FAULT-TOLERANT ATTITUDE DETERMINATION SYSTEM OF AN EARTH-POINTING SATELLITE

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Abstract. *This paper presents a scheme for fault-tolerant attitude determination of an Earth-pointing satellite using measurements of geomagnetic field, Sun direction and inertial angular velocity. A linearized Kalman filter is employed for attitude and angular velocity estimation and its innovation sequence is used as a residual for both fault detection and diagnosis. Detection is accomplished by means of a simple chi-square test. When the detection information becomes available, a module for diagnosing the fault is activated. The diagnosis problem is formulated through a multiple composite hypotheses testing whose optimal solution in the minimum probability of error sense is given by the maximum a posteriori decision rule. However, as in the practice the parameters of the fault are unknown, a common suboptimal approach is adopted that replaces the true deterministic parameters with their maximum likelihood estimates under the considered test statistics. As soon as the fault is diagnosed, the Kalman filter model is updated to better represent the new system behavior. The proposed scheme is evaluated by numerical simulation, that shows its capability to maintain small errors in attitude and angular velocity estimation when tackling abrupt additive changes on sensors.*

Keywords*: detection theory, Kalman filtering, fault diagnosis, attitude determination, satellite*

1. INTRODUCTION

The conventional Kalman filter assumes that an accurate stochastic dynamic model of the linear-Gaussian system is available. However, in practical situations, the model parameters may deviate from their true values. One of the causes for such deviation is the occurrence of faults in sensors. If such model changes are not taken into account, the filter performance may seriously degrade or even can show a diverging behavior. Traditionally, this problem is treated by augmenting the state vector with variables modeling the parameter errors as either constant or random biases, resulting in a high dimensional estimator prone to problems as lack of observability and low convergence rate. To avoid state augmentation, some approaches have been proposed in the literature.

Friedland (1969) presented the two-stage Kalman filter for estimating the states of linear-Gaussian systems subject to additive constant bias. Recently, by using an adaptive filtering procedure, this technique has been extended for nonlinear systems containing multiplicative unknown random parameters (Kim *et al.*, 2009).

Willsky and Jones (1976) integrated the Kalman filter with the generalized likelihood ratio test (GRLT) for estimating the states of linear systems subject to abrupt additive changes in the state equation that may occur at a sporadic time instant. In this method, the GLRT module monitors the innovation sequence of the filter to determine if a change has occurred and adjusts the filter accordingly. Deshpande *et al.* (2008) have extended this approach to treat nonlinear systems subject to additive faults in both state and measurement equations. A similar idea was considered in Kawauchi (1982) to diagnose abrupt changes in the attitude determination system (ADS) of a hypothetical satellite using rate gyros and star sensors. This method was shown by simulation as being capable of accommodating a fault by augmenting the filter state vector just with the variables associated with the parameters of the fault that has occurred.

A scheme for fault-tolerant attitude determination is proposed in the present paper. It is suitable for Earth-oriented rigid satellites moving in a low-Earth orbit and it relies on a suite of sensors containing three orthogonal rate gyros, three orthogonal magnetometers and two-axis Sun sensors. The scheme uses a Kalman filter based on a linearized model that, besides estimating attitude and angular velocity, its innovation is used as a residual for fault detection and diagnosis. The diagnosis module follows the Willsky and Jones' approach. As soon as the fault is diagnosed, the Kalman filter model is updated using the estimated fault to better represent the new system behavior. Simulation results shows that the proposed scheme is able to maintain small error in attitude and angular velocity estimation when tackling abrupt additive changes on sensors. The foregoing contribution is presented in a general way that becomes clear its applicability to other fault-tolerant linear state estimation problems.

This paper presents its method and results in the next 5 sections. Section 2 describes the issues that will be addressed here. Section 3 presents the linearized state-space model of the satellite angular motion with respect to an Earth-pointing reference frame. Section 4 develops a scheme for fault-tolerant attitude determination based on the linearized model. Section 5 presents the results of tests of the scheme on simulated data, and Section 6 presents the paper's conclusions.

2. PROBLEM STATEMENT

Consider the following two Cartesian coordinate systems (CCS): $S_b = \{x_b, y_b, z_b\}$ is the body fixed frame aligned with the principal axes of inertia and with origin at the center of mass of the satellite (CM); $S_o = \{x_o, y_o, z_o\}$ is the orbital reference CCS, whose origin is also at the CM. It rotates about its negative y-axis so that its z-axis always points towards the nadir. See the illustration in Fig. 1. The attitude of S_b with respect to S_o is described by the attitude matrix $\boldsymbol{D}_b^o$, which transforms representations of vectors from S_o to S_b. The angular velocity of S_b with respect to S_o is represented in S_b by $\boldsymbol{\omega}_b^{bo}$.

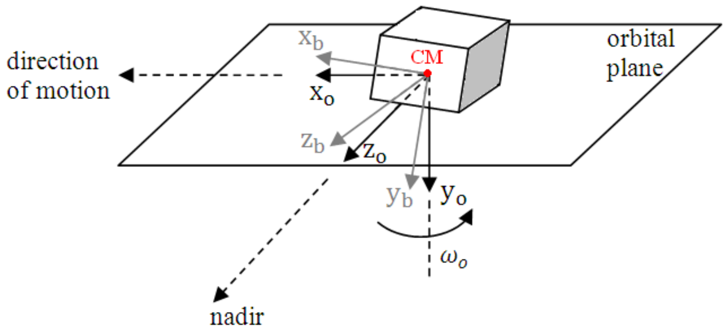


Figure 1. Cartesian coordinate systems

Measurements of the local geomagnetic field, $\tilde{\boldsymbol{b}}_b$, the Sun direction, $\tilde{\boldsymbol{s}}_b$, and the inertial angular velocity are considered available, $\tilde{\boldsymbol{\omega}}_b$, but their components may contain additive faults that is assumed here as step-wise. The subscript b denotes representation (projection) of vectors in S_b. Therefore, these measurements are modeled at discrete instants of time as:

$$\tilde{\mathbf{b}}_\mathrm{b}(k) = \mathbf{b}_\mathrm{b}(k) + \delta\mathbf{b}_\mathrm{b}(k) + \Delta\mathbf{b}_\mathrm{b}(k) \tag{1}$$

$$\tilde{\mathbf{s}}_\mathrm{b}(k) = \mathbf{s}_\mathrm{b}(k) + \delta\mathbf{s}_\mathrm{b}(k) + \Delta\mathbf{s}_\mathrm{b}(k)$$

$$\tilde{\boldsymbol{\omega}}_\mathrm{b}(k) = \boldsymbol{\omega}_\mathrm{b}(k) + \delta\boldsymbol{\omega}_\mathrm{b}(k) + \Delta\boldsymbol{\omega}_\mathrm{b}(k)$$

where $\boldsymbol{b}_b$, $\boldsymbol{s}_b$ and $\boldsymbol{\omega}_b$ are true values of the physical quantities; $\{\delta\boldsymbol{b}_b(k)\}$, $\{\delta\boldsymbol{s}_b(k)\}$ and $\{\delta\boldsymbol{\omega}_b(k)\}$ are assumed as Gaussian white sequences; finally, $\Delta\boldsymbol{b}_b$, $\Delta\boldsymbol{s}_b$ and $\Delta\boldsymbol{\omega}_b$ are the additive vectors of faults. The argument k will be omitted in the rest of the paper.

Problem 1 – Given $\tilde{\boldsymbol{b}}_b$, $\tilde{\boldsymbol{s}}_b$ and $\tilde{\boldsymbol{\omega}}_b$ and assuming that S_b does not deviate significantly from S_o yielding the use of a local linearized state and measurement models, then devise a scheme for attitude, $\boldsymbol{D}_b^o$, and angular velocity, $\boldsymbol{\omega}_b^{bo}$, estimation that avoids augmenting the state vector of the Kalman filter with the parameters of the faults. □

3. STATE SPACE MODEL OF THE SATELLITE ATTITUDE MOTION

Subsection 3.1 reviews the kinematic and dynamic equations of motion of a rigid satellite with respect to the orbital CCS, S_o. The attitude, $\boldsymbol{D}_b^o$, is parameterized by the Modified Rodrigues Parameters (MRP). Subsection 3.2 presents the measurement models, which relates the measurements in (1) to the attitude and angular velocity variables. Linearized models will be derived assuming the satellite does not deviate much from the nominal attitude given by S_o.

3.1. State equations

There exist many forms of parameterization of the attitude matrix $\boldsymbol{D}_b^o$ that obviates the problems associated with propagation of its nine elements by numerical integration (Santos, 2008). The most popular parameterizations are the four-component unit quaternion, the three-component Euler angles and the Modified Rodrigues Parameters (MRP) (Shuster, 1993). The quaternion has the advantage of providing global attitude representation, but that is not required here due to the small angle assumption for the attitude of S_b with respect to S_o. Among the three-component parameterizations, MRP is preferable because its kinematic equations avoid trigonometric functions. The nonlinear kinematic equation using MRP $\boldsymbol{m}_b^o$ is given by (Smith, 1982):

$$\dot{\mathbf{m}}_{\mathrm{b}}^{\mathrm{o}} = \frac{1}{4}\left[\left(1 - m_{\mathrm{b}}^{\mathrm{o}\,2}\right)\mathbf{I}_3 + 2[\mathbf{m}_{\mathrm{b}}^{\mathrm{o}} \times] + 2\mathbf{m}_{\mathrm{b}}^{\mathrm{o}}\mathbf{m}_{\mathrm{b}}^{\mathrm{o}\prime}\right]\boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bo}} \tag{2}$$

where m_b^o is the Euclidian norm of $\boldsymbol{m}_b^o$ and $[\boldsymbol{m}_b^o \times]$ is the cross-product matrix of $\boldsymbol{m}_b^o$. The relation between MRP and the corresponding attitude matrix is:

$$\mathbf{D}_{\mathrm{b}}^{\mathrm{o}}(\mathbf{m}_{\mathrm{b}}^{\mathrm{o}}) = \mathbf{I}_3 + \left[8[\mathbf{m}_{\mathrm{b}}^{\mathrm{o}} \times]^2 - 4\left(1 - m_{\mathrm{b}}^{\mathrm{o}\,2}\right)[\mathbf{m}_{\mathrm{b}}^{\mathrm{o}} \times]\right] / \left(1 + m_{\mathrm{b}}^{\mathrm{o}\,2}\right)^2 \tag{3}$$

Let $\boldsymbol{\omega}_b^{bi}$ represents the inertial angular velocity of S_b. The dynamic equation for a rigid satellite is given by (Wertz, 1978):

$$\dot{\boldsymbol{\omega}}_{\mathrm{b}}^{\mathrm{bi}} = \mathbf{J}^{-1}\left[\left(\mathbf{J}\boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bi}}\right) \times\right]\boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bi}} + \mathbf{J}^{-1}\left(\mathbf{T}_{\mathrm{b}}^{\mathrm{c}} + \mathbf{T}_{\mathrm{b}}^{\mathrm{d}}\right) \tag{4}$$

where $\boldsymbol{J}$ is the inertia matrix; $\boldsymbol{T}_b^c$ is the control torque vector, which can be produced by actuators such as magnetorquers and thrusters; and $\boldsymbol{T}_b^d$ is the disturbance torque vector representing, among others, gravity-gradient and residual magnetism.

The inertial angular velocity of the satellite can be rewritten as:

$$\boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bi}} = \boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bo}} + \boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{oi}}$$

$$\boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bi}} = \boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bo}} + \mathbf{D}_{\mathrm{b}}^{\mathrm{o}}(\mathbf{m}_{\mathrm{b}}^{\mathrm{o}})\boldsymbol{\omega}_{\mathrm{o}}^{\mathrm{oi}} \tag{5}$$

where $\boldsymbol{\omega}_o^{oi} = [0, -\omega_o, 0]'$ and ω_o is the orbital rate. For circular orbits, the orbital rate equals the mean angular motion $n_o = \sqrt{\mu/a^3}$, where μ is the Earth gravitational constant and a is the orbit semi-major axis.

Thus, substituting Eq. (5) into Eq. (4), using the kinematic equation of the attitude matrix (Wertz, 1978), $\dot{\boldsymbol{D}}_b^o = -\left[\boldsymbol{\omega}_b^{bo} \times\right]\boldsymbol{D}_b^o$, and assuming that $\boldsymbol{\omega}_{\boldsymbol{o}}^{\boldsymbol{oi}}$ is constant, the satellite dynamic equation with respect to the orbital CCS becomes:

$$\dot{\boldsymbol{\omega}}_{\mathrm{b}}^{\mathrm{bo}} = \left[\boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bo}} \times\right]\mathbf{D}_{\mathrm{b}}^{\mathrm{o}}(\mathbf{m}_{\mathrm{b}}^{\mathrm{o}})\boldsymbol{\omega}_{\mathrm{o}}^{\mathrm{oi}} + \mathbf{J}^{-1}\left[\left(\mathbf{J}\boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bo}} + \mathbf{J}\mathbf{D}_{\mathrm{b}}^{\mathrm{o}}(\mathbf{m}_{\mathrm{b}}^{\mathrm{o}})\boldsymbol{\omega}_{\mathrm{o}}^{\mathrm{oi}}\right) \times\right]\left(\boldsymbol{\omega}_{\mathrm{b}}^{\mathrm{bo}} + \mathbf{D}_{\mathrm{b}}^{\mathrm{o}}(\mathbf{m}_{\mathrm{b}}^{\mathrm{o}})\boldsymbol{\omega}_{\mathrm{o}}^{\mathrm{oi}}\right) + \mathbf{J}^{-1}\left(\mathbf{T}_{\mathrm{b}}^{\mathrm{c}} + \mathbf{T}_{\mathrm{b}}^{\mathrm{d}}\right) \tag{6}$$

Only the gravity-gradient disturbance torque is considered here. It can be modeled by (Wiesel, 1997):

$$\mathbf{T}_{\mathrm{b}}^{\mathrm{d}} = \frac{3\mu}{a^3}\left[\hat{\mathbf{R}} \times \left(\mathbf{J}\hat{\mathbf{R}}\right)\right] \tag{7}$$

where $\hat{\boldsymbol{R}}$ is the unit vector along the zenith.

By truncating the Taylor series expansion of the nonlinearities in Eq. (2) and Eq. (6) about the equilibrium point $\left[\bar{\boldsymbol{m}}_b^{o\prime}, \bar{\boldsymbol{\omega}}_b^{bo\prime}\right]' = \mathbf{0}$, the following linearized state equation is obtained:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{8a}$$

where $\boldsymbol{u} \triangleq \boldsymbol{T}_b^c$, $\boldsymbol{x} \triangleq \left[\boldsymbol{m}_b^{o\prime}, \boldsymbol{\omega}_b^{bo\prime}\right]'$ and,

$$\mathbf{B} = \begin{bmatrix} \mathbf{0}_{3\times3} \\ \mathbf{J}^{-1} \end{bmatrix} \tag{8b}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{3\times3} & \frac{1}{4}\mathbf{I}_3 \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \tag{8c}$$

where, by considering the inertia matrix as diagonal and defining $\sigma_x \triangleq (J_{22} - J_{33})J_{11}^{-1}$, $\sigma_y \triangleq (J_{11} - J_{33})J_{22}^{-1}$ and $\sigma_z \triangleq (J_{22} - J_{11})J_{33}^{-1}$, the matrices $\boldsymbol{A}_{21}$ and $\boldsymbol{A}_{22}$ are given by:

$$\mathbf{A}_{21} = \begin{bmatrix} 12\sigma_x\mu/a^3 & 0 & 0 \\ 0 & 12\sigma_y\mu/a^3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{8d}$$

$$\mathbf{A}_{22} = \begin{bmatrix} 0 & 0 & \omega_o(1-\sigma_x) \\ 0 & 0 & 0 \\ -\omega_o(1+\sigma_z) & 0 & 0 \end{bmatrix} \tag{8e}$$

Assuming that the satellite moves in a circular Keplerian orbit and it is a rigid body, then Eq. (8a) is time-invariant.

3.2. Measurement equations

The true values corresponding to the measurements are nonlinearly related with the state variables as follows:

$$\mathbf{b}_\mathrm{b} = \mathbf{D}_\mathrm{b}^\mathrm{o}(\mathbf{m}_\mathrm{b}^\mathrm{o})\mathbf{b}_\mathrm{o} \tag{9a}$$

$$\mathbf{s}_\mathrm{b} = \mathbf{D}_\mathrm{b}^\mathrm{o}(\mathbf{m}_\mathrm{b}^\mathrm{o})\mathbf{s}_\mathrm{o} \tag{9b}$$

$$\boldsymbol{\omega}_\mathrm{b} = \boldsymbol{\omega}_\mathrm{b}^\mathrm{bo} + \mathbf{D}_\mathrm{b}^\mathrm{o}(\mathbf{m}_\mathrm{b}^\mathrm{o})\boldsymbol{\omega}_\mathrm{o}^\mathrm{oi} \tag{9c}$$

Thus substituting Eq. (9) into Eq. (1) and linearizing it by truncating its Taylor series expansion about the equilibrium point $\left[\bar{\boldsymbol{m}}_b^{o\prime}, \bar{\boldsymbol{\omega}}_b^{bo\prime}\right]' = \mathbf{0}$, the desired measurement equation is obtained:

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{v} \tag{10}$$

where $\boldsymbol{y} = [\boldsymbol{y}_1', \boldsymbol{y}_2', \boldsymbol{y}_3']'$, $\boldsymbol{y}_1 \triangleq \tilde{\boldsymbol{b}}_b - \boldsymbol{b}_o$, $\boldsymbol{y}_2 \triangleq \tilde{\boldsymbol{s}}_b - \boldsymbol{s}_o$, $\boldsymbol{y}_3 \triangleq \tilde{\boldsymbol{\omega}}_b - \boldsymbol{\omega}_o^{oi}$, $\boldsymbol{v}$ are formed by the error (noise and fault) terms in Eq. (1) and the measurement matrix is given by,

$$\mathbf{C} = \begin{bmatrix} 4[\mathbf{b}_\mathrm{o}\times] & \mathbf{0}_{3\times3} \\ 4[\mathbf{s}_\mathrm{o}\times] & \mathbf{0}_{3\times3} \\ \mathbf{C}_{31} & \mathbf{I}_3 \end{bmatrix}$$

$$\mathbf{C}_{31} = 4\begin{bmatrix} 0 & 0 & -\omega_o \\ 0 & 0 & 0 \\ \omega_o & 0 & 0 \end{bmatrix}$$

Note that even though time notion was omited in Eq. (10), $\boldsymbol{C}$ is indeed a time-varying matrix due to its dependence on the vector representations in the orbital CCS, $\boldsymbol{b}_o$ and $\boldsymbol{s}_o$.

4. FAULT-TOLERANT ATTITUDE DETERMINATION

Figure 2 exhibits the fault-tolerant attitude determination scheme that is been proposed in this work. The modules of the scheme are described in the following subsections.

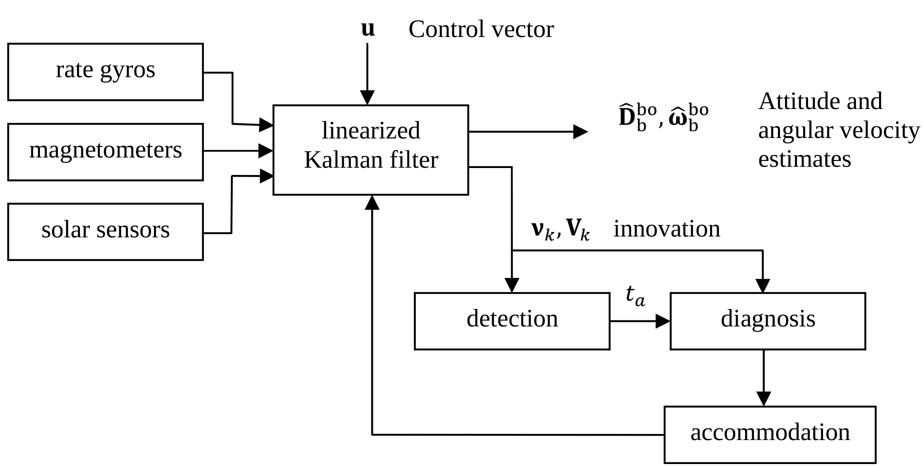


Figure 2. A scheme for fault-tolerant attitude determination

4.1. Linearized Kalman filter

A Kalman filter [see (Gelb, 1974) for details] that employs de discretized model corresponding to the *state equation* Eq. (8) and the *measurement equation* Eq. (10) is used to estimate MRP and angular velocity of the satellite with respect to S_o. Considering the motion is constrained within a region of the state space where those linearized equations are accurate approximations, then, for a fault-free system, the innovation $\{\boldsymbol{\nu}_k\}$ of the Kalman filter can be assumed as being a white sequence with zero mean and covariance $\boldsymbol{V}_k$ computed by the estimator (Anderson and Moore, 1982) as:

$$\mathbf{V}_k = \mathbf{C}_k \mathbf{P}_{k|k-1} \mathbf{C}_k' + \mathbf{R}_k \tag{11}$$

where $\boldsymbol{P}_{k|k-1} \in \mathfrak{R}^{n\times n}$ is the covariance of the predicted estimate of the state and $\boldsymbol{R}_k \in \mathfrak{R}^{m\times m}$ is the (assumed known) covariance of the measurement noise.

Even though measurements from rate gyros are available, the dynamic model of the satellite is being employed. Commonly, this is obviated by using angular velocity measurements in the integration of the attitude kinematic differential equation and, additionally, augmenting the state vector with gyro biases, whose estimates are used to correct those measurements (Lefferts *et al.*, 1982). However, in this case, as consequence of a bias at some gyro, the innovation sequence will exhibit a ramp-wise signature, for which the detection delay is larger.

4.2. Fault detection

Before activating the diagnosis module, detection need to be performed which says that something is going wrong in the system. This is accomplished here by simply realizing a chi-square test on the statistics:

$$\boldsymbol{\varepsilon}_k(\mathbf{M}^{\mathbf{d}}) = \sum_{j=k-\mathbf{M}^{\mathbf{d}}+1}^{k} \boldsymbol{\nu}_j{}' \mathbf{V}_j^{-1} \boldsymbol{\nu}_j \tag{12}$$

where M^d is the detection horizon. Under healthy condition, in which the innovation $\{\boldsymbol{\nu}_j\}$ is a zero-mean white Gaussian sequence, it can be shown that $\varepsilon_k(M^d)$ is a chi-square random variable with $M^d m$ degrees of freedom (Papoulis and Pillai, 2002),

$$\varepsilon_k(\mathrm{M^d}) \sim \chi^2_{(\mathrm{M^d}m)}$$

In this way, given a confidence level of $1-\alpha$, then the probability of detection is:

$$p(\varepsilon_k(\mathrm{M^d}) \in [0, \mathrm{r}_\alpha]) = 1 - \alpha$$

where r_α can be obtained from a chi-square table. Therefore, with probability of false alarm α, a fault is detected if $\varepsilon_{t_a}(M^d) > r_\alpha$. The time instant t_a is called alarm instant.

Before describing the diagnosis module, the signatures of each additive measurement fault belonging to a set of hypotheses are required to be known *a priori*. A signature formula is derived in the next subsection.

4.3. Fault signature on the innovation sequence

A common approach to fault diagnosis consists of the statistical processing of the innovation sequence of a state estimator (or a bank of state estimators) (Willsky and Jones, 1976), (Prakashi *et al.*, 2002), (Prakashi *et al.*, 2005), (Deshpande *et al.*, 2008). These methods require prior knowledge of the fault signatures on the innovation sequence for a set of hypothesized faults. Previous works on such approach have computed such signatures recursively. A new look into the problem is presented here that bases on a non-recursive formula for fault signatures, which is derived in the following by considering the system is linear-Gaussian and the state estimator is the Kalman filter.

Let the system be described by the following model:

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k + \boldsymbol{\Gamma}_k \mathbf{w}_k \tag{13a}$$

$$\mathbf{y}_k = \mathbf{C}_k \mathbf{x}_k + \mathbf{v}_k + b_f^l \mathbf{f}_k^l(t_f^l) \tag{13b}$$

where $\boldsymbol{x}_k \in \mathfrak{R}^n$ is the state vector and $\boldsymbol{y}_k \in \mathfrak{R}^m$ is the observed output vector; $\boldsymbol{A}_k$, $\boldsymbol{B}_k$, $\boldsymbol{\Gamma}_k$ e $\boldsymbol{C}_k$ are known matrices of appropriate dimensions; $\{\boldsymbol{w}_k\}$ and $\{\boldsymbol{v}_k\}$ are mutually independent Gaussian sequences with known parameters. These sequences are also independent of $\boldsymbol{x}_0$; $\boldsymbol{f}_k^l(t_f^l)$ is the l-th fault vector at instant k. t_f^l is the fault instant and b_f^l is the magnitude of the fault.

Lemma 1 – Let $\{\boldsymbol{\nu}_k\}$ be the innovation sequence generated by the Kalman filter using the model in Eq. (13) assuming that $\boldsymbol{f}_k^l(t_f^l) = \mathbf{0}$. It is well-known that if the system is indeed free from faults, then the innovation is a zero-mean Gaussian white sequence, whose covariance, $\boldsymbol{V}_k$, is computed in the filter by Eq. (11). However, considering that in fact the fault vector is different from zero and is not compensated in the filter, then at instant k, the innovation is given by:

$$\boldsymbol{\nu}_k = \boldsymbol{\nu}_k^{\mathrm{N}} + b_f^l \mathbf{G}_k^l(t_f^l) \tag{14a}$$

where $\{\boldsymbol{\nu}_k^N\}$ is a zero-mean white sequence with covariance $\boldsymbol{V}_k$ and,

$$\mathbf{G}_k^l(t_f^l) = -\mathbf{C}_k \sum_{j=1}^{k-t_f^l} \prod_{i=1}^{j-1} \mathbf{A}_{k-i}\left[\mathbf{I}_n - \mathbf{K}_{k-i}\mathbf{C}_{k-i}\right] \mathbf{A}_{k-j}\mathbf{K}_{k-j}\mathbf{f}_{k-j}^l(t_f^l) + \mathbf{f}_k^l(t_f^l) \tag{14b}$$

where $\boldsymbol{K}_k \in \mathfrak{R}^{n\times m}$ is the Kalman gain; $\boldsymbol{G}_k^l(t_f^l) \in \mathfrak{R}^m$ is the fault signature vector. □

proof. *The innovation vector at instant k is defined as $\boldsymbol{\nu}_k = \boldsymbol{y}_k - \hat{\boldsymbol{y}}_{k|k-1}$, where the measurement $\boldsymbol{y}_k$ is given in* Eq. *(13b) and $\hat{\boldsymbol{y}}_{k|k-1}$ is the expectation of $\boldsymbol{y}_k$ conditioned on all the measurements from the start of the estimation to the previous instant, $k-1$. This set of measurements is denoted by $\boldsymbol{y}_{1:k-1}$. The KF computes the predicted measurement $\hat{\mathbf{y}}_{k|k-1}$ by means of:*

$$\hat{\mathbf{y}}_{k|k-1} = \mathbf{C}_k \hat{\mathbf{x}}_{k|k-1} \tag{15a}$$

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1}(\mathbf{I}_n - \mathbf{K}_{k-1}\mathbf{C}_{k-1})\hat{\mathbf{x}}_{k-1|k-2} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} + \mathbf{A}_{k-1}\mathbf{K}_{k-1}\mathbf{y}_{k-1}^{\mathrm{N}} + \mathbf{A}_{k-1}\mathbf{K}_{k-1}\left(b_f^l \mathbf{f}_{k-1}^l(t_f^l)\right) \tag{15b}$$

where $\boldsymbol{y}_{k-1}^N \triangleq \boldsymbol{C}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{v}_{k-1}$, i. e., it is the part of the measurement that is not affected by the fault vector. Thus defining

$$\mathbf{a}_{k-1} = \mathbf{A}_{k-1}(\mathbf{I}_n - \mathbf{K}_{k-1}\mathrm{C}_{k-1}) \tag{16a}$$

$$\mathbf{b}_{k-1} = \mathbf{B}_{k-1}\mathbf{u}_{k-1} + \mathbf{A}_{k-1}\mathbf{K}_{k-1}\mathbf{y}_{k-1}^{\mathrm{N}} \tag{16b}$$

$$\mathbf{d}_{k-1} = \mathbf{A}_{k-1}\mathbf{K}_{k-1}\left(b_f^l \mathbf{f}_{k-1}^l(t_f^l)\right) \tag{16c}$$

then Eq. *(15b) can be written as*

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{a}_{k-1}\hat{\mathbf{x}}_{k-1|k-2} + \mathbf{b}_{k-1} + \mathbf{d}_{k-1} \tag{17}$$

Then, by recursively substituting Eq. *(17) in itself until the past instant t_f^l, gives rise to*

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{a}_{k-1}\left[\mathbf{a}_{k-2}\left\{\ldots\langle \mathbf{a}_{t+1}\left(\mathbf{a}_t\hat{\mathbf{x}}_{t|t-1} + \mathbf{b}_t + \mathbf{d}_t\right) + \mathbf{b}_{t+1} + \mathbf{d}_{t+1}\rangle \ldots\right\} + \mathbf{b}_{k-2} + \mathbf{d}_{k-2}\right] + \mathbf{b}_{k-1} + \mathbf{d}_{k-1} \tag{18}$$

Note that the terms in Eq. *(18) that makes it dependent on the fault vector are those containing $\boldsymbol{d}_t, \boldsymbol{d}_{t+1}, \ldots, \boldsymbol{d}_{k-1}$. Further, the term in $\boldsymbol{y}_k$ making it different from the healthy measurement $\boldsymbol{y}_k^N$ is $b_f^l \boldsymbol{f}_k^l(t_f^l)$. In this way, it appears a bias in the innovation vector $\boldsymbol{\nu}_k$ which is given by*

$$\mathbf{E}\{\boldsymbol{\nu}_k\} \triangleq b_f^l \mathbf{G}_k^l(t_f^l) = b_f^l \mathbf{f}_k^l(t_f^l) - \mathbf{C}_k(\mathbf{a}_{k-1}[\mathbf{a}_{k-2}\{\ldots\langle \mathbf{a}_{t+1}(\mathbf{d}_t) + \mathbf{d}_{t+1}\rangle \ldots\} + \mathbf{d}_{k-2}] + \mathbf{d}_{k-1}) \tag{19}$$

Finally, from Eq. *(19), by considering the definitions in* Eq. *(16), the fault signature vector $\boldsymbol{G}_k^l(t_f^l)$ can be written as* Eq. *(14b).* □

Remark 1 – It is noted that there exists a linear relation between the fault signature and its associated fault vectors, which would not be true if either system or filter was nonlinear. Even in the case of using an extended Kalman filter, as the Jacobian matrices depend on the estimates that in turn depend on faulty measurements, then it is not possible to write the fault signature as a linear combination of the fault vectors. □

4.3. Fault diagnosis

By hypothesizing a finite set of additive measurement faults for which signatures on the innovation sequence of the Kalman filter are given by Eq. (14), the fault diagnosis problem can be formulated as a multiple composite hypotheses testing as reviewed in the following. As mentioned earlier, the innovation sequence of the Kalman filter when the system is healthy is white and, at the time instant k, the innovation vector is $\boldsymbol{\nu}_k = \boldsymbol{\nu}_k^N \sim N(\mathbf{0}, \boldsymbol{V}_k)$. In this case, the joint probability density function (pdf) of the set of vectors $\boldsymbol{\nu}_{k-M+1:k} = \boldsymbol{\nu}_{k-M+1:k}^N \triangleq \{\boldsymbol{\nu}_{k-M+1}^N, \boldsymbol{\nu}_{k-M+2}^N, \dots, \boldsymbol{\nu}_k^N\}$ is given by:

$$p\left(\mathbf{v}_{k-\mathrm{M}+1:k}^{\mathrm{N}}\right) = \prod_{l=k-\mathrm{M}+1}^{k} N(\mathbf{0}, \mathbf{V}_l) \tag{20}$$

where M is the diagnosis horizon. The superscript N denotes normal (or healthy) condition.

In the presence of a fault that has not been compensated in the Kalman filter equations, the set $\boldsymbol{\nu}_{k-M+1:k}$ shows a deterministic signature [see Subsection 4.2] that shifts its mean value away from zero. Therefore, this set shall show the following joint pdf:

$$p(\mathbf{v}_{k-\mathrm{M}+1:k}) = \prod_{i=k-\mathrm{M}+1}^{k} N\left(b_f^l \mathbf{G}_i^l\left(t_f^l\right), \mathbf{V}_i\right) \tag{21}$$

where $b_f^l \mathbf{G}_i^l\left(t_f^l\right)$ is the signature at instant i of the hypothesized fault H_j.
In the following, the diagnosis is defined as a hypotheses testing.

Definition 1 – The diagnosis (isolation and estimation of the fault) problem can be defined by means of the following multiple composite hypotheses testing:

$$\mathrm{H}_0\colon \mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1} = \mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}^{\mathrm{N}} \tag{22}$$

$$\mathrm{H}_1\colon \mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1} = \mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}^{\mathrm{N}} + b_f^1 \mathbf{G}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}^1\left(t_f^1\right)$$

...

$$\mathrm{H}_h\colon \mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1} = \mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}^{\mathrm{N}} + b_f^h \mathbf{G}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}^h\left(t_f^h\right)$$

where H_0 models the absence of faults, while H_1 - H_h model possible faults; $b_f^l \boldsymbol{G}_{k-M+1:k}^l\left(t_f^l\right)$ is the signature of the l-th hypothesized fault H_l on the set of innovation vectors. □

The optimal solution (in the minimum probability of error sense) for the test in Eq. (22) consists of the Bayesian maximum *a posteriori* decision rule, which chooses hypothesis H_l as being that better explaining the occurred fault if (Kay, 1998):

$$p\left(\mathrm{H}_l \middle| \mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}\right) > p\left(\mathrm{H}_s \middle| \mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}\right), \quad \forall s \neq l \tag{23a}$$

$$p\left(\mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1} \middle| \mathrm{H}_l\right) p(\mathrm{H}_l) > p\left(\mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1} \middle| \mathrm{H}_s\right) p(\mathrm{H}_s), \quad \forall s \neq l \tag{23b}$$

where the likelihood function, $p\left(\boldsymbol{\nu}_{t_a-M:t_a+M-1} \middle| H_s\right)$, is given by:

$$p\left(\mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1} \middle| \mathrm{H}_s\right) = \prod_{i=t_a-\mathrm{M}}^{t_a+\mathrm{M}-1} N\left(b_f^s \mathbf{G}_i^s\left(t_f^s\right), \mathbf{V}_i\right) \tag{23c}$$

and the prior probabilities, $p(H_s)$, $s = 1, \dots, h$, are assumed to be known.

It is not possible to hypothesize the parameters (magnitude and fault instant) for each fault, which make difficult to solve Eq. (23b) explicitly. A suboptimal approach (known as generalized likelihood ratio test (GLRT)) to overcome this problem consists of replacing the true parameters by their maximum likelihood (ML) estimates. In the latter case, the detector is given by next theorem.

Theorem 1 – The GLRT for diagnosing additive measurement faults in linear-Gaussian systems modeled by Eq. (13) is given by:

Detect H_l if

$$\mathrm{T}_l\left(\mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}\right) > \mathrm{T}_s\left(\mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}\right), \quad \forall s \neq l \tag{24}$$

where

$$\mathrm{T}_s\left(\mathbf{v}_{t_a-\mathrm{M}:t_a+\mathrm{M}-1}\right) = ln\left(p(\mathrm{H}_s)\right) - \frac{\left(\hat{b}_f^s\right)^2}{2}\sum_{i=t_a-\mathrm{M}}^{t_a+\mathrm{M}-1} \mathbf{G}_i^s\left(\hat{t}_f^s\right)' \mathbf{V}_i^{-1}\mathbf{G}_i^s\left(\hat{t}_f^s\right) + \hat{b}_f^s \sum_{i=t_a-\mathrm{M}}^{t_a+\mathrm{M}-1} \mathbf{G}_i^s\left(\hat{t}_f^s\right)' \mathbf{V}_i^{-1}\mathbf{v}_i$$

$$\hat{t}_f^s = \arg\max_{t_f^s \in [t_a-\mathrm{M}:t_a]} \varsigma(t_f^s)$$

$$\varsigma\left(t_f^s\right) = -\frac{\left(\hat{b}_f^s\left(t_f^s\right)\right)^2}{2}\sum_{i=t_f^s}^{t_a+\mathrm{M}-1} \mathbf{G}_i^s\left(t_f^s\right)' \mathbf{V}_i^{-1}\mathbf{G}_i^s\left(t_f^s\right) + \hat{b}_f^s\left(t_f^s\right) \sum_{i=t_a-\mathrm{M}}^{t_a+\mathrm{M}-1} \mathbf{G}_i^s\left(t_f^s\right)' \mathbf{V}_i^{-1}\mathbf{v}_i$$

$$\hat{b}_f^s\left(t_f^s\right) = \frac{\sum_{i=t_a-\mathrm{M}}^{t_a+\mathrm{M}-1} \mathbf{G}_i^s\left(t_f^s\right)' \mathbf{V}_i^{-1}\mathbf{v}_i}{\sum_{i=t_a-\mathrm{M}}^{t_a+\mathrm{M}-1} \mathbf{G}_i^s\left(t_f^s\right)' \mathbf{V}_i^{-1}\mathbf{G}_i^s\left(t_f^s\right)}$$

$$\hat{b}_f^s \triangleq \hat{b}_f^s\left(\hat{t}_f^s\right) \qquad \square$$

The proof of the Theorem 1 is omitted here, but similar derivations can be found in Kay (1998).

4.4. Fault accommodation

Since diagnosis has finished at instant $k + M - 1$ producing an estimate of the actual fault, $\hat{b}_f^l \hat{\boldsymbol{f}}_k^l\left(\hat{t}_f^l\right)$, the Kalman filter need to be compensated in order to yield accurate state estimation. This is accomplished by assuming that the hypothesis H_l is the actual one and that $E\left[b_f^l \boldsymbol{f}_k^l\left(t_f^l\right) \middle| \boldsymbol{y}_{0:k}\right] \cong \hat{b}_f^l \hat{\boldsymbol{f}}_k^l\left(\hat{t}_f^l\right)$, which make possible to approximate the filter equation for measurement prediction by:

$$\hat{\mathbf{y}}_{k+1|k} = \mathbf{C}_k \hat{\mathbf{x}}_{k+1|k} + \hat{b}_f^l \hat{\mathbf{f}}_k^l\left(\hat{t}_f^l\right) \tag{25}$$

where $\hat{\boldsymbol{y}}_{k+1|k} \triangleq E[\boldsymbol{y}_{k+1}|\boldsymbol{y}_{0:k}]$.

5. SIMULATION RESULTS

In this section, the scheme for fault-tolerant attitude determination previously developed is used to estimate the attitude and angular velocity of an Earth-pointing spacecraft using measurements from magnetometer, Sun sensors and rate-gyros. The satellite is simulated as moving in a low-Earth Keplerian circular orbit with height of $750\ km$ and inclination of 87°. The attitude dynamic and kinematic equations were integrated using Runge-Kutta 4 with steps of $0.001\ s$. The disturbing torque was composed by gravity-gradient and a random Gaussian signal with mean zero and covariance of $1.0 \times 10^{-5}\ \boldsymbol{I}_3\ Nm$. The geomagnetic field was modeled by the WMM 2005 (McLean *et al.*, 2004) and the Sun direction is modeled by a simple Keplerian model of the Earth rotation about the Sun (Vallado, 2004). Table 1 presents the covariances of the measurement noises that were assumed as zero-mean white Gaussian sequences.

Table 1. Covariances of the measurement noises

Measurement in $\mathrm{S_b}$	**Covariance**	**unit**
Geomagnetic field	$4.0 \times 10^{-14}\mathbf{I}_3$	T^2 [(1)]
Sun direction	$1.0 \times 10^{-4}\ \mathbf{I}_3$	$-$
Inertial ang. velocity	$1.0 \times 10^{-10}\ \mathbf{I}_3$	$(rad/s)^2$

[(1)] T denotes Tesla.

Only abrupt faults on geomagnetic field and angular velocity components were considered in this brief study. Those are listed along with their parameters (fault magnitude and fault instant) in Tab. 2. The components are assumed as that projected in the body coordinate system S_b.

Monte Carlo simulations consisting of 100 runs were realized for each fault in Tab. 2. The statistics of the estimates of fault magnitudes produced by the proposed scheme are shown in Tab. 3. It is noted that accurate fault estimation is accomplished.

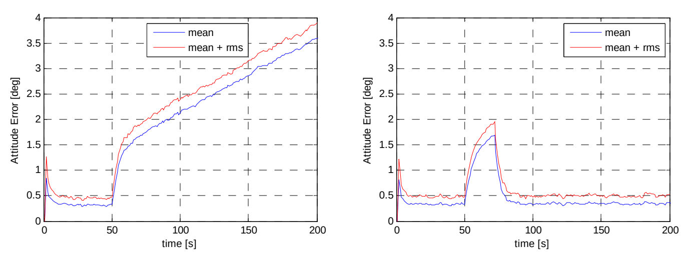
Figures 3 and 4, by taking the fault 1 into account, compare the fault-tolerant ADS (FTADS) with that obviating fault accommodation, which is called *basic* ADS. It was performed 100 runs of the estimation process. Figure 3 exhibits the angular error in degrees, while Fig. 4 shows the magnitude of the angular velocity error in degrees per hour. It is noted that both attitude and angular velocity estimations undergo a diverging behavior in the basic ADS as consequence of the fault. On the other hand, as soon as fault is diagnosed and reconfiguration of the Kalman filter is realized, the estimates of the FTADS become accurate, experiencing only a graceful degradation when compared with the performance that had been producing before the fault instant.

Table 2. Parameters of the simulated faults

Fault number	**Description**	**Magnitude**	**Instant** [s]
1	Bias in magnetometer x	$2.0 \times 10^{-6}\ T$	50
2	Bias in magnetometer y	$2.0 \times 10^{-6}\ T$	50
3	Bias in magnetometer z	$2.0 \times 10^{-6}\ T$	50
4	Bias in rate-gyro x	$5.0 \times 10^{-4}\ rad/s$	100
5	Bias in rate-gyro y	$5.0 \times 10^{-4}\ rad/s$	100
6	Bias in rate-gyro z	$5.0 \times 10^{-4}\ rad/s$	100

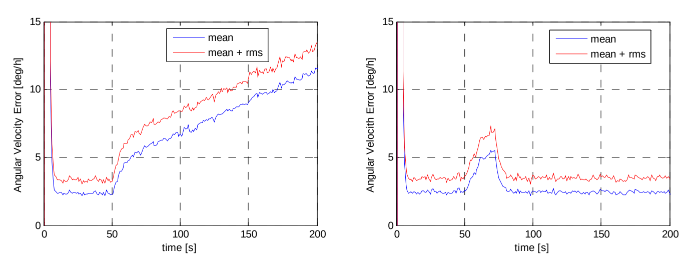
Table 3. Statistics of estimates of magnitude of faults using 100 runs

Fault number	**Mean**	**Std deviation**
1	$2.0084 \times 10^{-6}\ T$	$0.0737 \times 10^{-6}\ T$
2	$1.9820 \times 10^{-6}\ T$	$0.0739 \times 10^{-6}\ T$
3	$2.0000 \times 10^{-6}\ T$	$0.0778 \times 10^{-6}\ T$
4	$5.0198 \times 10^{-4}\ rad/s$	$0.0994 \times 10^{-4}\ rad/s$
5	$5.0164 \times 10^{-4}\ rad/s$	$0.0910 \times 10^{-4}\ rad/s$
6	$5.0158 \times 10^{-4}\ rad/s$	$0.0997 \times 10^{-4}\ rad/s$



(a) Basic ADS (b) Fault-Tolerant ADS

Figure 3. Magnitude of the attitude error in presence of fault 1



(a) Basic ADS (b) Fault-Tolerant ADS

Figure 4. Magnitude of the angular velocity error in presence of fault 1

6. CONCLUSION

A scheme for fault-tolerant attitude determination was proposed in this paper, which is applicable to Earth-oriented satellites moving in a low-Earth orbit and having in their suite of sensors three orthogonal magnetometers, three orthogonal rate gyros and Sun sensors. Fault-tolerance was reached by means of three modules. The first one is the *detection module*, which says that something is going wrong. The alarm generated by the detection module is used to activate the *diagnosis module*, which performs fault isolation and fault estimation through a generalized likelihood ratio test. Finally, as soon as the fault is diagnosed, the fault accommodation module is executed, whose task is to compensate the measurement prediction equation of the Kalman filter.

Simulation results showed that the proposed scheme is able to overcome additive measurement faults whose forms are step-wise, while obviating the augmentation of the state vector with measurement biases.

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