OPEN LOOP CONTROL OF FLEXIBLE BEAM PERIODIC MOTION VIA FREQUENCY RESPONSE ANALYSIS

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Abstract. *Manipulators and actuators are examples of mechatronic devices devoted to impose desired trajectories and motions in a machine. The trajectory is usually tracked by a feedback system, and controlled in closed loop. When the desired motions are periodic, one can take advantage of system inherent dynamic characteristics to design open loop controllers. For instance, flexible structures naturally present periodic vibration when subjected to excitation. Hence, one can find an excitation (force function) that makes the structure vibrate with the desired periodic motion. In this work, one proposes a procedure to find the force function that must be applied, in open loop, to the base of a cantilever beam, in order to have prescribed periodic motions in the free end of the beam. In this procedure, the necessary force function for the open loop controller is calculated in frequency domain, based on the frequency spectrum of the desired motion and the experimental frequency transfer function between the points of actuation and movement on the beam. By an inverse Fourier transformation, the force function is finally obtained in time domain. The advantage of this procedure lies on the fact that it can be applied to any structure that presents some flexibility. In this work, the procedure is implemented experimentally in a cantilever steel beam, where different types of periodic motion are tested and checked by acceleration measurements at the free end of the beam.*

Keywords: *open loop control, flexible structures, periodic motion, frequency response analysis*

1. INTRODUCTION

In the field of mechatronic devices, it is common the adoption of sensors and actuators, associated to control strategies, to obtain desired motions of the system. The control strategies are used to guarantee that the system performs the desired trajectory within acceptable tolerances. In most cases, the desired trajectory is point-to-point, i.e. the device must move from an initial position to a final position following the predetermined trajectory. However, there are systems where it is necessary to have periodic motions, where the point-to-point trajectory is repeated in time with a certain fundamental frequency. Examples of such systems are scanning mirrors, gyroscopes for motion sensing, electronic nose, atomic force microscope sensing, and 6 DOF motion simulators (Ma et al., 2007; Bucher, 2009).

The design of the control system becomes more challenging when the system is composed of a flexible structure. In this case, the control strategy must overcome and compensate the inherent dynamics of the structure in order to obtain the desired trajectory. Many different strategies can be found in literature to tackle the problem, being PD control (Gross and Tomizuka, 1994; Kim et al., 2003), LQR optimum control (Anisovich and Kriukov, 1982; Cai and Lim, 2006), H^{∞} feedback controller (Lenz et al., 1991), fuzzy logic control (Jnifene and Andrews, 2004), and two-time scale control (Lotfazar et al., 2008) just a few examples. In all these studies, closed loop control strategies are designed for obtaining non-periodic motion trajectories, aiming at suppressing any oscillatory motion of the system. In other words, one must design control strategies to eliminate the "annoying" tendency of dynamic systems, such as flexible structures, to vibrate during the trajectory.

However, considering periodic trajectories, one can take advantage of the natural vibratory behavior of dynamic systems. The problem is that this vibratory behavior is sinusoidal. If one wishes to obtain non-sinusoidal motions of a dynamic system, one will necessarily depend on the application of a control strategy. Fortunately, due to the vibratory nature of dynamic systems, one can apply open loop control strategies.

The application of open loop control strategies to flexible structures has been investigated in Laplace domain (Bhat et al., 1991), which requires a mathematical model of the system to determine the appropriated control signal. However, mathematical models are prone to inaccuracy due to non-linearities of the real system, difficult to measure parameters, and simplifying assumptions.

In this work, one proposes a method to find the necessary open loop control signal to impose a non-sinusoidal periodic motion on the tip of a flexible structure (cantilever beam). This method is based on the experimental frequency response analysis of the system, whose resultant input/output information is sufficient to calculate the control signal. Hence, no mathematical modeling of the system is necessary, and actuator and sensor dynamics can be incorporated into the input/output information of the frequency response analysis, thus forming a whole integrated system.

2. OPEN LOOP CONTROL SIGNAL DETERMINATION VIA FREQUENCY RESPONSE ANALYSIS

The dynamics of linear systems can be represented by input/output relationships called Frequency Response Functions (FRFs). By knowing these functions, one can infer what the system response (output) will be due to an excitation (input), in a given frequency. A usual representation of the FRFs is given by:

$$H(\omega) = \frac{X_o(\omega)}{X_i(\omega)} \tag{1}$$

where $X_i(\omega)$ is the input signal to the system (excitation), $X_o(\omega)$ is the output signal of the system (response), and $H(\omega)$ is the Frequency Response Function, all in frequency domain.

The objective of the open loop control in study is to make a specific position of the structure vibrate with a desired periodic motion (output). Hence, in this case, considering that the output is known, Eq. (1) can be rewritten as follows:

$$X_i(\omega) = \frac{X_o(\omega)}{H(\omega)} \tag{2}$$

Equation (2) estates that the necessary input signal to the system to make it vibrate with desired periodic motion is given by the ratio between the desired output itself and the system FRF (input/output relationship). In other words, the frequency spectrum of the input signal that makes the structure vibrate with the desired periodic output is given by the ratio between the frequency spectrum of the desired motion and the FRF, which contains the dynamic characteristics of the system.

Hence, by knowing the output periodic motion in time ($X_o(t)$), one can apply a Fourier Transform to obtain its frequency spectrum ($X_o(\omega)$). The frequency spectrum of the desired motion is then divided by the FRF of the system, which must be theoretically or experimentally identified a priori. As a result, one obtains the frequency spectrum of the input signal ($X_i(\omega)$), whose signal in time domain ($X_i(t)$) can be calculated by an Inverse Fourier Transform. The resultant input signal in time domain is the necessary open loop control signal to make the structure vibrate with the desired periodic motion. Figure 1 summarizes the procedure to calculate the necessary open loop control signal to obtain desired periodic motions on the system. This procedure is equivalent to the expression:

$$X_i(t) = \mathcal{F}^{-1}\left[\frac{\mathcal{F}[X_o(t)]}{H(\omega)}\right] \tag{3}$$

where $\mathcal{F}[.]$ is the Fourier Transform, and $\mathcal{F}^{-1}[.]$ is the Inverse Fourier Transform.

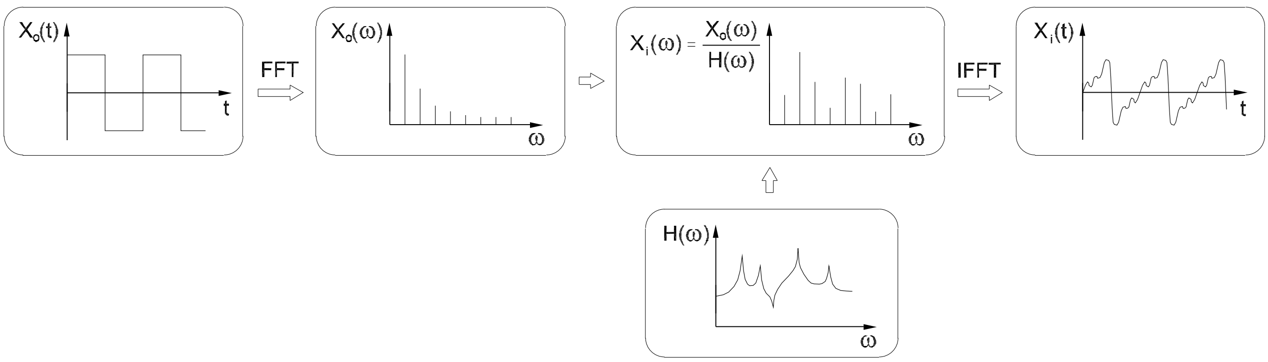


Figure 1. Procedure for the determination of the open loop control signal.

3. EXPERIMENTAL SET-UP

The procedure was tested in a cantilever steel beam, as shown in Fig. 2. An electrodynamic shaker B&K mod. 4809 (2) is connected to the bottom part of the beam, serving as the actuator. The actuator (shaker) is attached to the beam by a stinger and a load cell B&K mod. 8200 (3), which allows input force measurements. Two accelerometers B&K mod. 4383 (4 and 5) are mounted on the beam to measure the output accelerations in the positions of input force and beam tip.

The control signal is sent to the shaker amplifier through the analog output port of a NI PCI 6229 acquisition board, mounted in a microcomputer. The signals from the sensors (load cell and accelerometers) are acquired by analog input ports of the same acquisition board. The software MatLab, with help of the signal acquisition toolbox, is used to manage the whole process.

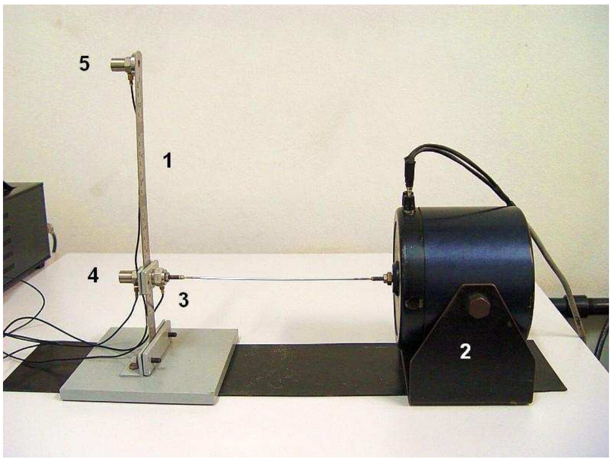


Figure 2. Experimental set-up for testing the open loop control system: 1) cantilever beam; 2) electrodynamic shaker; 3) load cell; 4) accelerometer in the input force position; 5) accelerometer in the beam tip position.

3.1 Identification of System FRF

As described in Section 2, the determination of the open loop control signal ($X_i(t)$) depends on the knowledge of the system FRF. Hence, an identification procedure of the system must be performed before tackling the problem of finding the control signal.

In order to obtain the system FRF, the system must be excited with a broad frequency band signal. In this work, a chirp signal is used, ranging from 5 to 200 Hz, period of 4 s and amplitude of 1 V. The chirp signal is repeated 10 times, totaling a sampling period of 40 s, with sampling frequency of 1 kHz. The chirp signal is sent to the shaker amplifier and the acceleration in the tip of the beam (output response of interest) is measured. With input and output signals in hand, one can calculate the frequency response function estimators H_1 and H_2, and associated coherence (Maia and Silva, 1997). The obtained results for the experimental set-up are shown in Fig. 3.

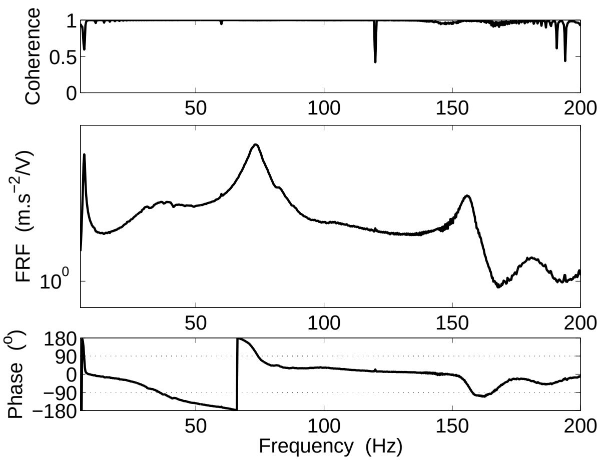


Figure 3. Coherence, amplitude and phase of the cantilever beam FRF (experimental results).

Figure 3 presents the experimentally identified FRF of the cantilever beam shown in Fig. 2 in terms of input control signal to the actuator (V) and output acceleration of the beam tip (m/s^2). One can see that there are a few resonances in the system, with different damping levels, which must be taken into account to find a proper open loop control signal. Because all calculation is done in frequency domain, this system dynamics is automatically considered in the analysis.

Considering that the input signal is the signal to the actuator (V), not the input force to the system, the actuator is considered part of the system and its dynamics is included in the FRF results. Similarly, due to the introduction of mass in the system, the acceleration sensors (accelerometers) must be also considered part of the system, and its dynamics included in the FRF results. Theses considerations simplify the determination of the open loop control signal, because the FRF relates the desired output (tip acceleration) to the input signal to the actuator (control signal).

4. EXPERIMENTAL RESULTS

In order to investigate the efficiency of the procedure, two different non-sinusoidal desired output periodic motions were tested: the triangular wave and the three level trapezoidal wave. In the first case, due to the simplicity of the triangular wave, the output signal in frequency domain is obtained via Fourier series. In the second case, the desired output signal is more complex, and calculation of the Fourier coefficients is tedious. In this case, an FFT algorithm is numerically applied to obtain the signal in frequency domain. After that, the open loop control signal is determined and implemented experimentally in the cantilever beam system.

4.1 Triangular Wave Motion via Fourier Series

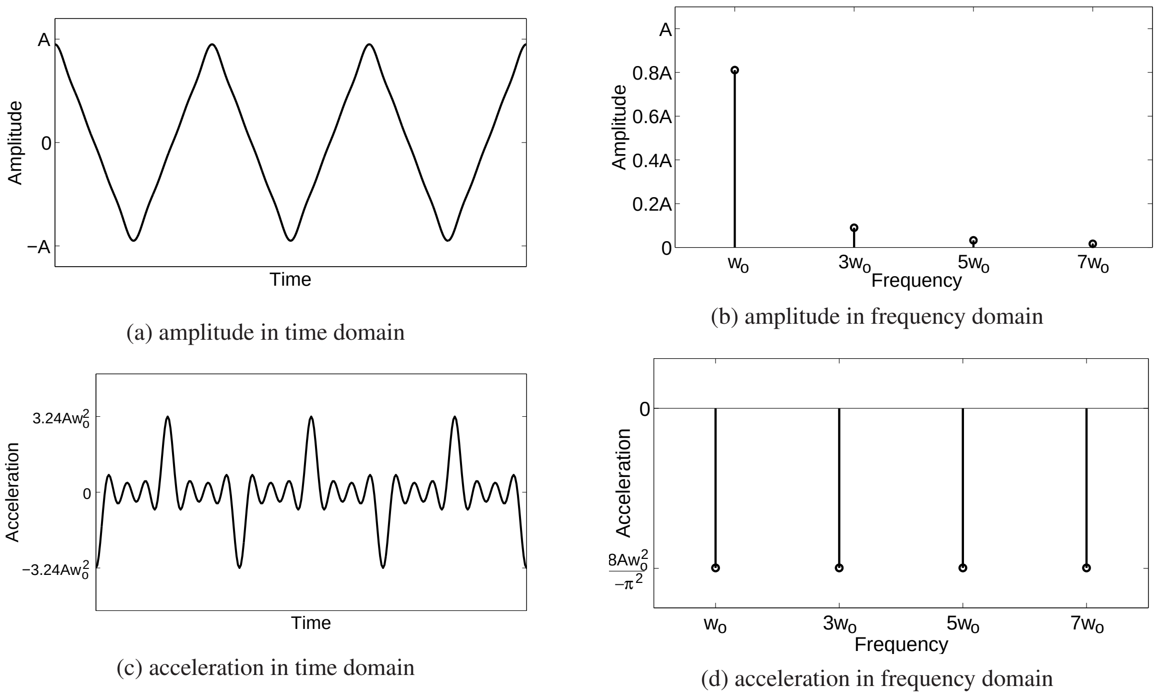
The Fourier series of a triangular wave is composed of the following coefficients:

$$\begin{cases} a_n = \dfrac{8A}{\pi^2 n^2} & \text{, for } n \text{ odd} \\ b_n = 0 & \text{, } \forall\ n \end{cases} \tag{4}$$

where n is the multiple of the fundamental frequency ω_o, and A is the wave amplitude. Hence, the second derivative of the triangular wave is given by:

$$\begin{cases} a_n = -\omega_o^2 \dfrac{8A}{\pi^2} & \text{, for } n \text{ odd} \\ b_n = 0 & \text{, } \forall\ n \end{cases} \tag{5}$$

As shown in Eqs. (4) and (5), the wave can be composed of an infinite number of terms. However, for practical reasons, the number of terms in the signal must be reduced. In this study, the triangular wave will be composed of four terms of the Fourier series (Fig. 4(b)), which gives a reasonably good triangular wave, as shown in Fig. 4(a). Hence, the associated acceleration of the triangular wave (Fig. 4(c)) is also composed of four terms of the Fourier series (Fig. 4(d)).



(a) amplitude in time domain

(b) amplitude in frequency domain

(c) acceleration in time domain

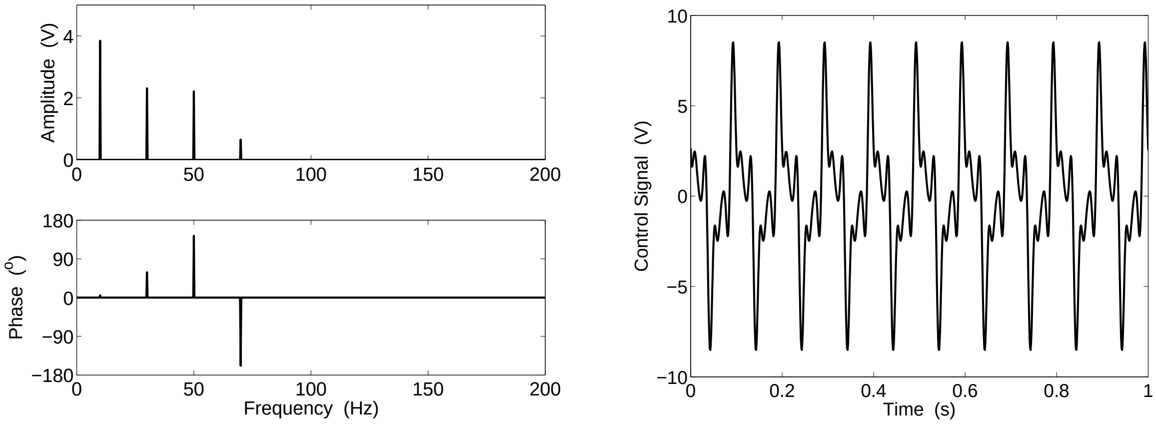
(d) acceleration in frequency domain

Figure 4. Triangular wave composed of four terms of the Fourier series (desired motion).

The desired motion of the beam tip will be given in terms of acceleration because the identified FRF of the system in study is related to acceleration (Fig. 3). Thus, in this case, the desired signal is the acceleration signal presented in Fig. 4. By applying Eq. (2), one has:

$$V_c(\omega) = \frac{A_o(\omega)}{H(\omega)} \tag{6}$$

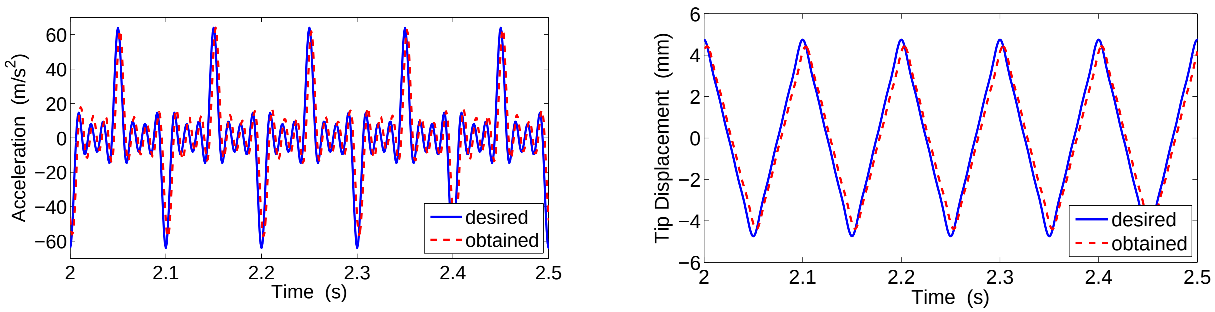
where $V_c(\omega)$ is the open loop control signal, in frequency domain, necessary to obtain the desired periodic motion in the system, $A_o(\omega)$ is the desired acceleration on the beam tip in frequency domain (Fig. 4(d)), and $H(\omega)$ is the FRF of the system, identified experimentally. The open loop control signal, obtained from Eq. (6) for a fundamental frequency of 10 Hz and maximum amplitude of 5 mm on the tip of the beam, is shown in Fig. 5, both in frequency and time domain.



(a) frequency domain (b) time domain

Figure 5. Open loop control signal for imposing the triangular wave in the tip of the beam ($\omega_o = 10$ Hz, $A = 5$ mm).

By sending, in open loop, the obtained control signal (Fig. 5(b)) to the actuator (shaker), one obtained the results shown in Fig. 6. As one can see, there is good agreement between the desired and the experimentally obtained periodic motion in the tip of the beam. After 2 s of excitation, transient vibration has already been attenuated and the resultant motion presents a 2 ms delay to the desired motion (Fig. 6). Discounting the delay, a maximum displacement error of 0.4 mm (8% error) is achieved experimentally between desired and obtained periodic motions, mainly in the peaks of the signal (Fig. 7).



(a) acceleration (b) displacement

Figure 6. Experimental results of the open loop control for a desired triangular wave in the tip of the beam ($\omega_o = 10$ Hz, $A = 5$ mm, 4 terms of the Fourier series).

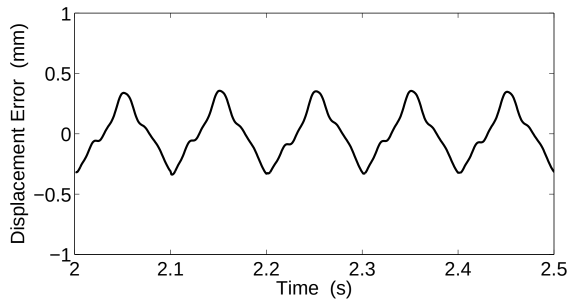
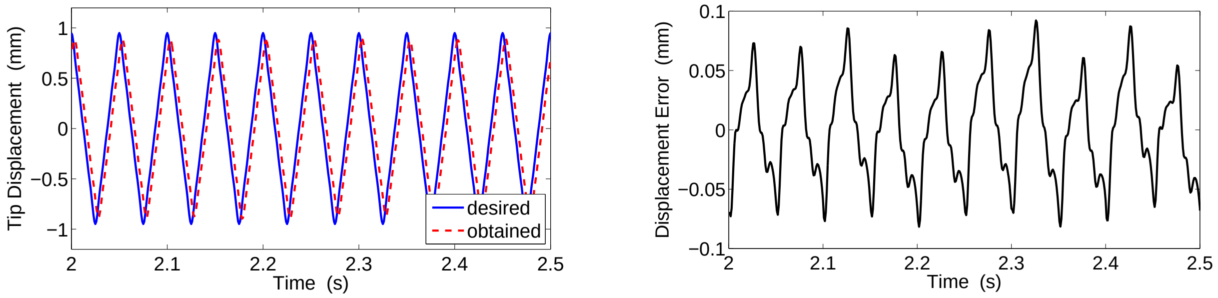


Figure 7. Error between desired and obtained displacements in the tip of the beam discounting signal delay ($\omega_o = 10$ Hz, $A = 5$ mm).

Testing the procedure for a fundamental frequency of 20 Hz and displacement amplitude of 1 mm, one obtained the results shown in Fig. 8. Again, good results are obtained after 2 s of excitation. Signal delay remains in 2 ms and displacement error discounting the delay is smaller than 10%, mainly in the peaks of the signal (Fig. 8(b)).



(a) displacement (b) displacement error without delay

Figure 8. Experimental results of the open loop control for a desired triangular wave in the tip of the beam ($\omega_o = 20$ Hz, $A = 1$ mm, 4 terms of the Fourier series).

In the case of the fundamental frequency of 20 Hz, it was necessary to reduce the tip displacement amplitude from 5 to 1 mm because of the resultant amplitude of the open loop control signal. The analog output port of the acquisition board requires that the control signal remain between -10 and +10 V to avoid saturation. This was only achieved by reducing the maximum desired amplitude in the tip of the beam to 1 mm. In this case, the higher the fundamental frequency of the desired periodic motion is, the higher the control signal will be, and the acquisition system limits the amplitude of the control signal to be used.

4.2 Three Level Trapezoidal Wave Motion via FFT Algorithm

The three level trapezoidal wave is shown in Fig. 9. The calculation of the Fourier coefficients of such wave is more laborious and represents a tedious work. Hence, a more practical, and general, way to obtain the coefficients is the use of the FFT algorithm, which can be applied to any periodic function however complex it is. Applying the FFT algorithm in the three level trapezoidal wave, one obtains the results shown in Fig. 10, where:

$$x(t) = \sum_{n=1}^{\infty} x_n(\omega) \cos\left(n\omega_o t + \phi_n\right) \quad (7)$$

where $x_n(\omega)$ is the n-th coefficient of the series (amplitude), and ϕ_n is the phase of the n-th coefficient. As one can see in Fig. 10, only odd terms are non-zero.

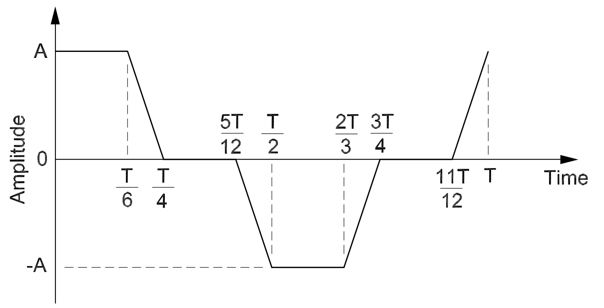


Figure 9. One period of the three level trapezoidal wave (desired motion).

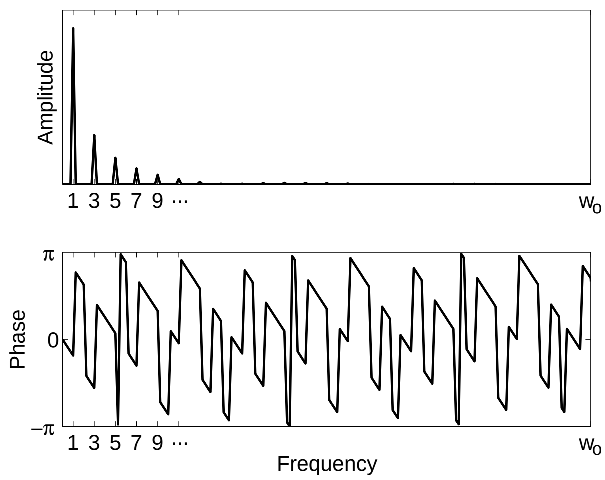
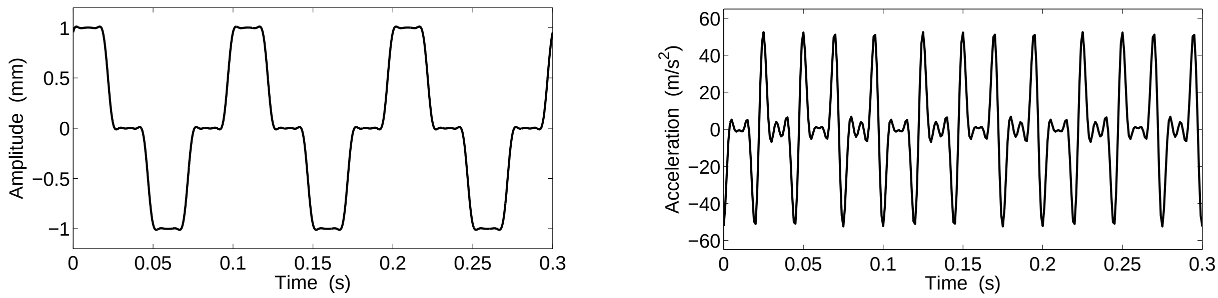


Figure 10. Amplitude and phase of the three level trapezoidal wave obtained via FFT algorithm.

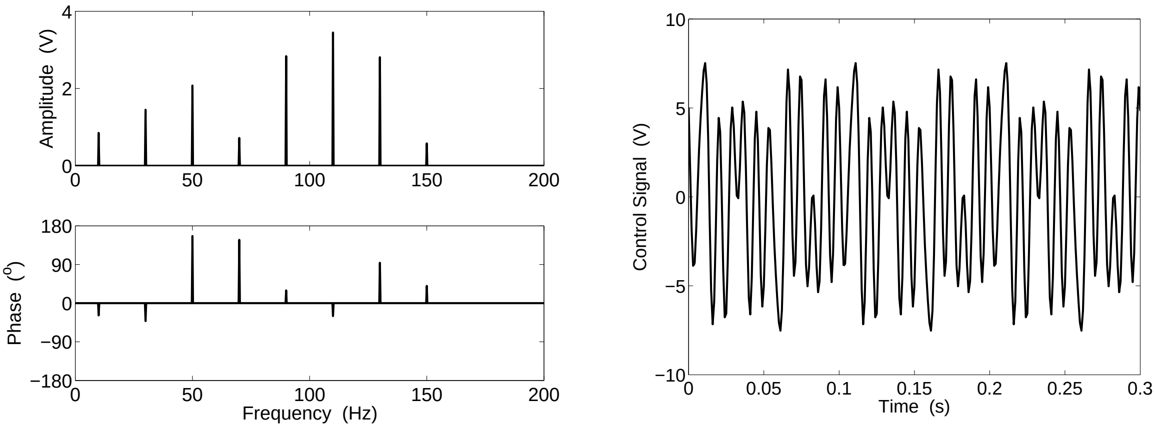
Again, the wave can be composed of an infinite number of terms of the Fourier series, and one will have to truncate the series due to practical reasons. Considering a wave with fundamental frequency of 10 Hz and amplitude of 1 mm, eight terms are used to have an acceptable wave form, as shown in Fig. 11(a). In this case, eight odd terms represents frequency components up to $15\times\omega_o$ (150 Hz), which is still within the frequency range of study (5 to 200 Hz).

The acceleration of the trapezoidal wave (Fig. 11(b)) is transformed to frequency domain by multiplying the Fourier coefficients $x_n(\omega)$ of the wave by $-n^2\omega_o^2$. Thus, applying Eq. (6) one obtains the open loop control signal for imposing the three level trapezoidal wave to the tip of the beam, as shown in Fig. 12.



(a) displacement (b) acceleration

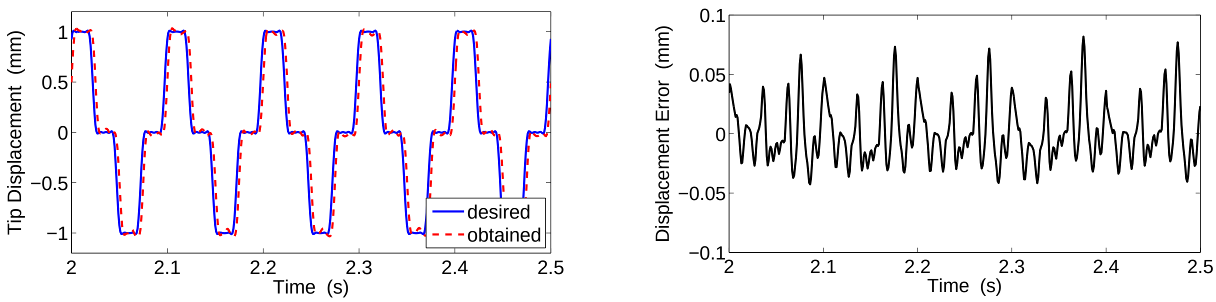
Figure 11. Three level trapezoidal wave composed of eight terms of the Fourier series ($\omega_o = 10$ Hz, $A = 1$ mm).



(a) frequency domain (b) time domain

Figure 12. Open loop control signal for imposing the three level trapezoidal wave in the tip of the beam ($\omega_o = 10$ Hz, $A = 1$ mm).

By sending, in open loop, the control signal (Fig. 12(b)) to the actuator, one obtained the results shown in Fig. 13. As one can see, good agreement is achieved between the desired and the experimentally obtained periodic motion in the tip of the beam. After 2 s of excitation, transient vibration has already been attenuated and, again, the resultant motion presents a 2 ms delay to the desired motion (Fig. 13(a)). Discounting the delay, a maximum displacement error of 0.08 mm (8% error) is obtained in the positive rising of the signal, whereas, in other parts of the signal, the error remained below 5% (Fig. 13(b)).



(a) displacement (b) displacement error without delay

Figure 13. Experimental results of the open loop control for a desired three level trapezoidal wave in the tip of the beam ($\omega_o = 10$ Hz, $A = 1$ mm, 8 terms of the Fourier series).

Testing the procedure for a fundamental frequency of 20 Hz, two important facts occur: the amplitude of the signal must be strongly reduced, down to 0.15 mm, in order to have a control signal within ±10 V, and the number of Fourier series terms must also be reduced because of the frequency range of study between 5 to 200 Hz. In the first case, the amplitude is too low and the actuator may not have enough resolution to control such small displacements. In the second case, considering that the Fourier series terms of the desired wave are odd multiples of the fundamental frequency, only five terms can be used in the range of 5 to 200 Hz (ω_o, $3\times\omega_o$, $5\times\omega_o$, $7\times\omega_o$, and $9\times\omega_o$). For this reason, the desired wave

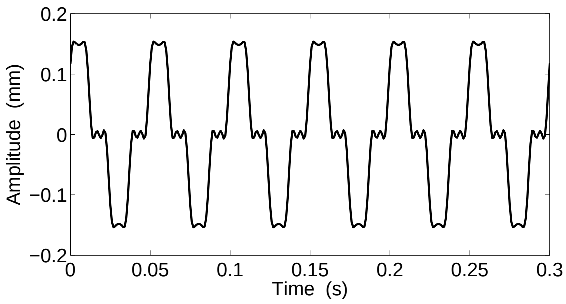
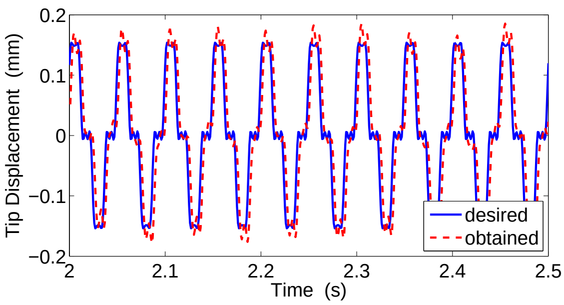


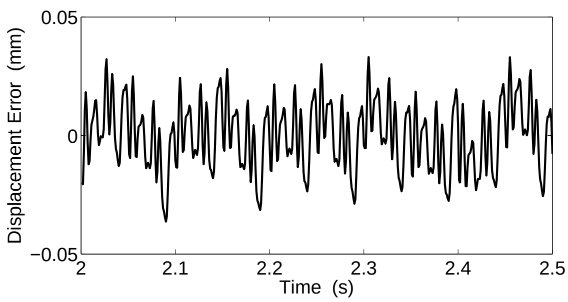
Figure 14. Three level trapezoidal wave composed of five terms of the Fourier series ($\omega_o = 20$ Hz, $A = 0.15$ mm).

is not as smooth as in the previous case, and oscillations are observed in the constant value levels of the signal (Fig. 14).

By applying Eq. (6) to calculate the open loop control signal and sending it to the actuator, one obtained the results shown in Fig. 15. Again, there is a signal delay of approximately 2 ms after 2 s of excitation, when transient dies out (Fig. 15(a)). However, the quality of the obtained periodic motion in the tip of the beam is not as good as before. Discounting the delay, the displacement error reaches 0.035 mm, which represents 23% of the signal amplitude (Fig. 15(b)), a much worse result when compared to the other studied cases. In fact, the measurement of such small displacement motions may be corrupted by noise, and the use of a smaller number of terms of the Fourier series may have had a negative impact on the obtained periodic motion.



(a) displacement



(b) displacement error without delay

Figure 15. Experimental results of the open loop control for a desired three level trapezoidal wave in the tip of the beam ($\omega_o = 20$ Hz, $A = 0.15$ mm, 5 terms of the Fourier series).

5. CONCLUSION

Periodic motion on flexible structures can be sustained by open loop control. In this work, the open loop control signal is obtained using information from the frequency response analysis of the system. Hence, no mathematical model is necessary to obtain the appropriated control signal, but the system FRF must be identified experimentally. Because FRFs represent input/output relationships of the system, the actuators and sensors can be analyzed all-together. In this case, the system + actuators + sensors are considered a whole integrated and sole system, whose input is the control signal and output is displacement/velocity/acceleration signals. As a consequence, the calculation of the necessary control signal is straightforward.

Theoretically, any periodic motion can be imposed to the flexible structure by the procedure presented in this work. However, experimental results show that small displacement motions may be difficult to control due to noise and actuator resolution. This limitation comes from the adopted hardware, which may vary from application to application. In fact, the restriction of ± 10 V in the analog output ports of the adopted acquisition board led to decreasingly small amplitudes as fundamental frequencies and number of Fourier series terms increased. An alternative to this drawback is to increase the gain of the actuator, which would result in larger forces to the same control signal. Inevitably in this case, a new FRF identification would be required because the input/output relationship would be changed.

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7. Responsibility notice