FLOQUET STABILITY ANALYSIS OF THE FLOW AROUND AN AIRFOIL

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Abstract. *This work is concerned with the three-dimensional instabilities of the flow around an airfoil profile in stalled configuration. The goal is to investigate how the flow becomes unstable in the wake, studying the influence of the Reynolds number based on the chord* (Re) *and the angle of attack* (α). *For each* Re *and* α, *the stability analysis of the two-dimensional flow is carried out with respect to three-dimensional infinitesimal perturbations, using the Floquet stability theory. The time-periodic base flow is computed with the spectral/hp element method. The results are the modulus of the Floquet multiplier versus the wavelength of the perturbation, showing the characteristic spanwise length of the three-dimensionality. It is possible to use these results in order to perform quasi-three-dimensional simulations of the flow past uniform wings, with great computational time saving and low resolution loss on the physics of the phenomena.*

Keywords: *Secondary wake transition, Floquet analysis, stalled airfoil*

1. INTRODUCTION

Flow past airfoils has obvious importance in aerospace applications. Although numerous studies concerned with the influence of angle of attack, Reynolds number, Mach number and geometry in the lift coefficient can be found in literature, the same can not be said about stability analysis of the wake. The main objective of this work is to investigate the secondary transition of the flow around an airfoil profile, obtaining the critical values of Re for which the wake becomes three-dimensional. The methodology used is the Floquet stability analysis of a base flow calculated using the spectral/hp elements method.

Barkley and Henderson (1996) used the Floquet theory to study the transition for the fixed circular cylinder and found two unstable modes. Mode A does not present a break in the spatio-temporal symmetry, has a characteristic length of 4 diameters and first occurs for $\mathrm{Re} = 188$. Mode B presents a break in the spatio-temporal symmetry, has characteristic length of 0.8 diameters and first occurs for $\mathrm{Re} = 259$.

Theofilis and Sherwin (2001) performed a stability analysis of the flow past a NACA 0012 airfoil profile. Main parameters are $\mathrm{Re} = 1000$ and $\alpha = 5^o$. The concern was the separation bubble formed on the trailing-edge, not the transition in the wake.

Abdessemed (2007) deals with the stability analysis of the flow past a low pressure turbine blade. Primary instabilities of the wake occurs for $\mathrm{Re} = 905$ and secondary instabilities were found to be indifferent to parameter change. It brings the question if it is possible to compare bluff geometries with aerodynamic ones.

Carmo et al. (2008) performed a similar study for two circular cylinders of diameter D in staggered arrangements with fixed streamwise separation of 5D and cross-stream separation varying from 0 to 3D. The influence of the second cylinder causes the vortex shedding pattern to be changed, and the flow presents unstable modes different from the single fixed cylinder geometry. Subharmonic modes can be noticed in the secondary transition of the wake.

This work is organized as follows. Section 2 shows a brief overview on the methods used in this paper. In Sec. 3, the convergence test performed is presented, along with the resulting two-dimensional mesh. Both aerodynamic and stability results are shown in Sec. 4, and the conclusions can be found in Sec. 5.

2. METHODOLOGY

2.1 Direct numerical simulations

We consider the flow of a viscous fluid past a NACA 0012 airfoil profile. The fluid is assumed to have constant dynamic viscosity μ and specific mass ρ. The incompressible flow depends on three parameters: the airfoil chord c, the free stream-velocity U and the kinematics viscosity $\nu = \mu/\rho$, so the control parameter of the problem can be taken as the Reynolds number based on chord $\mathrm{Re} = cU/\nu$. For any instant t, the fluid state is determined by the velocity field $\mathbf{u}(x, y, z, t)$ and the pressure field $p(x, y, z, t)$. Using U and c as reference values for velocity and length, we can write the non-dimensional Navier-Stokes equations that govern the problem.

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{u} \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0 \tag{2}$$

Karniadakis and Sherwin (1999) describes the spectral/hp elements method, which is used to solve numerically the incompressible Navier-Stokes equations.

2.2 Floquet stability theory

In stability analysis, the flow is considered as a combination of a base field $\mathbf{U}(x, y, t)$ and a three-dimensional perturbation $\mathbf{u}'(x, y, z, t)$, with $\mathcal{O}(\mathbf{u}') \ll \mathcal{O}(\mathbf{U})$. Hence, we can linearize the Navier-Stokes equation, obtaining:

$$\frac{\partial \mathbf{u}'}{\partial t} = -(\mathbf{U}\cdot\nabla)\mathbf{u}' - (\mathbf{u}'\cdot\nabla)\mathbf{U} - \nabla p' + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{u}' \tag{3}$$

$$\nabla\cdot\mathbf{u}' = 0 \tag{4}$$

where p' is the pressure perturbation. The inlet boundary condition for the perturbation is $\mathbf{u}' = 0$. Floquet theory deals with the stability of T-periodic base flow, and a more complete description of the method can be found in Iooss and Joseph (1990). The T-periodic, function of $\mathbf{U}$, linear operator $\mathbf{L}$ is mounted, and it evolves the perturbation from a instant t to the instant $t + T$. Therefore, we can understand the Eq. (3) as the eigen-problem shown in Eq. (5), restricted by the Eq. (4).

$$\frac{\partial \mathbf{u}'}{\partial t} = \mathbf{L}(\mathbf{u}') \tag{5}$$

The eigenvalues of the operator $\mathbf{L}$ are the so called Floquet exponents σ. The T-periodic solutions of Eq. (5) are a composition of solutions $\widetilde{\mathbf{u}}(x, y, z, t)\,\mathrm{e}^{\sigma t}$. Normally the stability is evaluated with respected of the Floquet multiplier $\mu \equiv \mathrm{e}^{\sigma t}$. If $|\mu| < 1$, then the perturbation is damped and asymptotically goes to zero; if $|\mu| > 1$, then the mode is unstable, and the perturbation grows in the flow.

Another simplification can be made: to assume the problem is homogenous in spanwise direction, so we can write the perturbation using the Fourier integral, as shown in Eq. (6).

$$\mathbf{u}'(x, y, z, t) = \int_{-\infty}^{\infty} \widehat{\mathbf{u}}(x, y, t)\,\mathrm{e}^{\imath\beta z}\, d\beta \tag{6}$$

As modes with different wavenumbers $\beta = 2\pi/\lambda$ do not couple and the base flow $\mathbf{U}$ is two-dimensional, perturbations of the form

$$\mathbf{u}'(x, y, z, t) = (\widehat{\mathbf{u}}\, \cos\beta z, \widehat{\mathbf{v}}\, \cos\beta z, \widehat{\mathbf{w}}\, \cos\beta z) \tag{7}$$

$$p'(x, y, z, t) = \widehat{p}\, \cos\beta z \tag{8}$$

remain under the same form under the operator $\mathbf{L}$.

The velocity and pressure components of the perturbation are functions of x, y and t, so the three-dimensional problem becomes several two-dimensional stability problems, each one with a characteristic wavenumber β. The numeric code used to perform the Floquet analysis computes the eigenvalues of the matrix $\mathbf{A}$ correspondent to the linear operator $\mathbf{L}$ over one period:

$$\{\mathbf{u}'\}_{n+1} = [\mathbf{A}]\,\{\mathbf{u}'\}_n \tag{9}$$

The Arnoldi method is used to evaluate the eigenvalues, computed in the Krylov space, set with dimension 20 in all cases of this work. The use of the Krylov space reduces the order of the matrix $\mathbf{A}$ without a significant change of the largest Floquet multiplier, which is the one that determines the stability of the sistem.

The concern of this work relies on the peaks of the curve $|\mu| \times \beta$, which are the most unstable modes, the ones that rule the growing of the perturbations of the linearized system. We are interested in the secondary transition of the wake, i.e. the Reynolds number for which the flow becomes three-dimensional.

Table 1. Influence of basis function order.

Basis function order	$\overline{C}_L$	C'_L	$\overline{C}_D$	C'_D	St
8th order	0.9230	0.9276	0.4115	0.4117	0.546
9th order	0.9231	0.9276	0.4115	0.4117	0.549
10th order	0.9231	0.9276	0.4115	0.4117	0.551

Table 2. Influence of the outflow boundary position.

Downstream length	$\overline{C}_L$	C'_L	$\overline{C}_D$	C'_D	St
$11c$	0.9208	0.9253	0.4107	0.4109	0.551
$15c$	0.9230	0.9275	0.4114	0.4116	0.551
$20c$	0.9231	0.9276	0.4115	0.4117	0.549
$25c$	0.9235	0.9280	0.4116	0.4118	0.549
$30c$	0.9235	0.9280	0.4116	0.4118	0.549

Table 3. Influence of the velocity inlet boundary position on each lateral.

Lateral length	$\overline{C}_L$	C'_L	$\overline{C}_D$	C'_D	St
$10c$	0.9231	0.9276	0.4115	0.4117	0.549
$15c$	0.9183	0.9227	0.4099	0.4101	0.549
$20c$	0.9148	0.9191	0.4085	0.4087	0.549
$25c$	0.9132	0.9175	0.4082	0.4084	0.549
$30c$	0.9116	0.9159	0.4075	0.4077	0.549
$35c$	0.9109	0.9152	0.4072	0.4075	0.550
$40c$	0.9105	0.9148	0.4071	0.4073	0.552
$45c$	0.9102	0.9145	0.4070	0.4072	0.552
$50c$	0.9100	0.9143	0.4069	0.4072	0.552

3. CONVERGENCE TEST

A convergence test was performed for reliability of the results and in order to optimize the mesh for two-dimensional aerodynamic calculations. The case studied in this work is the flow around the NACA 0012 airfoil. The methodology used is based on Carmo et al. (2008). The parameters tested were the extension of the computational domain, varying the length downstream and on each side, and the order of the basis function. It was used the mean and RMS values of the lift and drag coefficients, $\overline{C}_L$, C'_L, $\overline{C}_D$, C'_D, and the Strouhal number based on chord $\mathrm{St} = f\,c/U$ as reference values. The St was computed using a FFT of the temporal C_L signal. Results are shown in Tab. 1, Tab. 2 and Tab. 3.

The resulting mesh has extension of $10c$ upstream, $30c$ downstream and $50c$ on each side, with nearly 950 quadrilateral cells and a ninth-degree polynomial used as basis function. Figure 1 shows the computational grid. A unitary velocity condition was applied in the upstream and both lateral boundaries. Outflow condition was applied in the downstream boundary, i.e. sets all normal derivatives as zero. The airfoil is assumed to have a finite trailing-edge thickness of $0.005c$ and its boundary condition is set as viscous wall, satisfying non-slip condition.

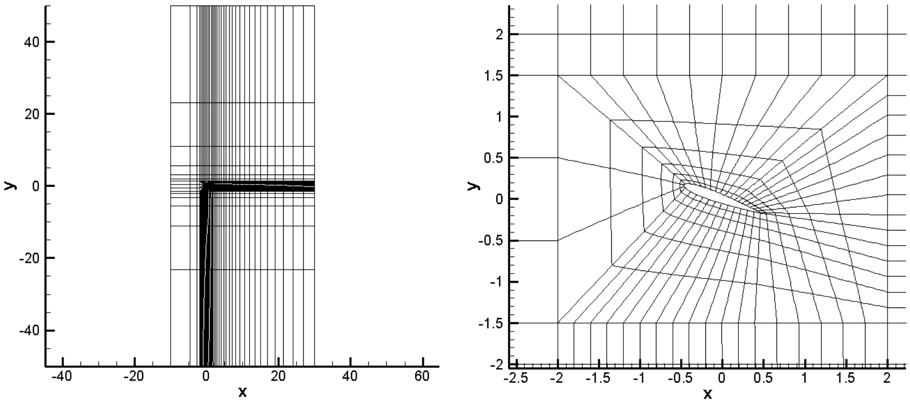
4. RESULTS

4.1 Base flow calculations

In order to obtain a two-dimensional periodic base flow, the flow around the NACA 0012 airfoil profile with an angle of attack $\alpha = 20^o$ was investigated. This high value of α causes the flow to detach and vortex shedding occurs, which is a classic periodic phenomenon. In aeronautics, this condition is referred to as stall, and in flight conditions it is not desirable, since there is an abrupt drop of lift.

In the present study, the range of Reynolds number investigated varied from 400 to 550. The time step was taken as $\Delta t = 0.0002$ time units in order to reduce the Courant-Friedrich-Lewis number (CFL) to values below 1, granting the convergence of the result. The aerodynamic results for each Re are shown in Tab. 4.

The base flow must be periodic, otherwise it is impossible to perform the Floquet stability analysis. We can see if this condition is accomplished by looking at the temporal signal of the lift coefficient and the contours of z component of



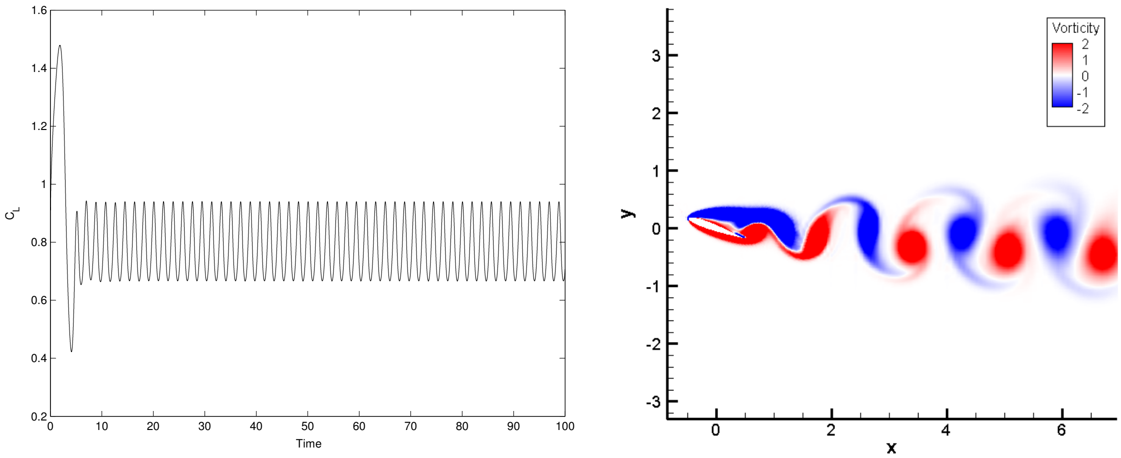
(a) (b)

Figure 1. NACA 0012 grid, $\alpha = 20^o$. (a) Computational domain. (b) Near airfoil detail.

Table 4. Aerodynamic results. NACA 0012, $\alpha = 20^o$.

Re	$\overline{C}_L$	C'_L	$\overline{C}_D$	C'_D	St
400	0.7414	0.7444	0.4051	0.4052	0.5034
450	0.7633	0.7677	0.4083	0.4085	0.5210
460	0.7670	0.7718	0.4091	0.4092	0.5286
470	0.7714	0.7764	0.4100	0.4101	0.5286
500	0.7836	0.7897	0.4128	0.4230	0.5286
550	0.8042	0.8117	0.4181	0.4183	0.5382

vorticity $\omega = \nabla \times \mathbf{u}$, both shown in Fig. 2.



(a) (b)

Figure 2. NACA 0012, $\mathrm{Re} = 500$, $\alpha = 20^o$. (a) Temporal C_L signal. (b) Instant contours of ω_z.

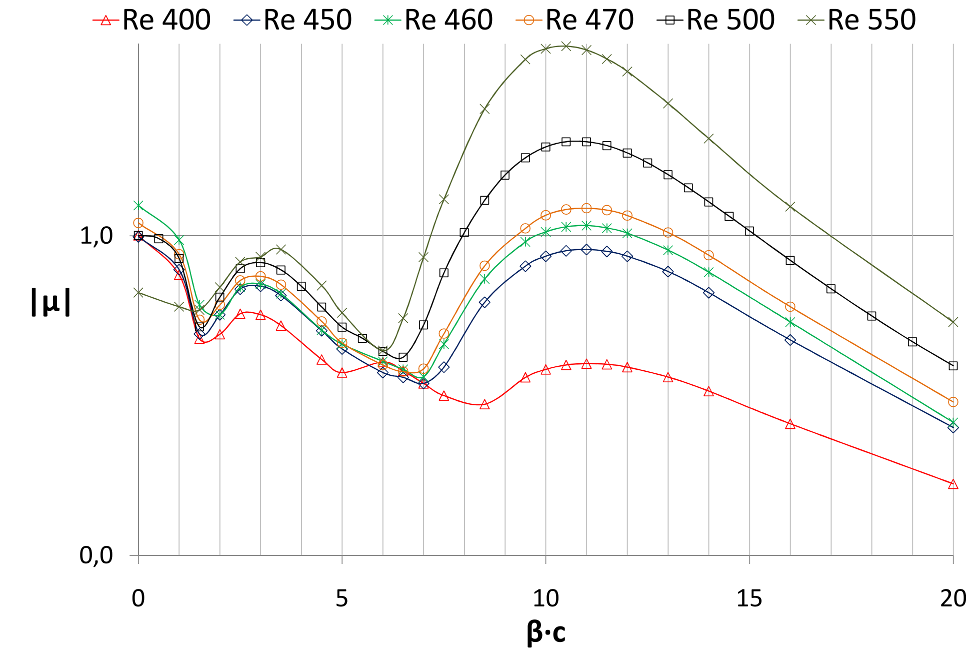
The vortex shedding pattern is the same for other Re values. Given the periodicity of the base flow, we can proceed to the application of Floquet stability analysis in this problem.

4.2 Stability analysis

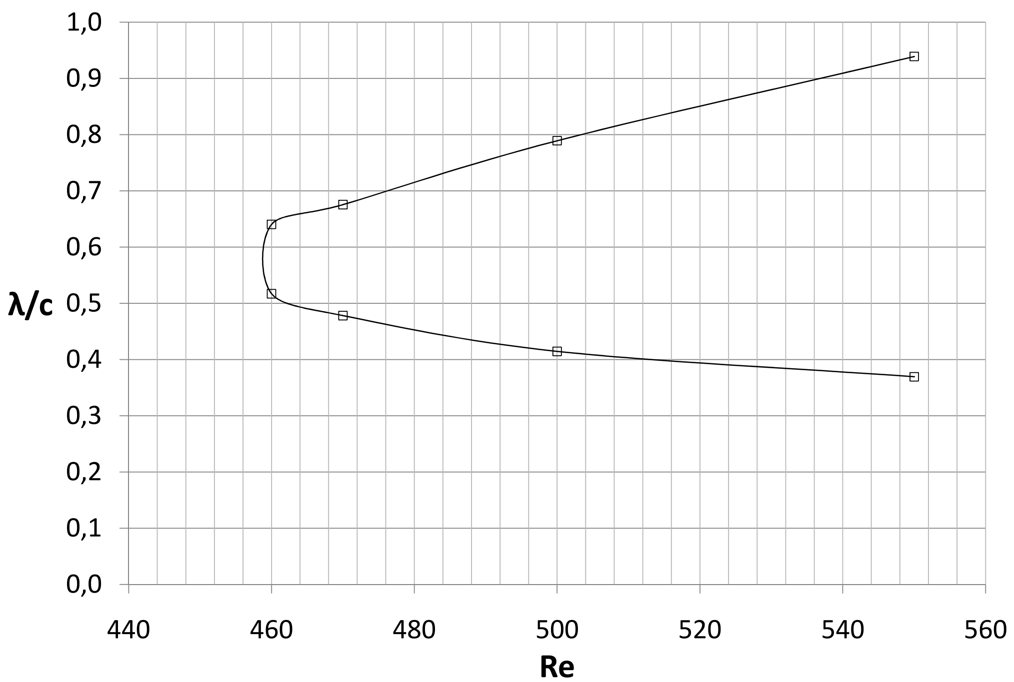
In stability analysis, a three-dimensional perturbation is evolved during a period of the base flow, and the growing rate of the mode is defined by the eigenvalues of the of the linear operator $\mathbf{L}$, which takes the perturbation along one period as seen in Sec. 2.2.

The study consists in finding the modulus of the Floquet multipliers ($|\mu|$) for each value of the wavenumber $\beta = 2\pi/\lambda$.

$|\mu| > 1$ determines an unstable mode. The results, for each Reynolds number studied, are shown in Fig. 3(a), in addition to the neutral stability curve.



(a)



(b)

Figure 3. NACA 0012, $\alpha = 20^o$. (a) Floquet multipliers spectrum. (b) Neutral stability curve.

An unstable mode that appears with $\mathrm{Re} \approx 460$, with characteristic wavenumber $\beta \approx 11$ (wavelength $\lambda \approx 0.57c$). Another peak is found at $\beta \approx 3$, and this mode should become unstable for higher values of Re. It is possible to reconstruct the three-dimensional flow using the base flow and the perturbations, as shown in Fig. 4, in order to get more information of the unstable mode structure. We can notice that the three-dimensional mode is originated in the near wake region, and is stronger in the vortices braids. It has period $2T$, where T is the period of the base flow, which indicates a subharmonic mode:

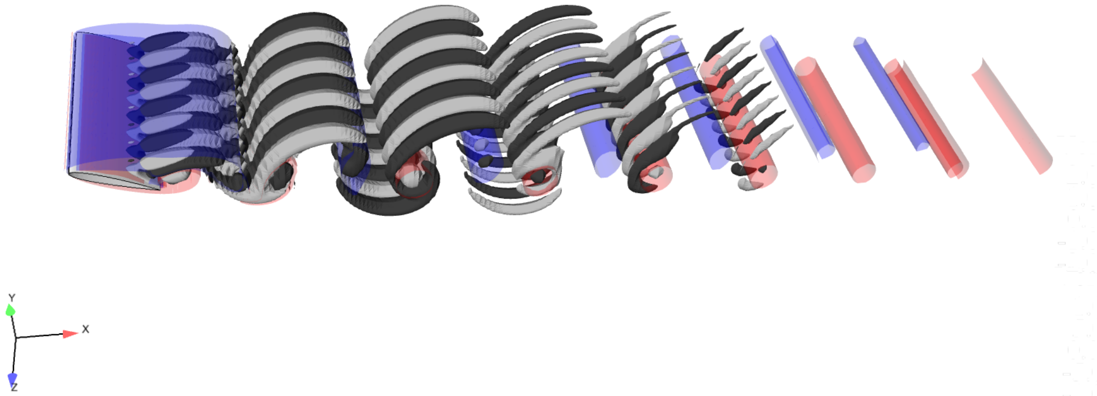


Figure 4. Reconstructed three-dimensional flow. Translucent surfaces are iso-surfaces of ω_z. Solid dark and light surfaces are iso-surfaces of ω_x. NACA 0012, $\mathrm{Re} = 500$, $\alpha = 20^o$, $\beta = 11$.

$$\omega_x(x, y, z, t) = \omega_x(x, y, z, t + 2T) \tag{10}$$

Mathematically, this characteristic is the result of real negative Floquet multipliers. Complex multipliers, as the ones found in the mode with $\beta \approx 3$, give rise to quasi-periodic modes, i.e. the three-dimensional mode has a different period than the base flow.

5. CONCLUSIONS AND PERSPECTIVES

The results presented show that the wake transition in the flow past an airfoil in stalled configuration is quite different from that on the circular cylinder case, beginning with the asymmetry of the geometry, even though both present a strong vortex shedding pattern. The unstable mode found is sub-harmonic, originated in near wake region and stronger in the vortices braids, regarding no semblances with modes A or B described in Barkley and Henderson (1996).

Further works intend to study the influence of the angle of attack α in the secondary transition of the wake. There is a motivation in verifying if, with lower values of α, the behavior of the instabilities can be compared with bluff geometries, such as the circular cylinder.

Since Floquet theory deals with a linearized problem, a non-linear analysis is required, in order to evaluate the final amplitude of the perturbations remaining in the flow. Also, complete three-dimensional direct numerical simulations will be performed in order to validate the method.

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8. Responsibility notice