ACTIVE VIBRATION CONTROL OF A CLAMPED BEAM BY MEANS OF PZT'S TRANDUCERS

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Abstract. *This article deals with the problem of active vibration control in structures by means of piezoelectric transducers. It was chosen a clamped beam sandwiched by PZT's sensor/actuator as a case of study. A mathematical model for beam is discretized by Finite Element Method (FEM), using ANSYS™ software. After the FEM modeling stage, the model obtained by ANSYS is validated through experimental modal analysis, where the real prototype beam is excited by an impulsive signal generated by a hammer and the dynamic response at different points are measured by a laser vibrometer. After this validation, the global stiffness and mass matrices are extracted through ANSYS. Using these matrices the dynamics of the system can be obtained and after a modal transformation represented by the state space approach. Based on the state space representation, a control system is proposed aiming to reduce the vibrations on the structure by perturbation rejection. For such, a LQR controller and a Luenberger state observer are designed. The control system proposed will be simulated on MATLAB™ and SIMULINK™ softwares and the results obtained will be analyzed and discussed.*

Keywords: *Active Vibration Control, Smart Materials, LQG, Finite Elements Method, Piezoelectric Transducers, Modal Analysis*

1. INTRODUCTION

The capability to measure and control vibrations is of great importance for several engineering areas as automobilist, aerospace, refrigerators, and petrochemical (Gani *et al.* 2003). Every system can be disturbed and taken off its normal functionality by noise action and undesirable vibrations. For this reason, the vibration control is considerate a high challenge in terms of technology.

There are basically two branches of vibration control: the active and the passive ones, where both aim to raise the vibration damping in a given structure. The passive approach is used to reduce the perturbation effects by dissipating vibratory energies. However, the passive control reduction ability is limited to a narrow band of frequencies, and it is hard to adapt the passive control when the system is subjected to different types of disturbances (Fuller *et al.* 2003).

Furthermore, intensive researches in recent decades have developed the technology of active control and the main advantage of this approach is to mitigate noise in various frequency ranges due to its high capacity of adaptation in disturbance rejection. But the active vibration control requires the inclusion of new components to create the control system like sensors, amplifies, and controllers. Among various options for measuring and acting the piezoelectric devices have been widely used (Wersing, 2002). The piezoelectric transducers are ideal for active control applications due to its electromechanical coupling capacity. This transducers have various advantages like the high actuation speed, reduced weight and size and wide band of linear activity. Laminated piezoelectric transducers have been widely used for active control of metallic plates and beams.

The system's model is essential for a good project of the controller. The mathematical modeling of a system can be obtained by several methods, and the Finite Elements Method (FEM) is the most highlighted. The model of mechanical structures equipped with piezoelectric transducers using FEM has been presented in several works (Piefort *et al*. 2003). The results obtained by the mathematical system modeling using the FEM are dynamic equations in which the spatial domain is discretized in nodal points of the system. Thereby the system can be written in a matricial form, which is advantageous to the computational purposes.

In general, dynamic models obtained in physical coordinates result in coupled dynamic equations, which involve a high computational cost in the system solution. Always used in the dynamic mechanics, it is possible to realize a linear transformation denominated Modal Analysis, that reduces a physical coordinates system to a generalized one (or modal system coordinate) in which the dynamic equations are uncoupled (Meirovitch, 1985), so the computational cost is reduced.

This article begins from the model of a sandwiched beam with piezoelectric transducers obtained by the MEF with tridimensional brick elements (Tzou and Tseng, 1990). Both the mechanical and electric parts are discretized in the spatial domain and the system equation is obtained leading to a state space model which is more adequate to the control purposes. A optimal control methodology is applied using the method *Linear-Quadratic and Gaussian* (LQG). A study case for a beam using hexagonal elements with piezoelectric transducers sandwiched is proposed to illustrate the obtained results.

2. MATHEMATICAL MODELING

Combining the Finite Element method (FEM) with the modal analysis, a mathematical model is obtained. This model is necessary to design a controller based on the models, as it is the case of the proposed control strategy shown in section 3. Nowadays the FEM has been used in several areas of engineering and physics, like electrostatics, electrodynamics, heat transfer, acoustics, vibrations, etc. For the particular case of active control the great advantage of the FEM is due to its ability to systematize the process of obtaining state space models for a generic structure with fixed piezoelectric transducers.

This procedure is made more easily because the global matrix of mass and stiffness can be extracted from a finite element model developed by commercial software such as ANSYS.

After the global matrix extraction a modal transformation is performed reducing the size of the system and choosing the desirable modes that will compose the final system model. Then the system can be written in the state space representation.

This methodology can be summarized as show in Fig. 1.

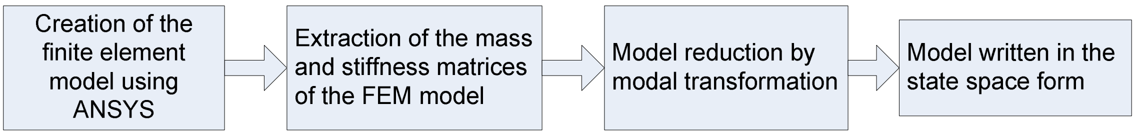

Figure 1. Modeling procedure

This methodology can be used in many mechanical structures, since the most simples to the complex ones. The structure chosen as study case in this work is a cantilevered beam with PZT patch fixed. The Fig. 2 shows the cantilevered beam and the transducers locations.

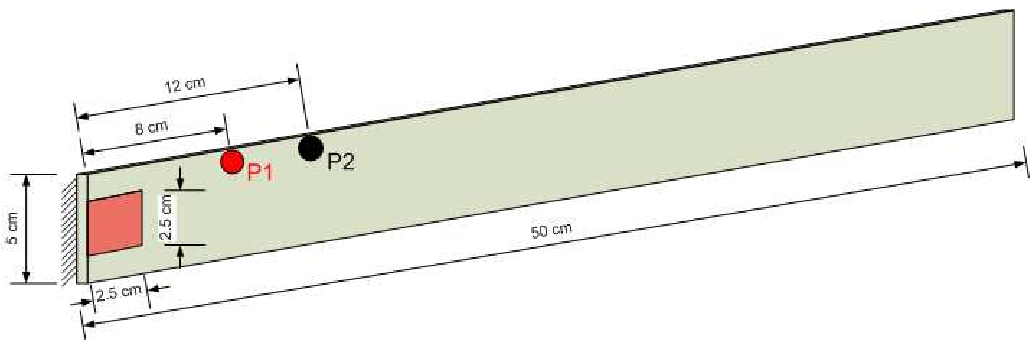

Figure 2. Transducers layout of cantilevered beam

Table 1 shows the beam and the transducers dimensions.

Table 1. Beam and transducers dimensions

	Length(m)	Width (m)	Thickness(m)	Material
Beam	0.5	0.05	0.003	Aluminium
Sensor	0.02	0.02	0.001	PZT-5A
Actuator	0.02	0.02	0.001	PZT-5A

The use of piezoelectric devices as transducers are possible due to the electromechanical coupling effect of the material, being mathematically described by the piezoelectric linear constitutive equations, written in its compact form:

$$\begin{cases} \vec{D} = [e]\boldsymbol{\varepsilon} + [\in^{\varepsilon}]\vec{E} \\ \boldsymbol{\sigma} = [c^E]\boldsymbol{\varepsilon} - [e]^T\vec{E} \end{cases} \tag{1}$$

In the model, $\sigma, \varepsilon, \vec{D}, \vec{E}$, are the mechanical tension, deformation, electric displacement and electric field respectively. The constitutive properties, $[c^E],[\in^{\varepsilon}],[e]$, are respectively the mechanical elasticity, dielectric permissivity and piezoelectric constants matrices.

2.1. Finite Elements

To the application of the FEM, the structure is described in a mesh of elements connected by nodes. In the present model it is used the brick element of eight nodes Fig. 3, as stated (Cook *et al.* 1995). In the classical mechanical problems like tension analysis, the node's degrees of freedom (d.o.f) are the displacement in the three dimensions. But when the analysis concern is the piezoelectric phenomenon we need to add one electric degree of freedom by node, the new d.o.f will be the voltage. For the proposes of this work it will be used two element: one to model the beam Fig. 3.b and another to represent the PZT transducers Fig. 3.a .

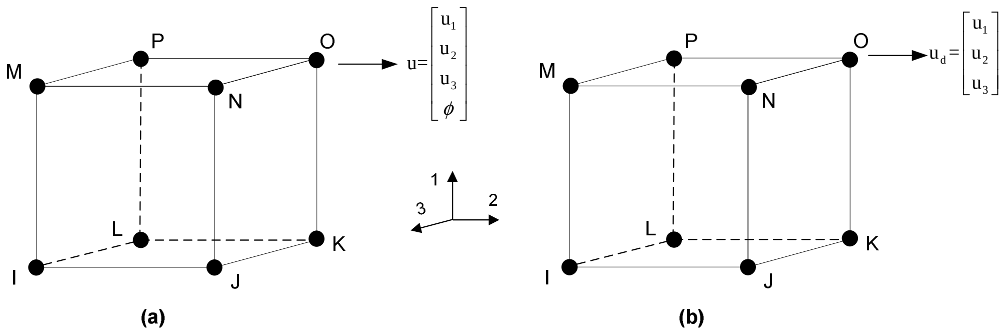

Figure 3. (a) Piezoelectric element, (b) Structural element

To obtain the global matrices that will compose the piezoelectric dynamic equation we can use the Lagrangian (Tzou and Tseng, 1990). For a piezoelectric body the Lagrangian can be written as**:**

$$\mathcal{L} = \int_{\mathcal{V}} (\mathcal{T} - \mathcal{U}) d\mathcal{V} \tag{2}$$

Where $\mathcal{L}$ is the Lagrangian, $\mathcal{T}$ the kinetic energy, $\mathcal{U}$ the potential energy and $\mathcal{V}$ the piezoelectric volume. Writing the kinetic energy through the material density and the potential energy using the piezoelectric tensor (Eq. 1) the Eq. 3 is obtained.

$$\mathcal{L} = \int_{\mathcal{V}} \left(\frac{1}{2}\rho\{\dot{\mathbf{u}}_{\mathbf{d}}\}'\{\dot{\mathbf{u}}_{\mathbf{d}}\} - \frac{1}{2}\left[\{\boldsymbol{\varepsilon}\}'\{\boldsymbol{\sigma}\} - \{\mathbf{E}\}'\{\mathbf{D}\}\right] \right) d\mathcal{V} \tag{3}$$

Where $\{\dot{\mathbf{u}}_{\mathbf{d}}\}$ is the velocity vector. Now, applying the virtual work theory and considering the action of external forces, electric charge and electric tension in the piezoelectric body, we will obtain the relation described by the following equation:

$$\delta\mathcal{W} = \int_{\mathcal{V}} \{\delta u\}'\{P_b\} d\mathcal{V} + \int_{A_1} \{\delta u\}'\{P_A\} dA_1 + \{\delta u\}'\{P_C\} - \int_{A_2} \phi q_A dA_2 \tag{4}$$

Where $\{P_b\}$ is the body force, A_1 and A_2 are the piezoelectric superficial areas, $\{P_A\}$ is the force distributed in the body surface, $\{P_C\}$ is the concentrated mechanical load and q_A is the electric load . Using the Lagrangian and the virtual work we obtain the piezoelectric dynamic equation through the variational Hamilton principle.

$$\int_{t_1}^{t_2} \delta\left(\mathcal{L}+\mathcal{W}^{e}\right)dt = 0 \tag{5}$$

Where t_1 e t_2 are the time interval where the time variation occurs. Replacing the Eq. 3 and Eq. 4 in Eq.5, Eq. 6 is obtained.

$$\begin{aligned}&\int_{\mathcal{V}}\left(\rho\{\delta\dot{\mathbf{u}}_{\mathbf{d}}\}'\{\dot{\mathbf{u}}_{\mathbf{d}}\}-\{\delta\boldsymbol{\varepsilon}\}'[c^{E}]\{\boldsymbol{\varepsilon}\}+\{\delta\boldsymbol{\varepsilon}\}'[e]^{T}\{\mathbf{E}\}-\{\delta\mathbf{E}\}'[e]\{\boldsymbol{\varepsilon}\}-\{\delta\mathbf{E}\}'[e]\{\mathbf{E}\}+\{\delta\mathbf{u}_{\mathbf{d}}\}'\{P_b\}\right)d\mathcal{V}+\\&+\int_{A_1}\{\delta u\}'\{P_A\}dA_1-\int_{A_2}\delta\phi q_A dA_2+\{\delta\mathbf{u}_{\mathbf{d}}\}'\{P_C\}=0\end{aligned} \tag{6}$$

To obtain the global matrices of mass and global stiffness we need to write the displacement vector $u_d=[u_1\ u_2\ u_3]$ and the voltage ϕ by means of nodal values. For this, a shape functions described by the matrices $[\mathbf{N}_{\mathbf{u}}]$ and $[\mathbf{N}_{\phi}]$ are used. Then, writing the variables system in its nodal form results that:

$$\begin{cases}\{\mathbf{u}_{\mathbf{d}}\}=[\mathbf{N}_{\mathbf{u}}]\{\mathbf{u}_{\mathbf{i}}\}\\ \{\phi\}=[\mathbf{N}_{\mathbf{u}}]\{\phi_{\mathbf{i}}\}\end{cases} \tag{7}$$

Where the index i denotes the nodal variables. The relation between the deformation and displacement can be established by the following equation:

$$\{\boldsymbol{\varepsilon}\}=[\partial]\{\mathbf{u}\} \quad ; \quad [\partial]=\begin{bmatrix}\frac{\partial}{\partial x} & 0 & 0\\ 0 & \frac{\partial}{\partial y} & 0\\ 0 & 0 & \frac{\partial}{\partial z}\\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0\\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x}\end{bmatrix} \tag{8}$$

Thus, the relation described by the Eq. 8 can be written by means of nodal variables leading to Eq. 9.

$$\begin{cases}\{\boldsymbol{\varepsilon}\}=[B_{\mathbf{u}}]\{\mathbf{u}_{\mathbf{i}}\}\\ [B_{\mathbf{u}}]=[\partial][\mathbf{N}_{\mathbf{u}}]\end{cases} \tag{9}$$

As in control system the variable is the electric tension, the electric field can be described by means of voltage. Thus, it can be written in the following way:

$$\{\mathbf{E}\}=-\nabla\phi \tag{10}$$

Writing the Eq. 9 by means of nodal variables we obtain the following relation for the electric field:

$$\begin{cases}\{\mathbf{E}\}=-[\mathbf{B}_{\phi}]\{\boldsymbol{\phi}_{i}\}\\ [\mathbf{B}_{\phi}]=\partial[\mathbf{N}_{\phi}]\end{cases} \tag{11}$$

Replacing the Eq. 11, Eq. 10, Eq. 9, Eq. 7 in the Eq. 6 and solving the integral the following piezoelectric dynamic equation is obtained.

$$\begin{bmatrix} [\mathbf{m_{uu}}] & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \{\ddot{\mathbf{u}}_\mathbf{i}\} \\ \{\ddot{\phi}_\mathrm{i}\} \end{bmatrix} + \begin{bmatrix} [\mathbf{k_{uu}}] & [\mathbf{k}_{\phi\mathbf{u}}] \\ [\mathbf{k}_{\mathbf{u}\phi}] & [\mathbf{k}_{\phi\phi}] \end{bmatrix} \begin{bmatrix} \{\mathbf{u_i}\} \\ \{\phi_\mathrm{i}\} \end{bmatrix} = \begin{bmatrix} \{\mathbf{f_i}\} \\ \{\mathbf{g_i}\} \end{bmatrix} \tag{12}$$

Where the matrices of Eq. 12 are obtained by the following equations:

$$[\mathbf{m_{uu}}]_{n_\mathbf{u}\times n_\mathbf{u}} = \int_{\mathcal{V}} \rho[\mathbf{N_u}]'[\mathbf{N_u}]\,d\mathcal{V} \tag{13}$$

$$[\mathbf{k_{uu}}]_{n_\mathbf{u}\times n_\mathbf{u}} = \int_{\mathcal{V}} [\mathbf{B_u}]'[\mathrm{c^E}][\mathbf{B_u}]\,d\mathcal{V} \tag{14}$$

$$[\mathbf{k}_{\mathbf{u}\phi}]_{n_\mathbf{u}\times n_\phi} = \int_{\mathcal{V}} [\mathbf{B_u}]'[e][\mathbf{B}_\phi]\,d\mathcal{V} \tag{15}$$

$$[\mathbf{k}_{\phi\phi}]_{n_\phi\times n_\phi} = \int_{\mathcal{V}} [\mathbf{B}_\phi]'[\in^\varepsilon][\mathbf{B}_\phi]\,d\mathcal{V} \tag{16}$$

$$\{\mathbf{f}_i\}_{n_\mathbf{u}\times 1} = \int_{\mathcal{V}} [\mathbf{N_u}]'\{\mathrm{P_b}\}\,d\mathcal{V} + \int_{\mathrm{A_1}} [\mathbf{N_u}]'\{\mathrm{P_A}\}d\mathrm{A_1} + [\mathbf{N_u}]'\{\mathrm{P_C}\} \tag{17}$$

$$\{\mathbf{g_i}\}_{n_\phi\times 1} = -\int_{\mathrm{A_2}} [\mathbf{N}_\phi]'\mathrm{q_A}\,d\mathrm{A_2} \tag{18}$$

$$[\mathbf{k}_{\mathbf{u}\phi}] = [\mathbf{k}_{\phi\mathbf{u}}]' \tag{19}$$

The index $n_\mathbf{u}$ and n_ϕ are the mechanical d.o.f numbers and electric d.o.f numbers and ρ is the material density.

2.1. Modal Analysis

The modal analysis aims to uncouple the system dynamic equations generated from the Eq. 11. Using a modal decomposition truncated in p vibration modes, the mechanical displacements are $\{\mathrm{u_i}\}=[\Psi]\{\mathrm{q(t)}\}$, in which $[\Psi]_{n_u\times p}$ are the modal forms and $\{\mathrm{q(t)}\}_\mathrm{p}$ are the displacements of the p vibration modes . The differential equation Eq. 12 can be written in its modal form:

$$\begin{cases} [\mathbf{m_{uu}}][\Psi]\{\ddot{\mathrm{q}}\} + [\mathbf{k_{uu}}][\Psi]\{\mathrm{q}\} + [\mathbf{k}_{\mathbf{u}\phi}]\{\phi_\mathrm{i}\} = \{\mathbf{f}_i\} \\ [\mathbf{k}_{\mathbf{u}\phi}][\Psi]\{\mathrm{q}\} - [\mathbf{k}_{\phi\phi}]\{\phi_\mathrm{i}\} = \{\mathbf{g_i}\} \end{cases} \tag{20}$$

Where the system's electric potential kwon both for the actuator (ϕ_a controlled) and the sensor ($\phi_s = 0$), the electric parcel does not influence the vibration modes and those are obtained by the classic eigenvalue problem of mechanical vibration, as in Eq. 21, in which the orthogonal properties of Eq. 22 are valid (Marinho, 2008).

$$\left([\mathbf{k_{uu}}] - \omega_i^2[\mathbf{m_{uu}}]\right)[\Psi] = 0 \tag{21}$$

$$\begin{cases} [\Psi]^T[\mathbf{m_{uu}}][\Psi] = \delta_{ij} \\ [\Psi]^T[\mathbf{k_{uu}}][\Psi] = \mathrm{diag}(\omega_i^2) \\ [\Psi]^T[C][\Psi] = \mathrm{diag}(2\xi_i\omega_i) \end{cases} \tag{22}$$

A low damping coefficient $[C][\Psi]\{\dot{q}\}$ can be added to the Eq. 20. Pre-multiplying the Eq. 20 plus the damping term by $[\Psi]^T$ and observing the orthogonal properties of Eq. 22 the system can be written as:

$$\begin{cases} \{\ddot{\mathrm{q}}\} + \mathrm{diag}(2\xi_i\omega_i)\{\dot{\mathrm{q}}\} + \mathrm{diag}(\omega_i^2)\{\mathrm{q}\} = [\Psi]^T\{\mathbf{f}_i\} - [\Psi]^T[\mathbf{k}_{\mathbf{u}\phi}]\{\phi_a\} \\ \{\mathrm{Q_s}\} = -[\mathbf{k}_{\phi\mathbf{u}}][\Psi]\{\mathrm{q}\} \end{cases} \tag{23}$$

In the first equation of Eq. 23 only the electric potential control $\{\phi_a\}$ are considered in the second equation only the electric charge of sensor are measured $\{Q_s\}$ and the sensor electric potential is zero ($\phi_s = 0$).

3. Modal Validation

Before making the state space realization, the FEM model was validated. For this purpose an experimental modal analysis was realized using an impact hammer and a laser doppler vibrometer (LDV). The beam was excited by the hammer in the point P1 Fig. 2 and the structure velocity was measured in several points using the LDV. So the structure simulation response was compared to the acquired through experimental procedure. The Fig. 4 shows the comparison between those responses for the point P2 Fig. 1 .

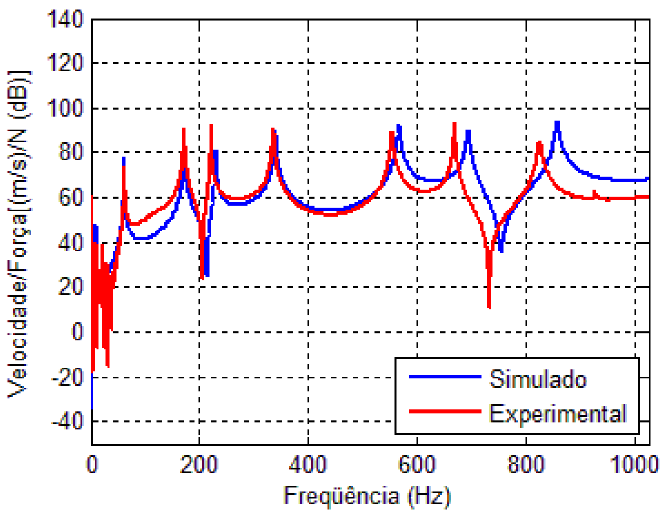

Figure 4. Comparison between simulated and experimental responses

As shown in the Fig. 4 the experimental and simulated responses are very close, and the difference between them occurs due to small differences between the structure and the model dimensions, and mainly due to the differences between the material physical properties used in the model and the real structure ones. It's possible to improve the mathematical model and reduce the error between the simulated and experimental responses, through model adjustments techniques. But, for the application purpose of this work the model obtained was considered satisfactory.

3. VIBRATION'S ACTIVE CONTROL

The noise reduction to an acceptable level is one of the main focuses in the vibration study. In this context, the active control has been a quite efficient alternative, being widely studied for several areas of engineering (Inman, 2006). The active control system is basically composed by a set of transducers connected to a controller.

3.1. State Space Realization

There are many ways to represent the dynamics of a given system. Among them, we can highlight the state approach, where it is possible to represent systems with a large number of inputs and outputs without raising meaningly its complexity. In this method any system, represented by n order differential equations, can be described as a set of first order differential equations, where the variables are known as states (Vasques *et al.* 2006).

In the case proposed for this study, the state variables were defined in generalized coordinate terms as being $X=\{q \quad \dot{q}\}^T$. The system realization is made by Eq. 24 and Eq. 25, resulting in the following representation:

$$\begin{cases} \dot{X}(t) = \mathbf{A}X(t) + \mathbf{B}_{MEC}F(t) + \mathbf{B}_{PZT}\Phi_a(t) \\ U(t) = \mathbf{C}_{MEC}X(t) \\ Q_s(t) = \mathbf{C}_{PZT}X(t) \end{cases} \tag{24}$$

The state space matrices of the equation Eq. 23 can be written as:

$$\begin{cases} \mathbf{A}_{[2N\times 2N]} = \begin{bmatrix} \mathbf{0}_{[N\times N]} & \mathbf{I}_{[N\times N]} \\ -\text{diag}(\omega_i^2,\ldots,\omega_N^2) & -2\,\text{diag}(\zeta_i\omega_i,\ldots,\zeta_N\omega_N) \end{bmatrix} \\ \mathbf{B}_{MEC[2Nxn_u]} = \begin{bmatrix} \mathbf{0}_{[n\times n_u]} \\ [\Psi]^T \end{bmatrix} \quad ; \qquad \mathbf{B}_{PZT[2N\,xn_\phi]} = \begin{bmatrix} \mathbf{0}_{[N\times n_\phi]} \\ -[\Psi]^T \left[\mathbf{k}_{\mathbf{u}\phi}\right][T_a]^T \end{bmatrix} \\ \mathbf{C}_{MEC[n_u\,x2N]} = \begin{bmatrix} [\Psi] & \mathbf{0}_{[n_u\times N]} \end{bmatrix} \quad ; \quad \mathbf{C}_{PZT[n_\phi\,x2N]} = \begin{bmatrix} -[T_s]^T\left[\mathbf{k}_{\phi\mathbf{u}}\right][\Psi] & \mathbf{0}_{[n_\phi\times N]} \end{bmatrix} \end{cases} \tag{25}$$

Where N are the modals numbers of the system, n_u and n_ϕ are respectively the numbers of mechanical freedom degrees and electric freedom degrees defined in the nodal space. Once represented the system by state space approach, it is possible to design and simulate the controller for reduce the vibrations originated by a disturbance source.

3.2. State Space Approach to Controller Design

Basically, the active control consists in the perturbation rejection problem. There are many control designs such as the Modal Control (Merovitch, 1985), the Positive Position Feedback (PPF), and the robust control techniques, as the H_2 and H_∞, that can be applied to the active control.

Regardless from the project strategy chosen, for this study the controller conception is based in the feedback topology, originating the control law $\{\Phi\} = -[k]\{X_F\}$, where $\{X_F\}$ are the states values estimated by the observer and $[k]$ is the gain matrix of controller. Thus, in accordance with separation principle the gain matrix of controller is designed based in the hypothesis of total state feedback and next it is designed a state observer to complete the implementation of the control system (Fuller *et al.* 1997). These two steps of the project were realized through the LQG technique.

To make the project of the controller's gain matrix, it was developed an optimal control strategy denominated *Linear-Quadratic Regulator* (LQR). This is a project methodology made for linear systems and it is based in the minimization of a quadratic criterion associated to the energy of state variables and control signals. The design challenge is to establish a commitment between the states energy and the signal control energy through the following cost function $J(\Phi)$ to be minimized:

$$J(\Phi) = \int_0^\infty \{X\}^T [Q]\{X\} + \{\Phi\}^T [R]\{\Phi\}\, dt \tag{26}$$

In which Q and R are positive defined weighting matrices, representing the desired balance between the states energy and the control signal energy respectively. As the states are related to the system vibration modes, the matrix Q can be chosen to prioritize the mitigation of a particular mode in preference to another.

One way to obtain the gain matrix k which minimizes the criterion showed by equation (9) is through Riccati's equation, as:

$$A^T P + PA - PB_{PZT} R^{-1} B_{PZT}{}^T P + Q = 0 \tag{27}$$

Thus, we can obtain a solution matrix P which satisfies the Eq. 25. So, once obtained the matrix P, the optimal gain $[k]$ can be obtained, as:

$$k = R^{-1} B_{PZT}{}^T P \tag{28}$$

To estimate the states, it was projected a special type of Luenberger observer known as Kalman's filter, which minimizes the estimation error variance. In a similar way to the LQR control gain obtainment, the gain L for the observer is obtained through the resolution of a Riccati equation, as follow.

$$SA^T + AS - SC_{PZT}{}^T R^{-1} C_{PZT} S + Q = 0 \tag{29}$$

So, once obtained the matrix S, the optimal gain L for the observer can be obtained, as:

$$L = SC_{PZT}{}^{T} R^{-1} \tag{30}$$

The same weighting matrices of the LQR project were used for the Kalman filter project. In general, the controlled system structure is shown in the Fig 5.

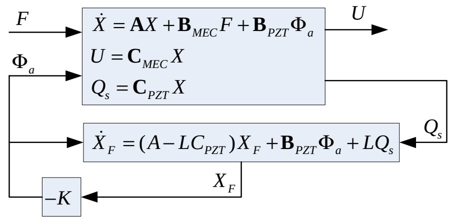

Figure 5. Topology of the control system

4. SIMULATION RESULTS

After the controller and the state observer were designed using Matlab, the control system was simulated to verify its performance. For this purpose a beam model composed by the first six vibration modes was used, the Tab. 2 compares the system open loop eigenvalues with the close loop ones.

Table 2. Comparation between the system eigenvalues

Open Loop Eigvalues (10^3)	Close Loop Eigevalues (10^3)
-0.0000 ± 0.0622i	-0.0144 ± 0.0607i
-0.0000 ± 0.3890i	-0.0844 ± 0.3804i
-0.0002 ± 2.1536i	-0.4067 ± 2.1198i
-0.0000 ± 0.7944i	-0.0000 ± 0.7944i
-0.0001 ± 1.0918i	-0.2226 ± 1.0726i
-0.0006 ± 3.5983i	-0.6266 ± 3.5152i

To evaluate the system disturb rejection capacity, a white noise has been chosen as perturbation input. This choice can be justified due to the white noise excite all structure frequency bands. Thus, by the simulation result is possible to evaluate the control system performance in a global way. The Fig. 6 compares the open loop bode diagram with the close loop one.

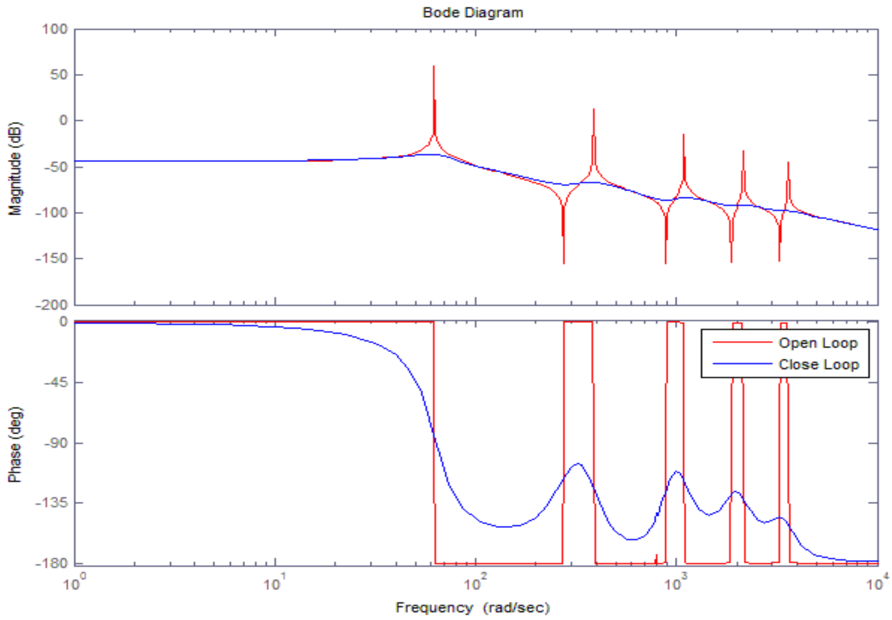

Figure 6. Bode diagram of open and close loop system

As shown in Fig. 6 the control improves the system damping reducing the resonance pikes. For an ideal PZT actuator it would be possible to decrease almost 100dB for the first resonance pike and, on average, 70dB for the others pikes. But, in practice the PZT actuator has operational limitations, thus the attenuation levels achieved by the control system can be inferior to the ones shown in Fig. 6.

To evaluate the actuator operational restrictions it was considered a control signal saturation of $\pm 200V$, and then the controlled and the open system responses were compared. Considering the disturbance input as a white noise, the acquired responses are shown in Fig. 7.

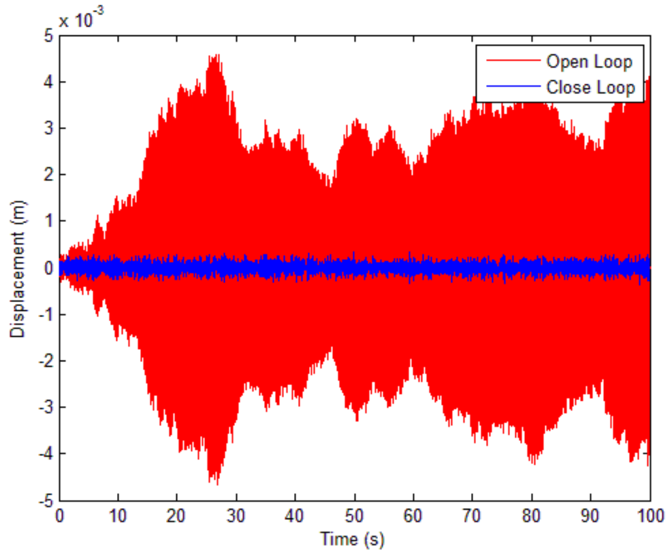

Figure 7. System response to a white noise disturb

The Fig. 7 shows that even with the restriction on the actuator the control system achieved improves considerably the structure ability to reject disturbance. This result can be better understood through the control signal history Fig. 8.

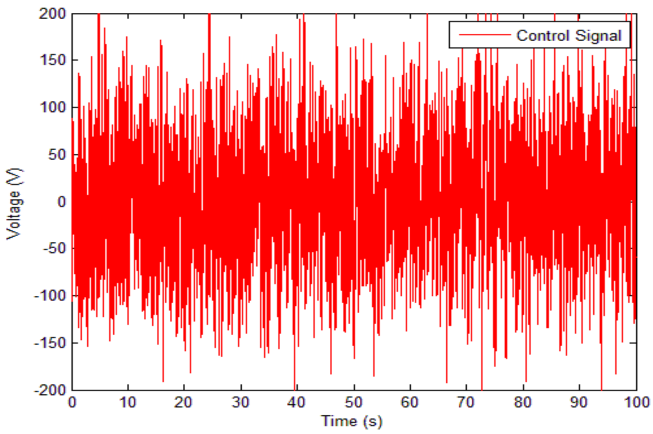

Figure 8. Control signal history

As shown in Fig. 8, the control signal rarely takes values greater than 200 V and when it occurs the signal keeps this level for a short time period. Then the actuator saturation does not influence the control system performance significantly. Instead of the some errors between the vibrations modes considered in the model and the experimental results, the LQG methods presents good performance due to its intrinsecals properties of robustness.

5. CONCLUSION

This article approached the active control problem by means of piezoelectric transducers. Due to the technology development of solid state actuators such as the piezoelectric, these devices have been widely used in the vibration area. They own various advantages when compared to the classic actuators, like its mechanical coupling capability, fast answer, reduced size/weight and driven by electric tension. However, these transducers are not like the conventional ones normally used for control purposes, they are spatially distributed on the structure. Due to the PZT transducers being patches bonded to the structure, the system modeling has to be done considering the set beam/PZT as a single structure. This peculiarity makes the mathematical modeling a critical stage in an active control design. Aiming to solve the modeling problem, this work has developed a practical methodology which used the FEM and the Modal Analysis as tools. A commercial software was used to solve the FEM problem and to obtain the system elementary matrices and to make easier the transducers specification. Besides, he has also made faster the model validation process through the comparison between experimental and practical modal analysis results. The joint use of Ansys and Matlab was a powerful computational tool to deal with the active control issue, allowing the states space model to be written from the reduced finite element equations.

The fundamental active control problem remains the disturbance rejection, for this purpose a controller was designed using the LQR control technique, which is based in the total state feedback. As not all states were measurable, an observer was designed to estimate all the states to the controller. The simulation results interpretation shows that the control system is successful for the disturbance rejection purposes, improving substantially the system damping and mitigating the white noise effect in the dynamic structure response. Even with high attenuation levels in practically all frequencies, the control system respected the actuator operational limits (± 200V) where the control signal has remained bellow 150V most of the time.

The concern of future works will be the control implementation in the prototype which model was validated in this study. For control techniques which system robustness is a project parameter like H_∞, it will be implemented and compared to the LQR.

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7. RESPONSIBILITY NOTICE

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