature, $T_\infty = 0$ corresponding to the freestream coolant.

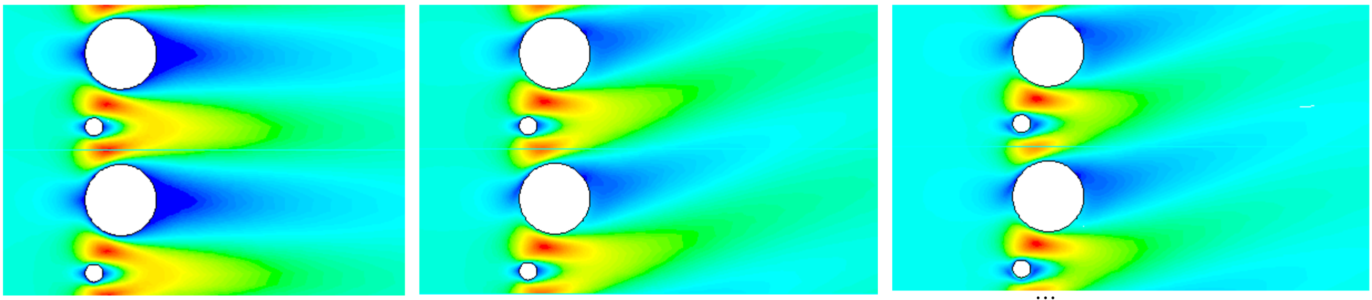

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Figure 11 Velocity contour of i) stationary cylinders, ii) co-rotating cylinders with the same leading edge and iii) counter-rotating cylinders with the same leading edge

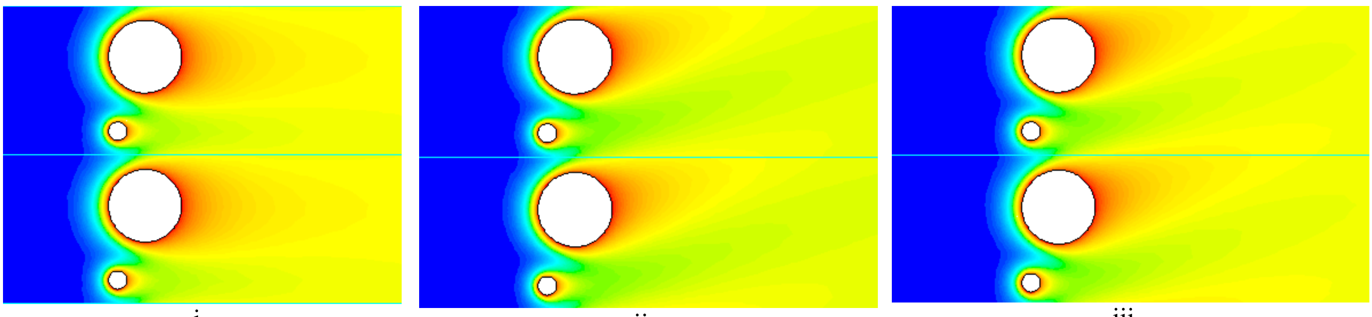

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Figure 12 Temperature contours of i) stationary cylinders, ii) co-rotating cylinders with the same leading edge and iii) counter-rotating cylinders with the same leading edge

4. CONCLUSION

Rotation is beneficial to the heat transfer at certain velocities for an imposed Bejan number and at lower velocities the heat transfer is not significant when compared to that of a stationary cylinder. What is more salient in this work is that at higher pressure drop numbers the optimal spacing between the cylinders decreases, the heat transfer rate density also increases with increase in pressure drop number, that is the Bejan number.

5. ACKNOWLEDGEMENTS

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